

A Methodology to Extract Intuitionistic Fuzzy Evidence from Fuzzy Information Based on the Non-Consonant Random Set

R.Irene Hepzibah¹, A.Nagoorgani² and G.Geethalakshmi³

¹Department of Mathematics, AVC College of Engineering, Mayiladuthurai
Tamil Nadu, India. e-mail: ireneraj74@gmail.com
Corresponding Author

²PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous)
Trichy-20, TamilNadu, India. e-mail: ganijmc@yahoo.co.in

³Department of Mathematics, K.S.K College of Engineering and Technology
Kumbakonam, Tamil Nadu, India. e-mail: geethavish294@gmail.com

Received 4 November 2014; accepted 4 December 2014

Abstract. Dempster Shafer Theory (DST) is a branch of Mathematics that concerns combination of empirical evidence in an individual mind in order to construct a coherent picture of reality and offers an alternative to traditional probabilistic theory for the mathematical representation of uncertainty. Natural language information could naturally be expressed by a membership function. Intuitionistic fuzzy sets, with independent memberships and non-memberships are generalization of fuzzy sets. Out of several higher order fuzzy sets, the Intuitionistic Fuzzy Sets (IFS) have been found to be highly useful to deal with vagueness. In this paper, we make an investigation to extract intuitionistic fuzzy evidence with non consonant focal elements based on random set theory and evidence theory. Some examples are provided here to show the robustness of the proposed method.

Keywords: Random set theory, Dempster-Shafer theory, intuitionistic fuzzy sets, fuzzy focal elements.

AMS Mathematics Subject Classification (2010): 03F55

1. Introduction

Probability theory is proposed only for randomness uncertainty. To overcome the constraint of probabilistic method, Dempster put forward a theory in 1976 and now it is known as Evidence Theory (or) Dempster- Shafer Theory (DST). The D-S theory of evidence one of the most popular uncertainty theories used in many areas, such as expert systems, pattern classification, information fusion [5], which was first developed by Dempster [3] and later extended and refined by Shafer [9]. In D-S theory, the information given by sensors or experts can be described by the focal elements on a frame of discernment and the corresponding Basic Probability Assignments (BPA) [1]. The determination of BPA is an important problem in the multi source information fusion.

Many practical source applications of multisource information fusion problems usually involve a kind of important information with intuitionistic fuzzy continuity and relativity, which usually comes from the natural language of experts or observers, like tallness or smallness, pleasure or pain, cold or hot etc. This kind of information could naturally been expressed by a non-membership function. So we need to extract intuitionistic fuzzy evidence from fuzzy information. In this paper, a method is proposed to extract intuitionistic fuzzy evidence with non consonants focal elements based on random set theory. Some examples are given to show the generality and the efficiency of this method.

2. Preliminaries

2.1. Dempster-Shafer theory (DST) [7]

Dempster-Shafer Theory (DST) is a mathematical theory of evidence. In a finite discrete space, Dempster-Shafer theory can be interpreted as a generalization of probability theory where probabilities assigned to sets as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In Dempster-Shafer Theory, evidence can be associated with multiple possible events. A frame of discernment (or simply a frame) usually denoted as Θ is a set of mutually exclusive and exhaustive propositional hypotheses one and only one of which is true [8]. Evidence theory is based on two dual non additive measures, namely Belief measure and Plausibility measure. There is one important function in Dempster-Shafer theory to define Belief measure and plausible measure which is known as Basic Probability Assignments. A function $m: 2^\Theta \rightarrow [0,1]$ is called Basic Probability Assignments on the set Θ if it satisfies the following conditions (i) $m(\phi)=0$ (ii) $\sum_{A \subseteq \Theta} m(A)=1$ where ϕ is an empty set and A is

any subset of Θ . The Basic Probability Assignment function (or mass function) is a primitive function. Given a frame, Θ , for each source of evidence, a mass function assigns a mass to every subset of Θ , which represents the degree of belief that one of the hypotheses in the subset is true, given the source of evidence. A subset A of a frame Θ is called the focal elements of m , if $m(A) > 0$. The lower bound, Belief for a set A is defined as the sum of all the basic probability assignments of the proper subsets (B) of the set of interest (A) $\{(ie) B \subseteq A\}$. The upper bound, Plausibility is the sum of all the basic probability assignments of set (B) that intersect the set of interest (A) $\{(ie) (B \cap A \neq \phi)\}$. Formally for all sets A that are elements of the power set ($A \in P(X)$), [Klir 1998],

$$Bel(A) = \sum_{B/B \subseteq A} m(B) \text{ and } Pl(A) = \sum_{B/B \cap A \neq \phi} m(B).$$

The two measures, Belief and Plausibility are non additive. This can be interpreted as not required for the sum of all the Belief measures to be one and similarly for the sum of all the Plausibility measures. Hence the, interval $[Bel(A), Pl(A)]$ is the range of belief A.

2.2. The Dempster rule of combination [4]

The Dempster rule of combination is critical to the original conception of the Dempster-Shafer theory. The measure of Belief and Plausibility are derived from the combined

basic assignments. Dempster's rule combines multiple belief functions through their basic probability assignments (m). These belief functions are defined on the same frame of discernment, but are based on independent assignments or bodies of evidence. The Dempster rule of combination is purely a conjunctive operation (AND). The combination rule results in a belief function based on conjunctive pooled evidence [Shafer 1986, pg 132]. The combination (is called the joint m_{12}) can be calculated from the aggregation of two Basic Probability assignments m_1 and m_2 in the following manner:

$$m(C) = \frac{\sum_{A \cap B = C} m_1(A)m_2(B)}{1 - \sum_{A \cap B = \phi} m_1(A)m_2(B)} \quad \text{where } C \neq \phi, m_{12}(\phi) = 0$$

2.3. Random set theory (RST) [6,9]

2.3.1. Random set

Let (Ω, A, P) be a probability space, Θ be a frame of discernment and its power set denoted as 2^Θ , then, a random set X is defined by a set valued mapping $X : 2^\Theta \rightarrow [0,1]$

A density $f : 2^\Theta \rightarrow [0,1]$, (ie) $f(A) = P\{\omega \in \Omega : X(\omega) = A\}, \forall A \subseteq \Theta$ (1)

Such that $f(\phi) = 0$ and $f(A) \geq 0$; and $\sum_{A \subseteq \Theta} m(A) = 1$ f determines a probability measure P

the corresponding distribution function F of the random set X is

$$F(A) = P\{\omega \in \Omega : X(\omega) \subseteq A\} = \sum_{B \subseteq A} f(B), \forall A \subseteq \Theta \quad (2)$$

A random set can also be described by a set of pairs

$$X = \left\{ (A, m(A)) / \forall A \subseteq \Theta, m(A) = P\{\omega \in \Omega : X(\omega) = A\}, \sum_{A \subseteq \Theta} m(A) = 1 \right\} \quad (3)$$

2.3.2. Trapping function

A function T defined as follows is called a trapping function

$$T(A) = P\{\omega \in \Omega : X(\omega) \cap A \neq \phi\} \forall A \subseteq \Theta \quad (4)$$

2.3.3. One-point covering function

A function γ_X defined as follows is called a one-point covering function of X for Non-membership function $\gamma_X(\theta) = P\{\omega \in \Omega : \theta \in X(\omega)\}, \forall A \subseteq \Theta$ (5)

It represents the relationship between any point in Θ and random set.

2.4. Intuitionistic fuzzy sets (IFS) [1]

An intuitionistic fuzzy sets (IFS) A is E is defined as an object of the following form

$$A = \{(x, \mu_A(x), \gamma_A(x)) / x \in E\} \quad (6)$$

where the functions $\mu_x : E \rightarrow [0,1]$ (7)

$$\gamma_x : E \rightarrow [0,1] \quad (8)$$

Define the degree of membership and the degree of Non-membership of the element $x \in E$, respectively and for every $x \in E$:

$$0 \leq \mu_A(X) + \gamma_A(X) \leq 1 \quad (9)$$

Obviously, each ordinary fuzzy set may be written as

$$\{(x, \mu_A(x), 1 - \mu_A(x)) / x \in E\} \quad (10)$$

2.4.1. Uncertainty

$$\text{The value of } \pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \quad (11)$$

is called the degree of non determinacy (or uncertainty) of the element $x \in E$ to the intuitionistic fuzzy set A .

2.4.2. Non-membership function

Let $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$ be a frame of discernment, an intuitionistic fuzzy set,

denoted $\tilde{A} \subseteq \Theta$ is defined by a Non-membership function $\gamma_{\tilde{A}} : \Theta \rightarrow [0,1]$

$$\gamma_{\tilde{A}}(\theta_i) \in [0,1], \forall \theta_i \in \Theta \quad (12)$$

A relationship between random set and intuitionistic fuzzy set has been proposed in [8], intuitionistic fuzzy set can be represented by random set through the one point covering function in equation (8) conversely, a random set can be induced by intuitionistic fuzzy set \tilde{A} [7]

$$X_{\tilde{A}}(\omega) = \{\theta_i / \gamma_{\tilde{A}}(\theta_i) \geq \epsilon(\omega)\} \quad (13)$$

Thus, $X_{\tilde{A}}$ is a non consonant random set. It is easy to validate that (3)

$$\gamma_{X_{\tilde{A}}}(\theta_i) = P\{\omega \in \Omega : \theta_i \in X_{\tilde{A}}(\omega)\} = \gamma_{\tilde{A}}(\theta_i) \quad (14)$$

In the following discussion, we give an example of how to represent fuzzy information using the intuitionistic fuzzy set theory. Let $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$ be a set of objects, and each of them can be described entirely by a set of attribute parameters, as

$$\phi = [x_1, x_2, \dots, x_k]^T \quad (15)$$

It includes the objects length, temperature, surface area, etc. Let us consider that the information about the parameter x_j is vague and let R_j be the range of this parameter ($x_j \in R_j$), R_j can be discrete or continuous, ordered or not. Let $\tilde{B} \subseteq R_j$ be the intuitionistic fuzzy set with a non - membership function $\gamma_{\tilde{B}}(x_i)$. The intuitionistic fuzzy set \tilde{B} must be transformed into an intuitionistic fuzzy set of $\tilde{A} \subseteq \Theta$ relatively to the parameter x_j (2).

$$\gamma_{\tilde{A}}(\theta_i) = \gamma_{\tilde{B}}(x_j^i) \quad (16)$$

where x_j^i is the j-th attribute value of θ_i . For example, the parameter x_j , could correspond to the length of the objects in Θ and the intuitionistic fuzzy set $\tilde{B} \subseteq R_j$ could correspond to “small length” (the unit of length in meter). In this case R_j is continuous and ordered. Each value from the interval from $[0,300]$ posses a Non-membership degree of the intuitionistic fuzzy subset “small length” (see fig.1). By using equation (10), we construct a new intuitionistic fuzzy set $\tilde{A} \subseteq \Theta$ where $\gamma_{\tilde{A}}(\theta_i)$ represents the degree of non – membership of each object in Θ to the intuitionistic fuzzy set” object

A Methodology to Extract Intuitionistic Fuzzy Evidence from Fuzzy Information ...
with small length” (see Figure.2).

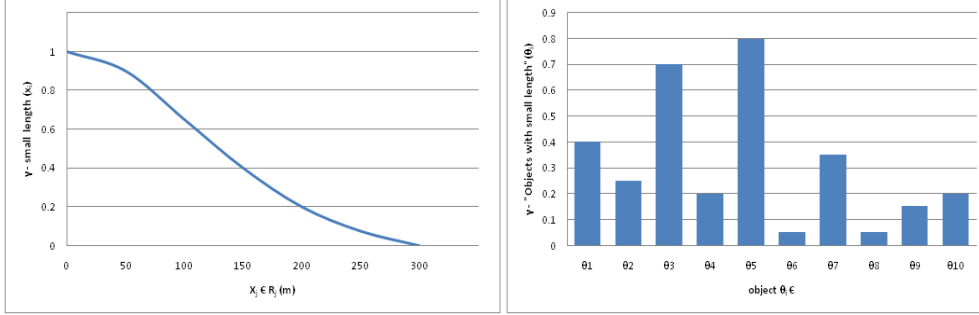


Figure 1: Non-membership of the intuitionistic fuzzy set "small length"

In fuzzy information fusion, focal elements are commonly according to the experience knowledge of experts or common sense. Consonant focal elements are only flexible and unpractical. Here we consider more general nonconsonant forms of focal elements and determines their BPA using by non-membership function is a meaningful and challenging problem. In this section, by merit of one-point covering function of non-membership function ,a non-consonant random set is constructed to represent a set of general focal elements given in advance, and then a linear group can be created to solve the BPA. Firstly, the definition of one-point covering function for non-membership function indicates. If $\tilde{A} \subseteq \Theta$ is the only focal elements to which Θ_i belongs, then

$$\gamma_X(\theta_i) = P\{\omega \in \Omega : X(\omega) = A_j\}, \forall \theta_i \in \Theta \quad (17)$$

If $A_1, A_2, A_3, \dots, A_k$ is a subset of parameter space are k focal elements and

$$\theta_i \in A_j (j = 1, 2, 3, \dots, k) \quad \gamma_X(\theta_i) = \sum_{j=1}^k P\{\omega \in \Omega : X(\omega) = A_j\}, \forall \theta_i \in \Theta \quad (18)$$

In particular, if $P\{\omega \in \Omega : X(\omega) = A_j\} = \Omega$, then $\gamma_X(\theta_i) = 1$ Secondly, let (Ω, A, P) be a probability space, and let $U = \{u_1, u_2, u_3, \dots, u_n\}$ be a finite space, here $u_i = \gamma_{\tilde{A}}(\theta_i) (i = 1, 2, 3, \dots, n)$, we construct a random set $X : \Omega \rightarrow 2^U$ and let $\gamma_X(u_i) = \gamma_{\tilde{A}}(\theta_i)$, according to equation (21), we have

$$\gamma_{\tilde{A}}(\theta_i) = \sum_{j=1}^L P\{\omega \in \Omega : X(\omega) = A_j\}, \forall \theta_i \in \Theta \quad (19)$$

where $E_1, E_2, E_3, \dots, E_k \subseteq U$ are $u_i \in E_j (j = 1, 2, 3, \dots, k)$, then, a non-consonant

random set $m : \Omega \rightarrow 2^\Theta$ can be induced by the intuitionistic fuzzy set \tilde{A} and random set X
 $X_{\tilde{A}}(\omega) = \{\theta_i / \gamma_{\tilde{A}}(\theta_i) \in X(\omega)\} \quad (20)$

$$\text{It is obvious that } m(A_j) = P\{\omega \in \Omega : X(\omega) = A_j\} = P\{\omega \in \Omega : X(\omega) = E_j\} \quad (21)$$

where $j = 1, 2, 2^2, 2^3, \dots, 2^n$, $A_j = \{\theta_i / \gamma_{\tilde{A}}(\theta_i) \in E_j\} \subseteq 2^\Theta$ and $E_j \subseteq U$. Finally, equation (19) can be written as equation (22) by the substitution of equation (21)

$$\sum_{j=1}^k m(A_j) = \gamma_{\tilde{A}}(\theta_i), \forall \theta_i \in \Theta \quad (22)$$

R.Irene Hepzibah, A.Nagoorgani and G.Geethalakshmi

Let $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$, $U = \{u_1, u_2, u_3, \dots, u_n\}$, here $u_i \in \gamma_{\bar{A}}(\theta_i)$, ($i = 1, 2, 3, \dots, n$), suppose there are 'm' different possible focal elements $E_1, E_2, E_3, \dots, E_m$ on, U and the corresponding focal elements $A_1, A_2, A_3, \dots, A_m \subseteq \Theta$, then a linear equation group to solve the BPA for the focal elements can be defined as equation (23) according to equation (22), $AX = B$

$$\text{where } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3m} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nm} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{pmatrix}, B = \begin{pmatrix} \gamma_{\bar{A}}(\theta_1) \\ \gamma_{\bar{A}}(\theta_2) \\ \gamma_{\bar{A}}(\theta_3) \\ \dots \\ \gamma_{\bar{A}}(\theta_n) \end{pmatrix} \text{ } a_{ij} \in \{0,1\} \text{ is a non-membership}$$

$$\text{indicator } a_{ij} = \begin{cases} 1, & \text{if } \theta_i \in A_j \\ 0, & \text{if } \theta_i \notin A_j \end{cases}$$

Then, let us discuss the solution of equation (23)

- (i) Rank(A) = Rank (A,B): linear equation (23) has no solution.
- (ii) Rank(A) = Rank (A,B)=m: linear equation (23) has a unique solution. So, x_1, x_2, \dots, x_m can be decided uniquely.

- If $x_1, x_2, \dots, x_m \geq 0$ and $x_1 + x_2 + \dots + x_m = 1$ are satisfied, then, $m(A_j) = x_j$ ($j = 1, 2, 3, \dots, m$)
- If $x_1, x_2, \dots, x_m \geq 0$ and $x_1 + x_2 + \dots + x_m \neq 1$ then a normalization process is

$$\text{needed in order to satisfy } \sum_{A \subseteq 2^{\Theta}} m(A) = 1 \text{ (ie) } m(A_j) = \left\{ x_j / \sum_{j=1}^m x_j \right\}, \text{ (j = 1, 2, 3, \dots, m)} \quad (24)$$

If $x_1, x_2, \dots, x_m \geq 0$ is not fulfilled, then there is no reasonable BPA for the corresponding focal elements.

- (iii) Rank(A) = Rank (A,B) < m : linear equation (23) have infinitely many solutions, suppose there are solutions such that $x_1, x_2, \dots, x_m \geq 0$.

We established an objective function to complete the minimum total uncertainty of one piece of evidence under some constraints, then an optimal solution can be obtained by solving an optimization problem (3) proposed a measure of total uncertainty for evidential reasoning. The proposed measure is a functional, TU_{PBH}

$$TU_{PBH} = - \sum_{A \subseteq 2^{\Theta}} m(A) \log_2 (m(A) / |A|) \quad (25)$$

Then, the objective function can be created as follows,

$$\text{Min} J(x_1, x_2, \dots, x_m) = - \sum_{j=1}^m \left\{ x_j / \sum_{i=1}^m x_i \right\} \log_2 \left[\left(x_j / \sum_{i=1}^m x_i \right) / |A_j| \right] \quad (26)$$

Its constraints are equation (23) and $0 \leq x_1, x_2, \dots, x_m \leq 1$ the optimal solution denoted as

$$(x_1^*, x_2^*, \dots, x_m^*) \text{ Can be obtained by solving the above optimization problem, then by normalization, we have } m(A_j) = \left\{ x_j^* / \sum_{i=1}^m x_i^* \right\}, \text{ (j = 1, 2, 3, \dots, m)} \quad (27)$$

A Methodology to Extract Intuitionistic Fuzzy Evidence from Fuzzy Information ...

Let the frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_m\}$, and its each elements non-memberships shown in fig 2. The following three examples illustrate how one applies equation (23) for computing the BPA.

3. Numerical illustrations

Example 3.1. Suppose there are 7 possible focal elements E_j ($j = 1, 2, 3, \dots, 7$) given by Expert A, as $E_1 = \{0.05, 0.8, 0.35\}$, $E_2 = \{0.8, 0.7, 0.25\}$, $E_3 = \{0.8, 0.2, 0.7\}$,

$E_4 = \{0.8, 0.15, 0.7, 0.35\}$, $E_5 = \{0.4, 0.1\}$, $E_6 = \{0.8, 0.4, 0.25\}$ $E_7 = \{0.4, 0.7, 0.25, 0.35\}$
According to equation (20), we have

$$A_1 = \{\theta_5, \theta_6, \theta_7, \theta_8\}, A_2 = \{\theta_2, \theta_3, \theta_5\}, A_3 = \{\theta_3, \theta_5\}, A_4 = \{\theta_3, \theta_5, \theta_7, \theta_9\},$$

$$A_5 = \{\theta_1, \theta_4, \theta_{10}\}, A_6 = \{\theta_1, \theta_2, \theta_5\}, A_7 = \{\theta_1, \theta_2, \theta_3, \theta_7\}$$
 Then according to equation

(23), corresponding linear equations can be established as $AX = B$ (28)

where,

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}, B = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix};$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix}$$

In this case, $\text{Rank}(A) = \text{Rank}(A,B) = 7 = m$ linear equation (23) has an unique solution, the unique solution is $X = [0.05 \ 0.05 \ 0.35 \ 0.15 \ 0.2 \ 0.05 \ 0.15]^T$ Then, we have $m(A_1) = 0.05$, $m(A_2) = 0.05$, $m(A_3) = 0.35$, $m(A_4) = 0.15$, $m(A_5) = 0.2$, $m(A_6) = 0.05$, $m(A_7) = 0.15$.

Example 3.2. Suppose there are 7 possible focal elements E_j ($j = 1, 2, 3, \dots, 7$) given by expert B, as

$$E_1 = \{0.05, 0.7\}, E_2 = \{0.8, 0.35, 0.2\}, E_3 = \{0.8, 0.4\}, E_4 = \{0.8, 0.4, 0.7, 0.25\}$$

$$E_5 = \{0.8, 0.2\}, E_6 = \{0.15, 0.7, 0.25\}, E_7 = \{0.7, 0.35\}$$

According to equation (15), we have

$$A_1 = \{\theta_3, \theta_6, \theta_8\}, A_2 = \{\theta_5, \theta_7, \theta_{10}\}, A_3 = \{\theta_1, \theta_5\}, A_4 = \{\theta_1, \theta_2, \theta_3, \theta_5\}, A_5 = \{\theta_4, \theta_5\},$$

$$A_6 = \{\theta_2, \theta_3, \theta_9\}, A_7 = \{\theta_3, \theta_7\}$$

Then according to equation (23), corresponding linear equations can be established as

$$AX = B \tag{29}$$

where,

$$A = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}, B = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix}$$

In this case, Rank(A) = Rank(A,B)=7= m linear equation (23) has an unique solution, the unique solution is $X = [0.05 \ 0.2 \ 0.3 \ 0.1 \ 0.2 \ 0.15 \ 0.15]^T$

Then, we have $m(A_1) = 0.043, m(A_2) = 0.174, m(A_3) = 0.261, m(A_4) = 0.087, m(A_5) = 0.174, m(A_6) = 0.130, m(A_7) = 0.130$.

Example 3.3. Suppose there are 8 possible focal elements E_j ($j = 1,2,3,\dots, 8$) given by expert C, as

$$E_1 = \{0.05,0.4,0.35\}, E_2 = \{0.8,0.2,0.7\}, E_3 = \{0.2,0.15\}, E_4 = \{0.8,0.15,0.7\}$$

$$E_5 = \{0.8,0.7\}, E_6 = \{0.25,0.35\}, E_7 = \{0.25,0.2\}, E_8 = \{0.8,0.4,0.25,0.35\}$$

According to equation (23), we have $A_1 = \{\theta_1, \theta_6, \theta_7, \theta_8\}, A_2 = \{\theta_3, \theta_4, \theta_5\},$
 $A_3 = \{\theta_4, \theta_9\}, A_4 = \{\theta_3, \theta_5, \theta_9\}, A_5 = \{\theta_3, \theta_5\}, A_6 = \{\theta_2, \theta_3, \theta_5, \theta_9\}, A_7 = \{\theta_1, \theta_{10}\},$
 $A_8 = \{\theta_1, \theta_2, \theta_5, \theta_7\}.$

Then according to equation (23), corresponding linear equations can be established as $AX = B$ (30)

where,

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix}, B = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.7 \\ 0.2 \\ 0.8 \\ 0.05 \\ 0.35 \\ 0.05 \\ 0.15 \\ 0.2 \end{pmatrix}$$

Here, Rank(A) = Rank(A,B)=7< m linear equation (23) has infinitely many solution, then the question of how to determine the solution.(ie) the solution which will give the least degrees of total uncertainty. Then according to equation (21) and its constraints, the optimization problem can be described as follows

$$MinJ(x_1, x_2, \dots, x_8) = -\sum_{j=1}^8 \left\{ x_j / \sum_{i=1}^8 x_i \right\} \log_2 \left[\left(x_j / \sum_{i=1}^8 x_i \right) / |A_j| \right]$$

(31)

such that $AX = B$ and $0 \leq x_1, x_2, \dots, x_8 \leq 1$.

The optimal solution is

A Methodology to Extract Intuitionistic Fuzzy Evidence from Fuzzy Information ...

$$(x_1^*, x_2^*, \dots, x_8^*) = [0.05 \quad 0.2 \quad 0 \quad 0.15 \quad 0.25 \quad 0.1 \quad 0.2 \quad 0.15]$$

Then according to equation (26) we have,

$$m(A_1) = 0.045, m(A_2) = 0.182, m(A_3) = 0, m(A_4) = 0.136, m(A_5) = 0.227, \\ m(A_6) = 0.091, m(A_7) = 0.182, m(A_8) = 0.136$$

It is very easy to see that the final focal elements are A_j , $j = 1, 2, 3, \dots, 8$. The Dempster's combination rule and other combination rules in D-S theory can be used for combination after the intuitionistic fuzzy evidence is extracted.

4. Conclusion

In this paper, a method is proposed to extract intuitionistic fuzzy evidence with non consonant focal elements based on random set theory. The examples summarized in this work show the generality and efficiency of the proposed method.

REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
2. J.F.Boldwin, The management of Fuzzy and probabilistic uncertainties for Knowledge based systems, in: Encyclopedia of AI (ed. S.A.Shapiro), John Wiley Second Edition, 1992, 528-537.
3. A.P.Dempster, Upper and lower probabilities induced by a multi-valued mapping, *Annals of Mathematical Statistics*, 38(1967) 325- 339.
4. M.Florea, A.L.Jouselme and D.Grenier, Combining belief functions and fuzzy membership functions, *Proceedings of SPIE*, 5099 (2003)113-122.
5. He You Wang Guohong and Guanxin, Information Fusion Theory with Applications, Belling: Publishing house of Electronics Industry, January 2010 (in Chinese).
6. I.R.Goodman, R.P.S.Mahler and H.T.Nguyen, Mathematics of Data Fusion, Kluwer Academic Publishers, 1997, 45-48.
7. R.Irene Hepzibah, A.Nagoorgani and G.Geethalakshmi, Modified Arithmetic operations of focal elements and their corresponding basic probability assignments in evidence theory, *Elixir Appl. Math.*, 74 (2014) 26798-26984.
8. P.Rajarajeswari and A.Sahaya Sudha, A new approach for ranking of fuzzy numbers using the incentre of centroids, *International Journal of Fuzzy Mathematical Archive*, 4(1) (2014) 52-60.
9. G.Shafer, A Mathematical Theory of Evidence, N.J, Princeton University press, 1976, 57-73.
10. Xu Xiaobin, Wen Chenglin and Li Zhiliang, A new method for constructing fuzzy Evidence based on the non consonant Random set, *Institute of information and control*, Hangzhoudianzi University, Hangzhou 3 10018, China.
11. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.