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k–Semi–Similar Interval–Valued Fuzzy Matrices

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Abstract. In this paper, we have introduced the concept of k-semi-similar interval-valued fuzzy matrices (IVFM) as a generalization of semi-similar IVFM and as a special case of semi-similar fuzzy matrices.

Keywords: k-semi-similar fuzzy matrices, semi-similar IVFM

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1. Introduction

In this paper, we deal with interval-valued fuzzy matrices (IVFM) that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0, 1]. Recently, the concept of IVFM a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [6], by extending the max-min operations on fuzzy algebra F=[0, 1], for elements a,b$\in$F, a+b=max{a,b} and a$\cdot$b=min{a,b} for intervals. In [4], Meenakshi and kaliraja have represented an IVFM as an interval matrix of its lower and upper limit fuzzy matrices. The concept of Pseudo-similar fuzzy matrices studied by Meenakshi in [3] is generalized for fuzzy matrices and IVFM in [5] and [1] respectively.

In section 2, some basic definitions and results required are given. In section 3, we have introduced the concept of k-semi-similar interval-valued fuzzy matrices (IVFM) as a generalization of semi-similar IVFM and as a special case of semi-similar fuzzy matrices [3].

2. Preliminaries

Definition 2.1. [3] $A\in F_m$ and $B\in F_n$ are said to be pseudo-similar and denote it by $A \cong B$ if there exist $X\in F_{mn}$ and $Y\in F_{nm}$ such that $A = XBY$, $B = YAX$ and $XYX = X$.

Definition 2.2. [3] $A\in F_m$ and $B\in F_n$ are said to be semi-similar and denote it by $A \approx B$ if there exist $X\in F_{mn}$ and $Y\in F_{nm}$ such that $A = XBY$ and $B = YAX$.

Lemma 2.1. [3] Let$A\in F_m$ and $B\in F_n$. Then the following are equivalent:

(i) $A \cong B$

(ii) There exist $X\in F_{mn}$, $Y\in F_{nm}$ such that $A = XBY$, $B = YAX$, $XYX = X$ and $YXY = Y$.

(iii) There exist $X\in F_{mn}$, $Y\in F_{nm}$ such that $A = XBY$, $B = ZAX$, $XYX = X = XZX$.

Definition 2.3. [4] An IVFM of order m×n is defined as $A=(a_{ij})_{m\times n}$, where $a_{ij}=[a_{ijL}, a_{ijU}]$, the ij[th] element of A is an interval representing the membership value. All the elements of an IVFM are intervals and all the intervals are subintervals of the interval [0, 1].

For $A=(a_{ij})_{m\times n}$ and $B=(b_{ij})_{m\times n}$, their sum A+B is defined by,

$A+B= A=(a_{ij}+ b_{ij})=([\,(a_{ijL}+ b_{ijL}),\ (a_{ijU}+ b_{ijU})])$ (2.1)and their product is defined by,

$AB=(c_{ij})=\sum_{k=1}^{n} a_{ik}b_{kj} = \left[\sum_{k=1}^{n}(a_{ikL}b_{kjL}),\quad \sum_{k=1}^{n}(a_{ikU}b_{kjU})\right]$ i=1,2,…..,m and j=1,2,….,p(2.2)

In particular if $a_{ijL}=a_{ijU}$ and $b_{ijL}=b_{ijU}$ then (2.2) reduces to the standard max-min composition of fuzzy matrices [1]. $A\le B \Leftrightarrow a_{ijL}\le b_{ijL}$ and $a_{ijU}\le b_{ijU}$.

Definition 2.4. [4] For a pair of fuzzy matrices $E=(e_{ij})$ and $F=(f_{ij})$ in F_{mn} such that $E\le F$, let us define the interval matrix denoted as [E, F], whose ij[th] entry is the interval with lower limit e_{ij} and upper limit f_{ij}, that is $([e_{ij}, f_{ij}])$. In particular for E=F, IVFM [E, E] reduces to $E\in F_{mn}$. For $A=(a_{ij})=[a_{ijL}, a_{ijU}]\in(\text{IVFM})_{mn}$, let us define $A_L=(a_{ijL})$ and $A_U=(a_{ijU})$. Clearly A_L and A_U belongs to F_{mn} such that $A_L\le A_U$ and from Definition (2.4) A can be written as $A=[A_L, A_U]$. For $A\in(\text{IVFM})_{mn}$, A^T denotes the transpose of A.

Lemma 2.2. [4] For $A=[A_L, A_U]\in(\text{IVFM})_{mn}$ and $B=[B_L, B_U]\in(\text{IVFM})_{np}$ the following hold:
(i) $A^T=[A_L^T, A_U^T]$
(ii) $AB=[A_L B_L, A_U B_U]$

Definition 2.4. [5] $A\in F_n$ is said to be right k-pseudo-similar to $B\in F_n$ and it is denoted by $A \underset{r}{\overset{k}{\cong}} B$ if there exist X, $Y\in F_n$ such that $A=XBY$, $B=YAX^k$, $X^kYX=X^k$ and $YXY=Y$.

Definition 2.5. [5] $A\in F_n$ is said to be left k-pseudo-similar to $B\in F_n$ and it is denoted by $A \underset{\ell}{\overset{k}{\cong}} B$ if there exist X, $Y\in F_n$ such that $A=X^kBY$, $B=YAX$, $XYX^k=X^k$ and $YXY=Y$.

Remark 2.1. In particular for k=1, Definitions 2.4 and 2.5 are identical. Hence k-pseudo-similar is reduced to pseudo-similar fuzzy matrices. However, both right and left k-pseudo-similarity relations are not symmetric as in the case of pseudo-similarity of fuzzy matrices.

3. k-semi-similar interval–valued fuzzy matrices (IVFM)

Definition 3.1. $A\in(\text{IVFM})_m$ and $B\in(\text{IVFM})_n$ are said to be semi-similar IVFM and it is denoted by $A\overset{I}{\approx}B$ if there exist $X\in(\text{IVFM})_{mn}$ and $Y\in(\text{IVFM})_{nm}$ such that $A=XBY$ and $B=YAX$.

Remark 3.1. In particular, for fuzzy matrices $A\in F_m$, $B\in F_n$, since $A_L=A_U$ and $B_L=B_U$, Definition 3.1 reduces to semi-similar fuzzy matrices (Definition [2.2]). Also we observe that $A \overset{I}{\approx} B \Leftrightarrow B \overset{I}{\approx} A$.

Definition 3.2. A∈(IVFM)$_n$ is said to be right k-semi-similar to B∈(IVFM)$_n$ and it is denoted by $A \underset{r}{\overset{I(k)}{\approx}} B$ if there exist X, Y∈(IVFM)$_n$ such that $A = XBY$ and $B = YAX^k$.

Definition 3.3. A∈(IVFM)$_n$ is said to be left k-semi-similar to B∈(IVFM)$_n$ and it is denoted by $A \underset{\ell}{\overset{I(k)}{\approx}} B$ if there exist X, Y∈(IVFM)$_n$ such that $A = X^k BY$ and $B = YAX$.

Remark 3.2. In particular, for fuzzy matrices A∈F$_m$, B∈F$_n$, since A_L=A_U and B_L=B_U, Definition 3.2 and 3.3 to the following:

Definition 3.4. A∈F$_n$ is said to be right k-semi-similar to B∈F$_n$ and it is denoted by $A \underset{r}{\overset{k}{\approx}} B$ if there exist X, Y∈F$_n$ such that $A = XBY$ and $B = YAX^k$.

Definition 3.5. A∈F$_n$ is said to be left k-semi-similar to B∈F$_n$ and it is denoted by $A \underset{\ell}{\overset{k}{\approx}} B$ if there exist X, Y∈F$_n$ such that $A = X^k BY$ and $B = YAX$.

Remark 3.3. For k=1, A_L=A_U and B_L=B_U Definitions 3.2 and 3.3 reduced to Definition 2.2.

Remark 3.4. It is clear that, k-pseudo-similar IVFM⇒k-semi-similar IVFM. But the converse is not true. This is illustrated in the following:

Example 3.1. Let us consider $X = \begin{bmatrix} (0.3,0.5) & (1,1) \\ (0.5,0.5) & (0.2,0.5) \end{bmatrix}$ and $Y = \begin{bmatrix} (1,1) & (1,1) \\ (0,0) & (0,0) \end{bmatrix}$.
For $A = \begin{bmatrix} (0.3,0.5) & (0.3,0.5) \\ (0.5,0.5) & (0.5,0.5) \end{bmatrix}$ and $B = \begin{bmatrix} (0.5,0.5) & (0.5,0.5) \\ (0,0) & (0,0) \end{bmatrix}$, $A = XBY$ and $B = YAX^2$.
Therefore A is right 2-semi-similar IVFM to B, but $X^2YX \neq X^2$ and $YXY \neq Y$. Hence A is not right 2-pseudo similar IVFM to B.

Theorem 3.1. Let A, B∈(IVFM)$_n$. Then the following are equivalent.

(i) $A \underset{r}{\overset{I(k)}{\approx}} B$

(ii) $B^T \underset{\ell}{\overset{I(k)}{\approx}} A^T$

(iii) $PAP^T \underset{r}{\overset{I(k)}{\approx}} PBP^T$ for some permutation matrix P=[P_L, P_U]∈(IVFM)$_n$ with P= P_L=P_U.

Proof: (i)⇔(ii): This is direct by taking transpose on both sides and by using $(A^T)^T = A$ and $(AX)^T = X^T A^T$.

(ii)⇔(iii): Suppose $A \underset{r}{\overset{k}{\approx}} B$ then $A = XBY$ and $B = YAX^k$.

$A = XBY \Rightarrow PAP^T = PXBYP^T = (PXP^T)(PBP^T)(PYP^T)$ (3.1)

$B = YAX^k \Rightarrow PBP^T = PYAX^k P^T = (PYP^T)(PAP^T)(PX^k P^T) = (PYP^T)(PAP^T)(PXP^T)^k$ (3.2)

Hence $PAP^T \overset{k}{\underset{r}{\approx}} PBP^T$.

Conversely, suppose $PAP^T \overset{k}{\underset{r}{\approx}} PBP^T$.

Pre multiply by P^T and post multiply by P in Equations 3.1 and 3.2, we get $A = XBY$ and $B = YAX^k$. Hence $A \overset{k}{\underset{r}{\approx}} B$.

Hence the proof.

Theorem 3.2. Let A, B∈(IVFM)$_n$. Then the following are equivalent.

(i) $A \overset{I(k)}{\underset{\ell}{\approx}} B$

(ii) $B^T \overset{I(k)}{\underset{r}{\approx}} A^T$

(iii) $PAP^T \overset{I(k)}{\underset{\ell}{\approx}} PBP^T$ for some permutation matrix P=[P$_L$, P$_U$]∈(IVFM)$_n$. with P= P$_L$=P$_U$.

Proof: Proof of the theorem is similar to Theorem 3.1 and hence omitted.

Remark 3.5. In particular, for k=1 Theorems 3.1 and 3.2 reduces to the following Theorem.

Theorem 3.3. Let A∈(IVFM)$_m$ and B∈(IVFM)$_n$. Then the following are equivalent.

(i) $A \overset{I}{\approx} B$

(ii) $A^T \overset{I}{\approx} B^T$

(iii) $PAP^T \overset{I}{\approx} QBQ^T$ for some permutation matrices P=[P$_L$, P$_U$]∈(IVFM)$_n$ with P= P$_L$=P$_U$ and Q=[Q$_L$, Q$_U$]∈(IVFM)$_n$ with Q= Q$_L$=Q$_U$.

Remark 3.6. In particular, for fuzzy matrices A∈F$_m$, B∈F$_n$, since A$_L$=A$_U$ and B$_L$=B$_U$ then Theorems 3.1 and 3.2 reduces the following theorems.

Theorem 3.4. Let A, B∈F$_n$. Then the following are equivalent.

(i) $A \overset{k}{\underset{r}{\approx}} B$

(ii) $B^T \overset{k}{\underset{\ell}{\approx}} A^T$

(iii) $PAP^T \overset{k}{\underset{r}{\approx}} PBP^T$ for some permutation matrix P∈F$_n$.

Theorem 3.5. Let A, B∈F $_n$. Then the following are equivalent.

(i) $A \overset{k}{\underset{\ell}{\approx}} B$

(ii) $B^T \overset{k}{\underset{r}{\approx}} A^T$

(iii) $PAP^T \overset{k}{\underset{\ell}{\approx}} PBP^T$ for some permutation matrix P∈F$_n$.

Remark 3.7, In particular, for k=1, for fuzzy matrices $A\in F_m$, $B\in F_n$, since $A_L=A_U$ and $B_L=B_U$ then Theorem (3.1) and (3.2) reduces the following theorem.

Theorem 3.6. [3] Let $A\in F_m$ and $B\in F_n$. Then the following are equivalent.

(i) $A \approx B$

(ii) $A^T \approx B^T$

(iii) $PAP^T \approx QBQ^T$ for some permutation matrices $P\in F_m$ and $Q\in F_n$.

Theorem 3.7. Let A, B, $C\in (IVFM)_n$. If $A \underset{r}{\overset{I(k)}{\approx}} B$ ($A \underset{\ell}{\overset{I(k)}{\approx}} B$) and $B \underset{r}{\overset{I(k)}{\approx}} C$ ($B \underset{\ell}{\overset{I(k)}{\approx}} C$) then $A \underset{r}{\overset{I(k)}{\approx}} C$ ($A \underset{\ell}{\overset{I(k)}{\approx}} C$) if there exist matrices X, Y, Z and L with $XL = LX$.

Proof: Since $A \underset{r}{\overset{I(k)}{\approx}} B$, by Definition [3.2] $A = XBY$ and $B = YAX^k$.

Since $B \underset{r}{\overset{I(k)}{\approx}} C$, by Definition 3.2 $B = LCZ$ and $C = ZBL^k$.

$A = XBY = X(LCZ)Y = (XL)C(ZY) = UCV$, where U=XL and V=ZY.

$C = ZBL^k = Z(YAX^k)L^k = (ZY)A(X^kL^k) = (ZY)A(XL)^k = VAU^k$.

Hence the proof.

Remark 3.8. In particular, for k=1 Theorem 3.7 reduces to the following theorem.

Theorem 3.8. Let $A\in (IVFM)_m$, $B\in (IVFM)_n$ and $C\in (IVFM)_p$. If $A \underset{r}{\overset{I}{\approx}} B$ and $B \underset{r}{\overset{I}{\approx}} C$ then $A \underset{r}{\overset{I}{\approx}} C$.

Remark 3.9. In particular, for fuzzy matrices $A\in F_m$, $B\in F_n$, since $A_L=A_U$ and $B_L=B_U$ then Theorem 3.7 reduces the following theorem.

Theorem 3.9. Let A, B, $C\in F_n$. If $A \underset{r}{\overset{k}{\approx}} B$ ($A \underset{\ell}{\overset{k}{\approx}} B$) and $B \underset{r}{\overset{k}{\approx}} C$ ($B \underset{\ell}{\overset{k}{\approx}} C$) then $A \underset{r}{\overset{k}{\approx}} C$ ($A \underset{\ell}{\overset{k}{\approx}} C$) if there exist matrices X, Y, Z and L with $XL = LX$.

Remark 3.10. In particular, for k=1, Theorem 3.9 reduces the following theorem.

Theorem 3.10. [3] Let $A\in F_m$, $B\in F_n$ and $C\in F_n$. If $A \approx B$, $B \approx C$ then $A \approx C$.

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