

On $(1,2)^*$ - π wg-Normal Spaces

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Abstract. This paper introduces the concepts of $(1,2)^*$ - π wg-normal, mildly $(1,2)^*$ - π wg - normal spaces and obtain some of its properties and relationships. Further, we obtain the characterizations of $(1,2)^*$ - π wg- normal spaces, mildly $(1,2)^*$ - π wg- normal spaces , properties of the forms of new bitopological functions and preservation theorems for mildly $(1,2)^*$ - π wg normal spaces in bitopological spaces.

Keywords: $(1,2)^*$ - π wg-closed set, almost $(1,2)^*$ - π wg-continuous function, $(1,2)^*$ - π wg-normal space, M- $(1,2)^*$ - π wg-open map, Mildly $(1,2)^*$ - π wg- normal space

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1. Introduction

Levine [10] initiated the investigation of g-closed sets in topological spaces. V. Zaitsev [20] introduced the concept of π -closed sets in topological space. Nagaveni[12] introduced and studied the concept of weakly generalized closed sets in topological spaces. First step in normality was taken by Vigilino [19] who defined semi normal spaces. Then Singal and Arya [17] introduced the class of almost normal spaces and proved that a space is normal if and only if it is both a semi-normal space and an almost normal space. Normality is an important topological property and hence it is of significance both from intrinsic interest as well as from applications view point to obtain factorizations of normality in terms of weaker topological properties. In recent years, many authors have studied several forms of normality [3,5,8,13]. On the other hand, the notions of p-normal spaces, s-normal spaces were introduced by Paul and Bhattacharyya [18], Maheshwari and Prasad [11] respectively. In 2007, ErdalEkici [4] introduced and studied a class of spaces called γ -normal spaces. The purpose of this paper is to introduce a new class of normal spaces, namely $(1,2)^*$ - π wg-normal spaces and obtain some of its characteristics. Throughout this paper, $(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2)$ and (Z, η_1, η_2) (briefly X, Y and Z) will denote bitopological spaces.

2. Preliminaries

Definition 2.1. [1,9,14] Let A be a subset of X. Then A is called $\tau_{1,2}$ -open if $A = A_1 \cup B_1$, where $A_1 \in \tau_1$, $B_1 \in \tau_2$. The complement of $\tau_{1,2}$ -open set [1,13] is $\tau_{1,2}$ -closed set. The

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family of all $\tau_{1,2}$ -open (resp. $\tau_{1,2}$ -closed) sets of X is denoted by $(1,2)^*$ - $O(X)$ and (resp. $(1,2)^*$ - $C(X)$).

Definition 2.2. [1,9,14] Let A be a subset of a bitopological space X . Then (i) $\tau_{1,2}$ - closure of A [1,13] denoted by $\tau_{1,2}$ - $cl(A)$ is defined by the intersection of all $\tau_{1,2}$ -closed sets containing A .

(ii) $\tau_{1,2}$ -interior of A [1,14]denoted by $\tau_{1,2}$ - $int(A)$ is defined by the union of all open sets contained in A .

Remark 2.1. [1,9,14] Notice that $\tau_{1,2}$ -open subsets of X need not necessarily form a topology. Now, we recall some definitions and results which are used in this paper.

Definition 2.3. A subset A of a bitopological space X is said to be

(i) $(1,2)^*$ -semi open [1,9,14] if $A \subset \tau_{1,2}$ - $cl(\tau_{1,2}$ - $int(A))$.

(ii) regular $(1,2)^*$ -open [14] if $A = \tau_{1,2}$ - $int(\tau_{1,2}$ - $cl(A))$.

(iii) $(1,2)^*$ - α -open [1,2] if $A \subset \tau_{1,2}$ - $int(\tau_{1,2}$ - $cl(\tau_{1,2}$ - $int(A))$).

(iv) $(1,2)^*$ - π - open [2] if A is the finite union of regular $(1,2)^*$ -open sets.

The complements of all the above mentioned open sets are called their respective closed sets. The family of all $(1,2)^*$ -open set [$(1,2)^*$ -semi open, $(1,2)^*$ - regular open, $(1,2)^*$ - α -open, $(1,2)^*$ - π -open]of X will be denoted by $(1,2)^*$ - $O(X)$ (resp. $(1,2)^*$ - $SO(X)$, $(1,2)^*$ - $RO(X)$, $(1,2)^*$ - $\alpha O(X)$, $(1,2)^*$ - $\pi O(X)$].

Definition 2.4. A subset A of a bitopological space X is said to be

(i) a $\tau_{1,2}$ - ω - closed [14]if $\tau_{1,2}$ - $cl(A) \subset U$ whenever $A \subset U$ and $U \in (1,2)^*$ - $SO(X)$.

(ii) a $(1,2)^*$ - generalized closed set [14] ($(1,2)^*$ - g closed set if $\tau_{1,2}$ - $cl(A) \subset U$ whenever $A \subset U$ and $U \in (1,2)^*$ - $O(X)$).

(iii) a $(1,2)^*$ - π wg- closed set [6] in X if $\tau_{1,2}$ - $cl(\tau_{1,2}$ - $int(A)) \subset U$ whenever $A \subset U$ and $U \in (1,2)^*$ - $\pi O(X)$.

Definition 2.5. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be

(i) $(1,2)^*$ - continuous [9] if $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X for every $\sigma_{1,2}$ - closed set V in Y .

(ii) $(1,2)^*$ - π -continuous [6,9]if $f^{-1}(V)$ is $(1,2)^*$ - π closed in X for every $\sigma_{1,2}$ - closed set V in Y .

(iii) completely $(1,2)^*$ - continuous [2]if $f^{-1}(V)$ is $(1,2)^*$ - regular open in X for every $\sigma_{1,2}$ -open set V in Y .

Definition 2.6. A map $f: X \rightarrow Y$ is said to be $(1,2)^*$ - R -map [15]if $f^{-1}(F)$ is $(1,2)^*$ -regular open in X for every $(1,2)^*$ -regular open set F of Y .

Definition 2.7. A Space X is said to be

(i) $(1,2)^*$ -Normal [16] if for any pair of disjoint $\tau_{1,2}$ -closed sets A and B , there exists disjoint $\tau_{1,2}$ -open sets U and V such that $A \subset U$ and $B \subset V$.

(ii) $(1,2)^*$ - S -Normal [13]if for any pair of disjoint $\tau_{1,2}$ -closed sets A and B , there exists $\tau_{1,2}$ -semi-open sets U and V such that $A \subset U$ and $B \subset V$.

(iii) $(1,2)^*$ - P -Normal [17]if for any pair of disjoint $\tau_{1,2}$ -closed sets A and B , there exists $\tau_{1,2}$ -pre-open sets U and V such that $A \subset U$ and $B \subset V$.

- (iv) Mildly (1,2)*-Normal[15] if for any two disjoint regularly $\tau_{1,2}$ -closed sets A and B , there exists two disjoint $\tau_{1,2}$ -open subsets U and V such that $A \subset U$ and $B \subset V$.
- (v) almost (1,2)*-Normal [15] if for any two disjoint $\tau_{1,2}$ -closed subsets A and B of X, one of which is regularly closed, there exists two disjoint $\tau_{1,2}$ -open subsets U and V of X such that $A \subset U$ and $B \subset V$.

3. On (1,2)*- π wg- continuity and closed maps

Definition 3.1. A function $f: X \rightarrow Y$ is said to be

- (i) (1,2)* - π wg- continuous[6] if every $f^{-1}(V)$ is (1,2)* - π wg-closed in (X, τ_1, τ_2) for every closed set V of Y.
- (ii) (1,2)* - π wg- irresolute[6] if $f^{-1}(V)$ is (1,2)* - π wg- closed in (X, τ_1, τ_2) for every (1,2)* - π wg -closed set V of Y.
- (iii) (1,2)*- π wg- closed [7] if for every $\tau_{1,2}$ -closed V of X, $f(V)$ is (1,2)*- π wg-closed in Y.
- (iv) M-(1,2)*- π wg-closed[6] if for every (1,2)*- π wg -closed V of X, $f(V)$ is (1,2)*- π wg- closed in Y.
- (v) almost (1,2)*- π wg- closed map[6] if for every (1,2)*-regular closed set V of X, $f(V)$ is (1,2)*- π wg-closed in Y.

The complement of the (1,2)*- π wg-closed map and M-(1,2)*- π wg-closed is (1,2)*- π wg-open map and M-(1,2)*- π wg-open map respectively.

Now, let us define the following definition:

Definition 3.2. A function $f: X \rightarrow Y$ is said to be almost (1,2)* - π wg- continuous if every $f^{-1}(V)$ is (1,2)* - π wg-closed in X for every (1,2)*-regular closed set V of Y.

Proposition 3.1. Let $f: X \rightarrow Y$ be a map. Then

- (i) Every (1,2)*- π wg-continuous function is almost (1,2)*- π wg-continuous.
- (ii) Every (1,2)*- π wg-irresolute function is almost (1,2)*- π wg-continuous.
- (iii) Every (1,2)*- π wg-irresolute function is (1,2)*- π wg-continuous.
- (iv) Every (1,2)*- π wg-closed map is almost (1,2)*- π wg-closed.
- (v) Every M-(1,2)*- π wg-closed map is (1,2)*- π wg-closed.
- (vi) Every M-(1,2)*- π wg-map is almost (1,2)*- π wg-closed.

Proof: Follows from the definitions.

Remark 3.1. The converse of the above implications need not be true as seen in the following examples.

Example 3.1. Let $X=Y= \{a,b,c\}$, $\tau_1 = \{\phi, X, \{b\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{a,b\}\}$, $\tau_{1,2}$ -open = $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma_1 = \{\phi, Y\}$, $\sigma_2 = \{\phi, Y, \{a\}\}$, $\sigma_{1,2}$ -open = $\{\phi, Y, \{a\}\}$.

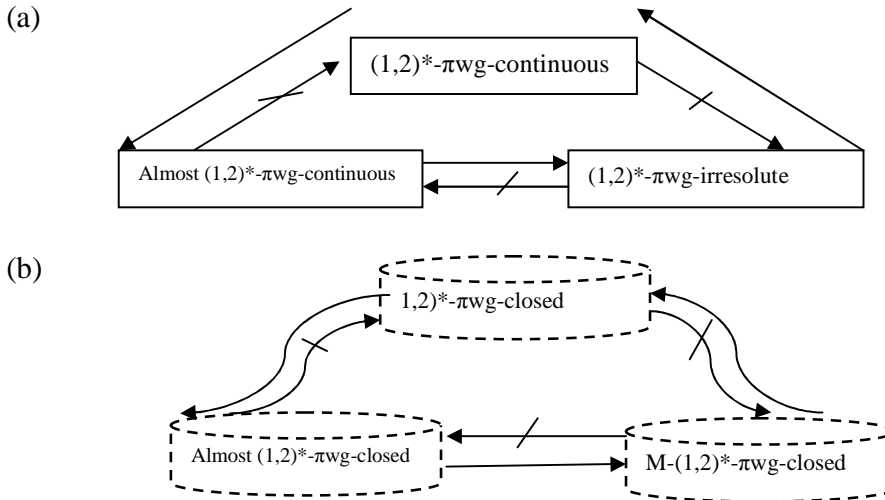
- (a) Let $f: X \rightarrow Y$ be an identity map. Then f is (1,2)*- π wg-continuous, almost (1,2)*- π wg-continuous but not (1,2)*- π wg-irresolute.
- (b) Let $f: X \rightarrow Y$ be defined by $f(a)=b$, $f(b)=c$, $f(c)=a$. Then f is almost (1,2)*- π wg-continuous, but not (1,2)*- π wg-continuous and (1,2)*- π wg-irresolute.

Example 3.2. Let $X=Y= \{a,b,c\}$, $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\tau_{1,2}$ -open = $\{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y, \{b\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{a,b\}\}$, $\sigma_{1,2}$ -open = $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.

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- (a) Let $f: X \rightarrow Y$ be an identity map. Then f is $(1,2)^*$ - π wg-closed and almost $(1,2)^*$ - π wg-closed in X but not M - $(1,2)^*$ - π wg-closed in X .
- (b) Let $f: X \rightarrow Y$ be defined by $f(a)=c, f(b)=a, f(c)=b$. Then f is almost $(1,2)^*$ - π wg-closed but not $(1,2)^*$ - π wg-closed and M - $(1,2)^*$ - π wg-closed.

Remark 3.2. The above relations are represented diagrammatically as follows:



3.1. $(1,2)^*$ - π wg-normal spaces

In this section, we introduce the notion of $(1,2)^*$ - π wg-normal spaces and study some of its properties.

Definition 3.1.1. A space X is said to be $(1,2)^*$ - π wg-Normal if for any two disjoint $\tau_{1,2}$ -closed subsets A and B of X , there exists two disjoint $(1,2)^*$ - π wg-open subsets U and V of X such that $A \subset U$ and $B \subset V$.

Definition 3.1.2. A space X is said to be Mildly $(1,2)^*$ - π wg-Normal if for the pair of disjoint regular $\tau_{1,2}$ -closed subsets A and B of X , there exists a pair of disjoint $(1,2)^*$ - π wg-open subsets U and V of X such that $A \subset U$ and $B \subset V$.

Theorem 3.1.1. Every $(1,2)^*$ -normal space is $(1,2)^*$ - π wg-normal.

Proof: Let the space X be a $(1,2)^*$ -normal space, then for the pair of disjoint $\tau_{1,2}$ -closed sets A and B , there exists disjoint $\tau_{1,2}$ -open sets U and V such that $A \subset U$ and $B \subset V$. Since every $\tau_{1,2}$ -open set is $(1,2)^*$ - π wg-open, the space X is $(1,2)^*$ - π wg-normal.

Remark 3.1.1. The converse of the above need not be true as seen in the following example.

Example 3.1.1. Let $X=\{a,b,c,d\}$, $\tau_1 = \{\emptyset, X, \{b,c,d\}\}$, $\tau_2 = \{\emptyset, X, \{c,d\}, \{a,c,d\}\}$, $\tau_{1,2}$ -open $=\{\emptyset, X, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$, $\tau_{1,2}$ -closed $=\{\emptyset, X, \{a,b\}, \{b\}, \{a\}\}$. Let $A=\{a\}$, $B=\{b\}$. Then the space X is $(1,2)^*$ - π wg-normal but not an $(1,2)^*$ -normal space.

Theorem 3.1.2. For a space X , the following are equivalent.

1. X is $(1,2)^*$ - π wg-normal
2. For every pair of disjoint $\tau_{1,2}$ -open sets U and V whose union is X , there exists $(1,2)^*$ - π wg -closed sets A and B such that $A \subset U, B \subset V$ and $A \cup B = X$.
3. For every $\tau_{1,2}$ -closed set F and every $\tau_{1,2}$ -open set G containing F , there exists a $(1,2)^*$ - π wg-open set U such that $F \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset G$.

Proof: (1) \Rightarrow (2): Let U and V be a pair of disjoint $\tau_{1,2}$ -open sets such that $X = U \cup V$.

Then $(X-U) \cap (X-V) = \emptyset$. Since X is $(1,2)^*$ - π wg-normal, there exists disjoint $(1,2)^*$ - π wg-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$.

Let $A = X-U_1, B = X-V_1$. Then A and B are $(1,2)^*$ - π wg-closed sets such that $A \subset U, B \subset V$ and $A \cup B = X$.

(2) \Rightarrow (3): Let F be a $\tau_{1,2}$ -closed set and G be a $\tau_{1,2}$ -open set containing F . Then $X - F$ and G are $\tau_{1,2}$ -open sets whose union is X . Then by (2), there exists $(1,2)^*$ - π wg-closed sets W_1 and W_2 such that $W_1 \subset X-F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X-W_1, X-G \subset X-W_2$ and $(X-W_1) \cap (X-W_2) = \emptyset$. Let $U = X-W_1$ and $V = X-W_2$. Then U and V are disjoint $(1,2)^*$ - π wg-open sets such that $F \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset X-V \subset G$.

(3) \Rightarrow (1): Let A and B be any two disjoint $\tau_{1,2}$ -closed subsets of X . Then $A \subset X-B$.

Put $G = X-B$. Then G is the $\tau_{1,2}$ -open set containing A . By part (3), there exists a $(1,2)^*$ - π wg-open sets U and V such that $A \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset G$. It follows that $B \subset X - (1,2)^*$ - π wg-cl(U). Let $V = X - (1,2)^*$ - π wg-cl(U). Then V is a $(1,2)^*$ - π wg-open set and $U \cap V = \emptyset$. Therefore, X is $(1,2)^*$ - π wg-normal.

Theorem 3.1.3. The following are equivalent for a space X .

1. X is Mildly $(1,2)^*$ - π wg-normal.
2. For every pair of regular open sets U and V , whose union is X , there exists $(1,2)^*$ - π wg-closed sets A and B such that $A \subset U$ and $B \subset V$ and $A \cup B = X$.
3. For any $(1,2)^*$ -closed set A and every $(1,2)^*$ -regular open set B containing A , there exists $(1,2)^*$ - π wg-open set U such that $A \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset B$.

Proof: (1) \Rightarrow (2): Let U and V be a pair of disjoint regular $(1,2)^*$ -open sets such that $X = U \cup V$. Then $X-U$ and $X-V$ are disjoint regular $(1,2)^*$ -regular closed sets. Since X is mildly $(1,2)^*$ - π wg-normal, there exists disjoint $(1,2)^*$ - π wg-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$. Let $A = X-U_1$ and $B = X-V_1$. Then A and B are $(1,2)^*$ - π wg-closed sets such that $A \subset U$ and $B \subset V$ and $A \cup B = X$.

(2) \Rightarrow (3): Let A be a $(1,2)^*$ - regular closed set in X and B be an $(1,2)^*$ - regular open set of X containing A . Then $X-A$ and B are $(1,2)^*$ -regular open sets of X whose union is X . Then by (2), there exists $(1,2)^*$ - π wg-closed sets W_1 and W_2 such that $W_1 \subset X-A$ and $W_2 \subset B, W_1 \cup W_2 = X$. Then $A \subset X-W_1, X-B \subset X-W_2$ and $(X-W_1) \cap (X-W_2) = \emptyset$. Let $U = X-W_1$ and $V = X-W_2$. Then U and V are disjoint $(1,2)^*$ - π wg- open sets such that $A \subset U \subset X-V \subset B$ implies $A \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset X-V \subset B$.

(3) \Rightarrow (1): Let A and B be any two disjoint $(1,2)^*$ -regular closed set of X . Then $A \subset X-B$. Put $G = X-B$. Then G is an $(1,2)^*$ - regular open set containing A . By (3), there exists a $(1,2)^*$ - π wg-open set U in X such that $A \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset (X-B)$. It follows that $B \subset X - (1,2)^*$ - π wg-cl(U). Let $V = X - (1,2)^*$ - π wg-cl(U). Then V is an $(1,2)^*$ - π wg-open set and $U \cap V = \emptyset$. Hence the space X is Mildly $(1,2)^*$ -normal.

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Theorem 3.1.4. If $f : X \rightarrow Y$ is an M - $(1,2)^*$ - π wg-open, $(1,2)^*$ -continuous and $(1,2)^*$ - π wg-irresolute function from a $(1,2)^*$ - π wg-normal space X onto a space Y , then Y is $(1,2)^*$ - π wg-normal.

Proof: Let A be a $\sigma_{1,2}$ -closed subset of Y and B be an $\sigma_{1,2}$ -open set containing A . Then by $(1,2)^*$ -continuity of f , $f^{-1}(A)$ is $\tau_{1,2}$ -closed in X and $f^{-1}(B)$ is an $\tau_{1,2}$ -open in X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is $(1,2)^*$ - π wg-normal, by theorem 3.6, there exists a $(1,2)^*$ - π wg-open set U in X such that $f^{-1}(A) \subset U \subset (1,2)^*$ - π wg-cl(U) $\subset f^{-1}(B)$. Then $f(f^{-1}(A)) \subset f(U) \subset f((1,2)^*$ - π wg-cl(U)) $\subset f(f^{-1}(B))$. Since f is M - $(1,2)^*$ - π wg-open and $(1,2)^*$ - π wg-irresolute surjection, we obtain $A \subset f(U) \subset (1,2)^*$ - π wg-cl($f(U)$) $\subset B$. Again by theorem 3.1.2, the space Y is $(1,2)^*$ - π wg-normal.

Lemma 3.1.1. A function $f: X \rightarrow Y$ is an M - $(1,2)^*$ - π wg-closed map if and only if for each subset A in Y and for each $(1,2)^*$ - π wg-open set U in X containing $f^{-1}(A)$, there exists a $(1,2)^*$ - π wg-open set V containing A such that $f^{-1}(V) \subset U$.

Proof: Suppose that f is M - $(1,2)^*$ - π wg-closed. Let A be a subset of Y and $U \subset (1,2)^*$ - π wgO(X) containing $f^{-1}(A)$. Put $V = Y - f(X - U)$, then V is a $(1,2)^*$ - π wg-open set of Y such that $A \subset V$ and $f^{-1}(V) \subset U$. Now on the other hand, let F be any $(1,2)^*$ - π wg-closed set of X and let $A = Y - f(F)$. Then $U = X - F$ is $(1,2)^*$ - π wg-open set in X . Now, by hypothesis, there exists a $(1,2)^*$ - π wg-open set V containing $A = Y - f(F)$ and $f^{-1}(V) \subset X - F$. Therefore, $f(F) \supset Y - V$. Also, $f^{-1}(V) \subset U = X - F$ implies that $F \subset X - f^{-1}(V)$. The $f(F) \subset f(X - f^{-1}(V)) = Y - V$. Hence $f(F) = Y - V$ and $f(F)$ is $(1,2)^*$ - π wg-closed in Y . This shows that f is M - $(1,2)^*$ - π wg-closed.

Theorem 3.1.5. If $f : X \rightarrow Y$ is an M - $(1,2)^*$ - π wg-closed, $(1,2)^*$ -continuous function from a $(1,2)^*$ - π wg-normal space X onto a space Y , then Y is $(1,2)^*$ - π wg-normal.

Proof: Let F_1 and F_2 be disjoint $\sigma_{1,2}$ -closed sets of Y . Since f is $(1,2)^*$ -continuous function, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $\tau_{1,2}$ -closed sets in X . Since X is $(1,2)^*$ - π wg-normal, there exist disjoint $(1,2)^*$ - π wg-open sets U and V such that $f^{-1}(F_1) \subset U$ and $f^{-1}(F_2) \subset V$. By lemma 3.9, there exist $(1,2)^*$ - π wg-open sets A and B of X such that $F_1 \subset A$, $F_2 \subset B$, $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Also, A and B are disjoint and hence Y is $(1,2)^*$ - π wg-normal.

Theorem 3.1.6. If $f: X \rightarrow Y$ is an M - $(1,2)^*$ - π wg-open, $(1,2)^*$ - R -map and $(1,2)^*$ - π wg-irresolute surjection from a mildly $(1,2)^*$ - π wg-normal space X onto a space Y , then Y is mildly $(1,2)^*$ - π wg-normal.

Proof: Let A be a $(1,2)^*$ -regular closed set and B be a $(1,2)^*$ -regular open set in Y containing A . Since f is an R -map, $f^{-1}(A)$ is a $(1,2)^*$ -regular closed set contained in the $(1,2)^*$ -regular open set $f^{-1}(B)$. Since X is Mildly $(1,2)^*$ -normal, there exists a $(1,2)^*$ - π wg-open set V such that $f^{-1}(A) \subset V \subset (1,2)^*$ - π wg-cl(V) $\subset f^{-1}(B)$ implies that $A \subset f(V) \subset (1,2)^*$ - π wg-cl(V). Since f is an M - $(1,2)^*$ - π wg-open surjection, Y is mildly $(1,2)^*$ - π wg-normal.

Theorem 3.1.6. If $f: X \rightarrow Y$ is as $(1,2)^*$ - π wg-irresolute, almost $(1,2)^*$ -closed injection and Y is $(1,2)^*$ - π wg-normal, then X is mildly $(1,2)^*$ - π wg-normal.

Proof: Let A and B be disjoint regular $(1,2)^*$ -closed set in X . Since f is an almost $(1,2)^*$ -closed injection, $f(A)$ and $f(B)$ are disjoint $\sigma_{1,2}$ -closed sets in Y . Since Y is $(1,2)^*$ - π wg-

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normal, there exists $(1,2)^*$ - π wg-open sets U and V in Y such that $f(A) \subset U$ and $f(B) \subset V$ and $U \cap V = \emptyset$. Since f is $(1,2)^*$ - π wg-irresolute, $f^1(U), f^1(V)$ are $(1,2)^*$ - π wg-open sets in X such that $A \subset f^1(U)$, $B \subset f^1(V)$ and $f^1(U) \cap f^1(V) = \emptyset$. Hence X is mildly $(1,2)^*$ - π wg-normal.

Theorem 3.1.7. If $f: X \rightarrow Y$ is a completely $(1,2)^*$ -continuous, M - $(1,2)^*$ - π wg-open surjection and X is mildly $(1,2)^*$ -normal, then Y is $(1,2)^*$ - π wg-normal.

Proof: Let A and B be disjoint $\sigma_{1,2}$ -closed subsets of Y . Since f is completely $(1,2)^*$ -continuous, $f^1(A)$ and $f^1(B)$ are disjoint regular closed subsets of X . Since X is mildly $(1,2)^*$ - π wg-normal, then there exists disjoint $(1,2)^*$ - π wg-open sets U and V in X such that $f^1(A) \subset U$ and $f^1(B) \subset V$. Since f is an M - $(1,2)^*$ - π wg-open surjection, $f(U)$ and $f(V)$ are disjoint $(1,2)^*$ - π wg-open sets in Y . It follows that Y is $(1,2)^*$ - π wg-normal.

3.2. Almost $(1,2)^*$ - π WG-normal spaces

Definition 3.2.1. A space X is said to be almost $(1,2)^*$ - π wg-normal if for each $\tau_{1,2}$ -closed set A and each $(1,2)^*$ -regular closed set B such that $A \cap B = \emptyset$, there exists disjoint $(1,2)^*$ - π wg-open sets U and V such that $A \subset U$ and $B \subset V$.

Example 3.2.1. Let $X = \{a, b, c\}, \tau_1 = \{\emptyset, X, \{b\}\}, \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}, \tau_{1,2}$ -open = $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}, \tau_{1,2}$ -closed = $\{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$. Here we can find for disjoint $\tau_{1,2}$ -closed set $\{b\}$ and $(1,2)^*$ -regular closed set $\{a, c\}$, $(1,2)^*$ - π wg-open sets contained $\{b\}$ and $\{a, c\}$. Hence the space X is almost $(1,2)^*$ - π wg-normal.

Theorem 3.2.1. For a space X , the following are equivalent.

- 1) X is almost $(1,2)^*$ - π wg-normal
- 2) For every pair of sets U and V one of which is $\tau_{1,2}$ -open set and the other is $(1,2)^*$ -regular open, whose union is X , there exists $(1,2)^*$ - π wg-closed sets G and H such that $G \subset U, H \subset V$ and $G \cup H = X$.
- 3) For every $\tau_{1,2}$ -closed set A and every $(1,2)^*$ -regular open set B containing A , there is a $(1,2)^*$ - π wg-open set V such that $A \subset V \subset (1,2)^*$ - π wg-cl(V) $\subset B$.

Proof: (1) \Rightarrow (2)

Let U be an $\tau_{1,2}$ -open set and V be $(1,2)^*$ -regular open set such that $X = U \cup V$. Then $X - U$ is $\tau_{1,2}$ -closed and $X - V$ is $(1,2)^*$ -regular closed. Since X is almost $(1,2)^*$ - π wg-normal, there exists $(1,2)^*$ - π wg-open sets U_1 and V_1 such that $X - U \subset U_1$ and $X - V \subset V_1$.

Let $G = X - U_1$ and $H = X - V_1$. Then G and H are $(1,2)^*$ - π wg-closed sets such that $G \subset U, H \subset V$ and $G \cup H = X$.

(2) \Rightarrow (3) and (3) \Rightarrow (1) are obvious.

Some characterizations of $(1,2)^*$ - π wg-normal spaces

Theorem 3.2.2. If $f: X \rightarrow Y$ is a $(1,2)^*$ -continuous surjection, M - $(1,2)^*$ - π wg-closed map and X is $(1,2)^*$ - π wg-normal, then Y is $(1,2)^*$ - π wg-normal.

Proof: Let M_1 and M_2 be any disjoint $\sigma_{1,2}$ -closed sets of Y . Since f is $(1,2)^*$ -continuous, $f^1(M_1)$ and $f^1(M_2)$ are disjoint $\tau_{1,2}$ -closed sets of X . Since X is $(1,2)^*$ - π wg-normal, there exist disjoint $(1,2)^*$ - π wg-open sets U_1, U_2 of X such that $f^1(M_i) \subset U_i$ for $i = 1, 2$. Put $V_i = Y - f(X - U_i)$. Since f is M - $(1,2)^*$ - π wg-closed, $f(X - U_i)$ ($i = 1, 2$) is $(1,2)^*$ - π wg-closed in Y

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implies V_i is $(1,2)^*$ - π wg-open in Y , $M_i \subset V_i$ and $f^{-1}(V_i) \subset U_i$ for $i = 1, 2$. Since $U_1 \cap U_2 = \emptyset$ and f is surjective, we have $V_1 \cap V_2 = \emptyset$. This shows that Y is $(1,2)^*$ - π wg-normal.

Corollary 3.2.1. If $f: X \rightarrow Y$ is a $(1,2)^*$ -closed π wg-irresolute injection and Y is $(1,2)^*$ - π wg-normal, then X is $(1,2)^*$ - π wg-normal.

Proof: Obvious.

Theorem 3.2.3. If $f: X \rightarrow Y$ is an almost $(1,2)^*$ - π wg-continuous, $(1,2)^*$ -rc-preserving or $(1,2)^*$ -almost closed injection and Y is $(1,2)^*$ -mildly normal (or) $(1,2)^*$ -normal, then X is Mildly $(1,2)^*$ - π wg-normal.

Proof: Let A and B be any disjoint regular $(1,2)^*$ -closed sets of X . Since f is $(1,2)^*$ -rc-preserving, $f(A)$ and $f(B)$ are disjoint regular $(1,2)^*$ -closed sets of Y . By the mild $(1,2)^*$ -normality of Y , there exists disjoint $\sigma_{1,2}$ -open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Now, put $G = \sigma_{1,2}\text{-int}(\sigma_{1,2}\text{-cl}(U))$, $H = \sigma_{1,2}\text{-int}(\sigma_{1,2}\text{-cl}(V))$. Then G and H are disjoint regular $(1,2)^*$ -open sets such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost $(1,2)^*$ - π wg-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $(1,2)^*$ - π wg-open sets containing A and B respectively. Hence X is Mildly $(1,2)^*$ - π wg-normal.

Lemma 3.2.1. A function $f: X \rightarrow Y$ is almost $(1,2)^*$ - π wg-closed if and only if for each subset B of Y and each $U \subset (1,2)^*$ -RO(X) containing $f^{-1}(B)$, there exists a $(1,2)^*$ - π wg-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Suppose that f is almost $(1,2)^*$ - π wg-closed map. Let A be a subset of Y and $U \subset (1,2)^*$ -RO(X) containing $f^{-1}(A)$. Put $V = Y - f(X - U)$, then V is a $(1,2)^*$ - π wg-open set of Y such that $A \subset V$ and $f^{-1}(V) \subset U$. Now on the other hand, let F be any $(1,2)^*$ -regular closed set of X and let $B = Y - f(F)$. Then $U = X - F$ is $(1,2)^*$ -regular open set in X . Now, by hypothesis, there exists a $(1,2)^*$ - π wg-open set V containing $B = Y - f(F)$ and $f^{-1}(V) \subset X - F$. Therefore, $f(F) \supset Y - V$. Also, $f^{-1}(V) \subset U = X - F$ implies that $F \subset X - f^{-1}(V)$.

Then $f(F) \subset f(X - f^{-1}(V)) = Y - V$. Hence $f(F) = Y - V$ and $f(F)$ is $(1,2)^*$ - π wg-closed in Y . This shows that f is almost $(1,2)^*$ - π wg-closed.

6. Conclusion

Here, a study made on the analysis of $(1,2)^*$ - π wg-normal spaces and obtained their characterizations. This shows that there is a future scope for further research by using $(1,2)^*$ - π wg-closed sets.

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