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Gracefulness of Some Super Graphs of KC_4 -Snake

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Abstract. In this paper we introduce a new definition called the complete *m*-points projection on some projected vertices of a graph and then we prove that the complete *m*-points projection on some projected vertices of kC_4 -snake is graceful.

Keywords: Graphs, Complete *m*-points projection, kC_n -snake

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

A function *f* is called a *graceful labeling* of a graph *G* with *m* edges if *f* is an injection from the vertex set of *G* to the set $\{0, 1, 2, ..., m\}$ such that, when each edge *xy*is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct.

Rosa [6] introduced such labeling in 1967 and named it as a β-valuation of graph while Golomb [5] independently introduced such labeling and called it as graceful labeling. Acharya [1] has constructed certain infinite families of graceful graphs from a given graceful graph while Rosa [7] and Golomb [5] have discussed gracefulness of complete bipartite graphs and Eulerian graphs. Sekar [8] has proved that the splitting graph (the graph obtained by duplicating the vertices of a given graph altogether) of C_n admits graceful labeling for $n \equiv 1, 2 \pmod 4$ .A kC_n -snakeis a connected graph with *k* blocks, each of the block is isomorphic to the cycle C_n, such that the block-cut-vertex graph is a path. Following Chartrand, Lesniak [4], by a block-cut-vertex graph of a graph *G* we mean the graph whose vertices are the blocks and cut-vertices of *G* where two vertices are adjacent if and only if one vertex is a block and the other is a cut-vertex belonging to the block. We also call a kC_n -snake as a cyclic snake. This graph was first introduced by Barrientos [3] and he proves that kC_4 -snakes are graceful and later it was discussed by Badr [2] as generalization of the concept of triangular snake introduced by Rosa [6]. A kC_n -snake contains $M = nk$ edges and $N = (n-1)k+1$ vertices. Among these vertices, $k-1$ vertices have degree 4 and the other vertices of degree 2. Let $u_1, u_2, ... u_{k-1}$ be the consecutive cut-vertices of *G*. Let d_i be the distance between $u_i$and

u_{i+1}in G for$1 \le i \le k-2$ the string $(d_1, d_2, \ldots d_{k-2})$ of integers characterizes the graph G in the class of n-cyclic snakes. For example we can construct two different $3C_4$-snake from a $2C_4$-snake, the first is with string- 1 (Figure 1.2) and the second is with string- 2 (Figure 1.3).

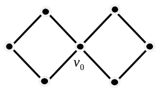

Figure 1.1:

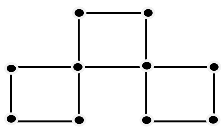

Figure 1.2:

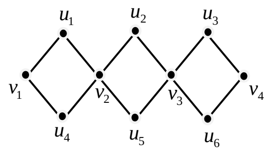

Figure 1.3:

Figure 1.1. $2C_4$-snake with a cut vertex
Figure 1.2. $3C_4$-snake with string- 1
Figure 1.3. $3C_4$-snake with string- 2
In this paper, we consider kC_4-snake with string- 2.

2. Gracefulness of some super graphs of KC_4 -Snake

Definition 2.1. The *complete m*-points *projection* $(m \ge 1)$ on some projected vertices (say l) of a graph *H(p,q)* is the Super graph *G(N,M)* of *H(p,q)* by adding *m* isolated vertices (N_m) to the vertices set of *H(p,q)* and adding complete bipartite edges (ml *edges*) between the sets *A*&*B*, where *A* is the set of newly added *m* isolated vertices (N_m) and *B* is the set of the l -projected vertices of the graph *H(p,q)*. So the number of vertices of the super graph *G* is $N = p + m$ and the number of edges of the super graph *G* is $M = q + ml$.

Definition 2.2. The *adjoint vertices* of kC_4-snake is the set of union of cut vertices and two non-adjacent vertices of cut vertices of kC_4-snake. The *disjoint vertices* of kC_4-snake is the set of union of adjacent vertices of cut vertices of kC_4-snake. So there are $k+1$adjoint vertices and $2k$ disjoint vertices. In Figure 1.3, v_1, v_2, v_3 & v_4 are the adjoint vertices and u_1, u_2, u_3, u_4, u_5 & u_6 are the disjoint vertices of $3C_4$-snake.

Example 2.1. Let *H(p,q)*be the graph K_5 (see Figure 2.1) and let any two vertices of *H(p,q)* be the projected vertices (say v_1 & v_3). Suppose the chosen null graph is N_3(see Figure 2.2), that is $m = 3$. Then the complete 3-points projection on the projected vertices v_1 & v_3 of *H(p,q)* from N_3 will be visible as in Figure 2.3.

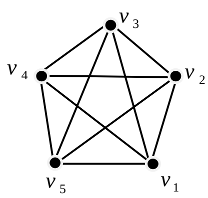

Figure 2.1:

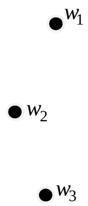

Figure 2.2:

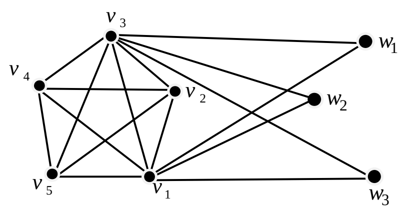

Figure 2.3:

Theorem 2.1. The complete *m*-points projection $(m \geq 1)$ on the adjoint vertices of kC_4-snake is graceful.

Proof: Let *H* be the kC_4-snake with *k* blocks, so the number of vertices of *H* is $3k+1$ and the number of edges of *H*is $4k$. Let *G* be the Super graph of *H* such that *G* is the complete *m*-points projection on the adjoint vertices of kC_4-snake. Let $N = m+3k+1$ be the number of vertices of *G* and $M = (m+4)k+m$ be the number of edges of *G*. [Refer Figure 2.4]. To prove *G* is graceful it is enough to prove that the *M* edges of *G* having the edge values as $\{M, M-1, M-2, ..., 3, 2, 1\}$. Name the $k+1$ adjoint vertices by $\{v_1, v_2, \ldots, v_{k+1}\}$, $2k$ disjoint vertices by $\{u_1, u_2, \ldots, u_k, u_{k+1}, \ldots, u_{2k}\}$ and the *m* isolated vertices by $\{w_1, w_2, \ldots, w_m\}$ as described in Figure 2.4.

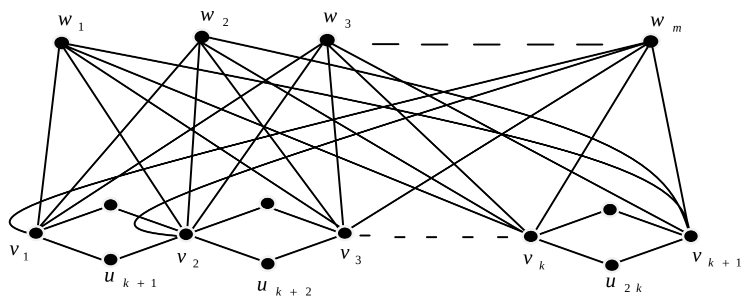

Figure 2.4:

Define: $f(v_i) = mk+m+4k+1-i, \quad 1 \leq i \leq k+1$

$f(u_i) = mk+m-1+i, \quad 1 \leq i \leq k$

$f(u_{k+i}) = mk+m+2k-1+i, \quad 1 \leq i \leq k$

$f(w_i) = (k+1)(i-1), \quad 1 \leq i \leq m$

From the above vertex labeling, the sets $\{ f(w_i) \;/\; 1 \leq i \leq m \}$, $\{ f(u_i) \;/\; 1 \leq i \leq k \}$and $\{ f(u_{k+i}) \;/\; 1 \leq i \leq k \}$ form a monotonically increasing sequence and the set $\{ f(v_i) \;/\; 1 \leq i \leq k+1 \}$ form a monotonically decreasing sequence.

Observe that $\max\{\{ f(w_i) \;/\; 1 \leq i \leq m \} \cup \{ f(u_i) \;/\; 1 \leq i \leq k \} \cup \{ f(u_{k+i}) \;/\; 1 \leq i \leq k \}\}$
$< \min \{ f(v_i) \;/\; 1 \leq i \leq k+1 \}$

Therefore the labels of all vertices of *G* are distinct.

- Let A_1 denote the set of $m\,k+m$ edges $\{w_1v_1, w_1v_2, \ldots w_1v_{k+1}, w_2v_1, w_2v_2, \ldots w_2v_{k+1}, \ldots, w_mv_1, w_mv_2, \ldots w_mv_{k+1}\}$ of *G*.
- Let A_2 denote the set of $2k$ edges $\{v_1u_1, u_1v_2, v_2u_2, \ldots v_ku_k, u_kv_{k+1}\}$ of *G*.
- Let A_3 denote the set of $2k$ edges $\{v_1u_{k+1}, u_{k+1}v_2, v_2u_{k+2}, \ldots v_ku_{2k}, u_{2k}v_{k+1}\}$ of*G*.

We give below the edge values in the sets $A_1, A_2 \& A_3$ and we denote these sets respectively by $A_1', A_2' \& A_3'$.

$A_1' = \{M,\ M-1,\ M-2,\ .\ .\ .\ .\ ,\ 4k+1\}$

$A_2' = \{4k,\ 4k-1,\ .\ .\ .\ .\ ,\ 2k+1\}$

$A_3' = \{2k,\ 2k-1,\ .\ .\ .\ .\ ,\ 3,\ 2,\ 1\}$

Observe that the values in the sets A_1, A_2 & A_3 are all distinct and $A_1' \cup A_2' \cup A_3' = \{M, M-1, M-2, ..., 3, 2, 1\}$. Hence *G* is graceful.

Theorem 2.2. The complete *m*-points projection $(m \geq 1)$ on the disjoint vertices of kC_4-snake is graceful.

Proof: Let *H* be the kC_4-snake with *k* blocks, so the number of vertices of *H* is $3k+1$ and the number of edges of *H* is $4k$. Let *G* be the super graph of *H* such that *G* is the complete *m*-points projection on the disjoint vertices of kC_4-snake. Let $N = m+3k+1$ be the number of vertices of *G* and $M = 2mk+4k$ be the number of edges of *G*. [Refer Figure 2.5]. To prove *G* is graceful it is enough to prove that the *M* edges o f *G* having the edge values as $\{M, M-1, M-2, ..., 3, 2, 1\}$. Name the $k+1$adjoint vertices by $\{v_1, v_2, . . . , v_{k+1}\}$, $2k$ disjoint vertices by $\{u_1, u_2, . . . , u_k, u_{k+1}, . . . , u_{2k}\}$ and the *m* isolated vertices by $\{w_1, w_2, . . . , w_m\}$ as described in Figure 2.5.

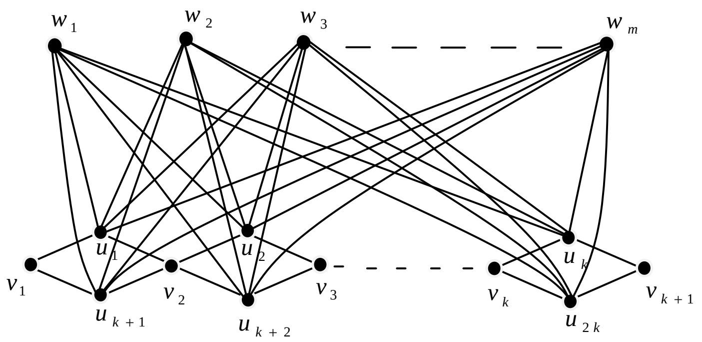

Figure 2.5:

Define

$f(v_i) = mk - 1 + i, \quad 1 \leq i \leq k+1$

$f(u_i) = 2mk + 4k + 1 - i, \quad 1 \leq i \leq k$

$f(u_{k+i}) = mk + 2k + 1 - i, \quad 1 \leq i \leq k$

$f(w_i) = k(i-1), \quad 1 \leq i \leq m$

From the above vertex labeling, the sets $\{f(w_i)\ /\ 1 \leq i \leq m\}$ and $\{f(v_i)\ /\ 1 \leq i \leq k+1\}$ form a monotonically increasing sequence and the sets $\{f(u_i)\ /\ 1 \leq i \leq k\}$ and $\{f(u_{k+i})\ /\ 1 \leq i \leq k\}$ form a monotonically decreasing sequence. Observe that

$\max\{\{f(w_i)\ /\ 1 \leq i \leq m\} \cup \{f(v_i)\ /\ 1 \leq i \leq k+1\}\} < \min\{\{f(u_i)\ /\ 1 \leq i \leq k\} \cup \{f(u_{k+i})\ /\ 1 \leq i \leq k\}\}$

Therefore the labels of all vertices of *G* are distinct.

- Let A_1 denote the set of $m\,k$ edges $\{w_1u_1, w_1u_2, \ldots\ w_1u_k\ , w_2u_1, w_2u_2, \ldots\ w_2u_k\ , \ .\ .\ .\ , w_mu_1, w_mu_2, \ldots\ w_mu_k\}$ of G.
- Let A_2 denote the set of $2k$ edges $\{v_1u_1, u_1v_2, v_2u_2, \ldots\ v_ku_k, u_kv_{k+1}\}$ of G.
- Let A_3 denote the set of $m\,k$ edges $\{w_1u_{k+1}, w_1u_{k+2}, \ldots\ w_1u_{2k}, w_2u_{k+1}, w_2u_{k+2}, \ldots \ldots w_2u_{2k}, \ .\ .\ .\ , w_mu_{k+1}, w_mu_{k+2}, \ldots\ w_mu_{2k}\}$ of G.
- Let A_4 denote the set of $2k$ edges $\{v_1u_{k+1}, u_{k+1}v_2, v_2u_{k+2}, \ldots\ v_ku_{2k}, u_{2k}v_{k+1}\}$ of G.

We give below the edge values in the sets A_1, A_2, A_3 & A_4 and we denote these sets respectively by A_1', A_2', A_3' & A_4'

$$A_1' = \{M,\ M-1,\ M-2,\ .\ .\ .\ ,\ mk+4k+1\}$$
$$A_2' = \{mk+4k,\ mk+4k-1,\ .\ .\ .\ ,\ mk+2k+1\}$$
$$A_3' = \{mk+2k,\ mk+2k-1,\ .\ .\ .\ ,\ 2k+1\}$$
$$A_4' = \{2k,\ 2k-1,\ .\ .\ .\ ,\ 3,\ 2,\ 1\}$$

Observe that the values in the sets A_1, A_2, A_3 & A_4 are all distinct and $A_1' \cup A_2' \cup A_3' \cup A_4' = \{M, M-1, M-2, \ldots, 3, 2, 1\}$. Hence G is graceful.

Examples 2.2.
1. The complete 4-points projection on the adjoint vertices of $2C_4$-snake.
[Refer Figure 2.6]
2. The complete 3-points projection on the disjoint vertices of $4C_4$-snake.
[Refer Figure 2.7]

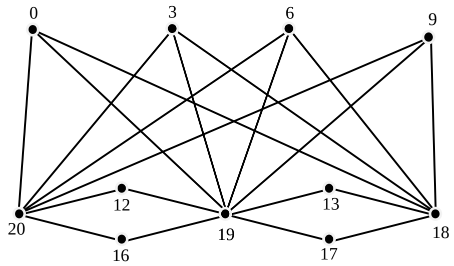

Figure 2.6 :

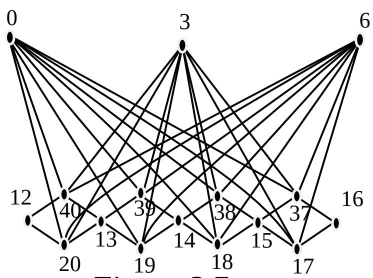

Figure 2.7 :

3. Conclusion
In Theorems 2.1 and 2.2 we have shown that the complete *m*-points projection $(m \geq 1)$ on the adjoint vertices of kC_4-snake and the complete *m*-points projection $(m \geq 1)$ on the disjoint vertices of kC_4-snake are graceful.

Remark. 'The complete *m*-points projection' can be used as a powerful operation to get larger graphs from a given graph. In obtaining the complete *m*-points projection from a given graph, the super graph can be extended to an infinite size and length.

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