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On β^*- Connectedness and β^*- Disconnectedness and their Applications

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Abstract. In this paper, by using β^*-closed sets we study the concept of β^*- separated sets. With this concept we study the notion of β^*-connected sets and strongly β^*-connected sets. We give some properties of such concepts with some β^*-separation axioms and compact spaces. Finally, we construct a new topological space on a connected graph.

Keywords: β^* -separated sets, β^* -connected sets

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1. Introduction

Connectedness [1] is a well-known notion in topology. Numerous authors studied connectedness. In [2], P-spaces and external disconnectednessare studied. Connectedness in [4–6] are used to expand some topological spaces. In [13], authors proved that neither first countable nor C˜ech-complete spaces are maximal Tychonoff connected. Many other topologists defined and studied connectedness in bitopological spaces [3, 12]. It is important to study some types of connectedness in digital spaces. A point with integer coordinates is called a digital point. The problem of finding a topology for the digital plane and the digital 3-space is of importance in image processing and more generally in all situations where spatial relations are modeled on a computer. In all these applications it is essential to have a data structure on the computer which shares as many as possible features with the real topological situation. Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. The concept of Hausdorff spaces is almost an integral part of compactness. Investigations into the properties of cut points of topological spaces which are connected, compact and Hausdorff date back to the 1920s. Connectedness together with compactness with the assumption of Hausdorff has been studied in [15] from the view point of cut points. In [7], authors studied some types of connected topological spaces. Recently Palanimani [9] introduced and studied a new class of sets called β^* -closed sets in topological spaces. Since then these concepts have used to define and investigate many topological properties. The aim of this paper is to study β^* -connectedness. Also digital spaces are examined in the context of these new concepts. However, our main interest shall be digital spaces that are also topological spaces.

2. Preliminaries

Throughout the present paper, the space (X, τ) and (Y, σ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated.
Here we present some of the definitions, which are used in our study.

Definition 2.1. A subset A of a topological spaces (X, τ) is called a

(i) generalized closed (briefly, g-closed) [8] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(ii) β^*-closed [9] is $Cl(Int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

The Complements of the above mentioned closed sets are their respective open sets. We denote the collection of all g-closed (resp. β^*-closed) sets by $GC(X)$ (resp. $\beta^* C(X)$). We set $GC(X, x) = \{U : x \in U \in GC(X)\}$ (resp. $\beta^* C(X, x) = \{U : x \in U \in \beta^* C(X)\}$. The β^* closure of a set A, denoted by $\beta^* Cl(A)$, is the intersection of all β^*-closed sets containing A. $\beta^* Cl(A)$ is the smallest β^* -closed set containing A. The β^* -interior of a set A denoted by $\beta^* Int(A)$, is the union of all β^* -open sets contained in A. $\beta^* Int(A)$ is the largest β^* -open set contained in A. The family of all β^*-open (resp. β^* -closed) sets in a space X will be denoted by $\beta^* O(X)$ (resp. $\beta^* C(X)$) .

Proposition 2.1. [9] (i) The union of any family of β^* -open sets is a β^* -open set.
(ii) The intersection of an open and a β^* -open set is a β^* -open set.

Lemma 2.1. [9] The β^*-closure of a subset A of X, denoted by $\beta^* Cl(A)$, is the set of all x $\in X$ such that $O \cap A \neq \phi$ for every $O \in \beta^* O(X, x)$, where $\beta^* O(X, x) = \{U : x \in U \in \beta^* O(X, \tau)\}$.

Definition 2.2. The β^*-boundary of a set A of a space X is defined by $\beta^*\text{-}bd(A) = \beta^* Cl(A) \cap \beta^* Cl(X - A)$.

Definition 2.3. A space X is said to be β^* -connected if X cannot be expressed as the union of two disjoint nonempty β^* -open sets of X.

Lemma 2.2. Let A be a subset of a topological space X. Then $A \in \beta^* O(X)$ if and only if $\beta^* Cl(A)$ is β^* -clopen in X (i.e., β^* -open and β^* -closed).

Definition 2.4. [11] A subset $N \subseteq X$ is called a β^*-neighborhood (briefly β^* -nbd) of a point $x \in X$ if there exists a β^* -open set $U \subseteq N$ such that $x \in U \subseteq N$.

3. β* - Separateness and β*-connectedness

Definition 3.1. Two subsets A and B in a space X are said to be β^* -separated if and only if $A \cap \beta^* Cl(B) = \phi$ and $\beta^* Cl(A) \cap B = \phi$. From the fact that $\beta^* Cl(A) \subset Cl(A)$, for every subset A of X, every separated set is β^*-separated. But the converse may not be true as shown in the following example.

Example 3.1. Let $X = \{a,b,c,d\}$ with a topology $\tau = \{X, \phi, \{b\}, \{c,d\}, \{b,c,d\}\}$. The subsets $\{c\}$, $\{a,d\}$ are β^* -separated but not separated.

Remark 3.1. Each two β^* -separated sets are always disjoint, since $A \cap B \subseteq A \cap \beta^* Cl(B) = \phi$. The converse may not be true in general.

Example 3.2. In Example 3.1, β^*O(X) = {X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d},{b, c, d}}. The subsets $\{b,c\}$, $\{a,d\}$ are disjoint but not β^* -separated.

Theorem 3.1. Let A and B be nonempty sets in a space X. The following statements hold:

(i) If A and B are β^* -separated and $A_1 \subseteq A$ and $B_1 \subseteq B$, then A_1 and B_1 are so.

(ii) If $A \cap B = \phi$ such that each of A and B are both β^* -closed (β^* -open), then A and B are β^* -separated.

(iii)If each of A and B are both β^*-closed (β^*-open) and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G are β^* -separated.

Proof: (i) Since $A_1 \subset A$, then $\beta^* Cl(A_1) \subset \beta^* Cl(A)$. Then $B \cap \beta^* Cl\,(A) = \phi$ implies $B_1 \cap \beta^* Cl(A) = \phi$ and $B_1 \cap \beta^* Cl(A_1) = \phi$. Similarly, $A_1 \cap \beta^* Cl(B_1) = \phi$. Hence A_1 and B_1 are β^* -separated.

(ii) Since $A = \beta^* Cl(A)$ and $B = \beta^* Cl(B)$ and $A \cap B = \phi$, then $\beta^* Cl(A) \cap B = \phi$ and $\beta^* Cl(B) \cap A = \phi$. Hence A and B are β^* -separated. If A and B are β^* -open, then their complements are β^* -closed.

(iii) If A and B are β^*-open, then $X - A$ and $X - B$ are β^*-closed. Since $H \subset X - B$, $\beta^* Cl(H) \subset \beta^* Cl(X - B) = X - B$ and so $\beta^* Cl(H) \cap B = \phi$. Thus $G \cap \beta^* Cl\,(H) = \phi$. Similarly, $H \cap \beta^* Cl(G) = \phi$. Hence H and G are β^* -Separated.

Theorem 3.2. The sets A and B of a space X are β^* -separated if and only if there exist U and V in $\beta^* O(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$.

Proof: Let A and B be β^* -separated sets. Set $V = X - \beta^* Cl(A)$ and $U = X - \beta^* Cl(B)$. Then $U, V \in \beta^* O(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$. On the other hand, let $U, V \in \beta^* O(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$. Since $X - V$ and $X - U$ are β^* -closed, then $\beta^* Cl(A) \subset X - V \subset X - B$ and $\beta^* Cl(B) \subset X - U \subset X - A$. Thus $\beta^* Cl\,(A) \cap B = \phi$ and $\beta^* Cl(B) \cap A = \phi$.

Definition 3.2. A point $x \in X$ is called a β^* -limit point of a set $A \subset X$ if every β^* -open set $U \subseteq X$ containing x contains a point of A other than x.

Theorem 3.3. Let A and B be nonempty disjoint subsets of a space X and E = A∪B. Then A and B are β^*-separated if and only if each of A and B is β^* -closed (β^* -open) in E.

Proof: Let A and B are β^* -separated sets. By Definition 3.1., A contains no β^* -limit points of B. Then B contains all β^* -limit points of B which are in A∪B and B is β^* -closed in A∪B. Therefore B is β^* -closed in E. Similarly A is β^* -closed in E.

Definition 3.3. A subset S of a space X is said to be β^* -connected relative to X if there is not exist two β^* -separated subsets A and B relative to X and S = A∪B. Otherwise, S is said to be β^* -disconnected.

By Definition 3.3., one can show that each β^* -connected set is connected. The converse may not be true in general as shown in the below examples. In other words, each disconnected isβ^* -disconnected.

Example 3.3. Any space with indiscrete topology is connected but not β^*-connected since β^* -open sets establish a discrete topology.

Example 3.4. Let$X = \{a, b, c, d\}$ with a topology$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. The subset $\{a, b, c\}$is connected but not β^* -connected.

Theorem 3.4. Let A and B be subsets in a space X such that $A \subset B \subset \beta^* Cl(A)$. If A is$\beta^*$-connected, then B is β^*-connected.

Proof: If B is β^*-disconnected, then there exist two β^*-separated subsets U and V relative to X such that $B = U \cup V$.Then either $A \subseteq U$or$A \subseteq V$. Without loss of generality, let $A \subseteq U$. As $A \subseteq U \subseteq B, \beta^* Cl_B(A) \subseteq \beta^* Cl_B(U) \subset \beta^* Cl(U)$.Also$\beta^* Cl_B(A) = B \cap \beta^* Cl(A) = B \supseteq \beta^* Cl(U)$.This implies to$B = \beta^* Cl(U)$. So U and V are not β^*-separated and B isβ^*-connected.

Theorem 3.5. If E is β^* -connected, then$\beta^* Cl(E)$ is β^* - connected.

Proof: By contradiction, suppose that $\beta^* Cl(E)$ is β^*-disconnected. Then there are two nonempty β^* -separated sets G and H in X such that $\beta^* Cl(E) = G \cup H$.

Since $E = (G \cap E) \cup (H \cap E)$ and $\beta^* Cl(G \cap E) \subset \beta^* Cl(G)$ and $\beta^* Cl(H \cap E) \subset \beta^* Cl(H)$ and $G \cap H = \phi$, then $(\beta^* Cl(G \cap E)) \cap H = \phi$. Hence $(\beta^* Cl(G \cap E)) \cap (H \cap E) = \phi$. Similarly $(\beta^* Cl(H \cap E)) \cap (G \cap E) = \phi$. Therefore Eis β^* -disconnected.

Lemma 3.1. Let $A \subseteq B \cup C$ such that A be a nonempty β^*-connected set in a space Xand B, C are β^* -separated. Then only one of the following conditions holds:

(i)$A \subseteq B$and$A \cap C = \phi$,(ii)$A \subseteq C$and $A \cap B = \phi$.

Proof: Since $A \cap C = \phi$, then $A \subseteq B$. Also, if$A \cap B = \phi$, then $A \subseteq C$. Since $A \subseteq B \cap C$, then both $A \cap B = \phi$ and $A \cap C = \phi$cannot hold simultaneously.

Similarly, suppose that $A \cap B \neq \phi$and $A \cap C \neq \phi$, then, by Theorem 3.5.(i), $A \cap B$ and $A \cap C$ are β^*-separated such that $A = (A \cap B) \cup (A \cap C)$ which contradicts with the β^*-connectedness of A. Hence one of the conditions (i) and (ii) must be hold.

Definition 3.4. [10], [11] A function $f : X \to Y$ is said to be:

(i) β^* -continuous if the inverse image of each open set in Y is β^* -open in X.

(ii) β^* -open if the image of each open set in X is β^* -open in Y.

(iii) β^* -closed if the image of each closed set inX is β^* -closed in Y.

Lemma 3.2. Let$f : X \to Y$ be a β^*-continuous function. Then $\beta^* Cl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$, for each $B \subseteq Y$.

Proof: Let A be subset of (X, τ). Let $B = f(A)$ be subset of Y. Then Cl(B) is closed inY.Since f is β^*-continuous, $f^{-1}(Cl(B))$ is β^*-closed in X and $A \subset f^{-1}(f(A)) \subset f^{-1}(Cl(B))$ that is $f^{-1}(Cl(B))$ is β^* -closed subset of XcontainingA. By Definition of β^*-closed sets implies β^*-$Cl(A) \subseteq f^{-1}(Cl(B))$. Hence $\beta^* Cl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$.

Theorem 3.6. For a β^*-continuous function $f: X \to Y$, if K is β^*-connected in X, then f(K) is connected in Y

Proof: Suppose that f(K) is disconnected inY. There exist two separated sets P and Q of Y such that$f(K) = P \cup Q$. Set $A = K \cap f^{-1}(P)$ and$B = K \cap f^{-1}(Q)$. Since$f(K) \cap P \neq \phi$, then $K \cap f^{-1}(P) \neq \phi$ and so $A \neq \phi$. Similarly$B \neq \phi$, Since$P \cap Q = \phi$, then $A \cap B = K \cap f^{-1}(P \cap Q) = \phi$and so$A \cap B = \phi$. Since f is β^*-continuous, then by Lemma 3.2., $\beta^* Cl\left(f^{-1}(Q)\right) \subset f^{-1}(Cl(Q))andB \subset f^{-1}(Q)$, then $\beta^* Cl(B)) \subset f^{-1}(Cl(Q))$.
Since $P \cap Cl(Q) = \phi$, then $A \cap f^{-1}\left(Cl(Q)\right) \subset f^{-1}(P) \cap f^{-1}(Cl(Q)) = \phi$ and then $A \cap \beta^* Cl(B) = \phi$. Thus A and B are β^* -separated.

Corollary 3.1. For a β^* -continuous function $f: X \to Y$, if K is disconnected in X, then f(K) is β^*- disconnected in Y.

Proof: Obvious.

Theorem 3.7. For a bijectiveβ^* -closed $f: X \to Y$, if K is β^*-connected in$Y$, then $f^{-1}(K)$ is connected in X.

Proof: The proof is similar to that of Theorem 3.6. Thus we omit it.

Definition 3.5. A function $f: X \to Y$is said to be:

(i) β^* -Irresolute if for each point $x \in X$ and each β^* -open set V of Ycontainingf(x), there exists aβ^* -open set U of X containing x such thatf (U) $\subset$V.
(ii) β^* -Irresolute [10] if $f^{-1}(V) \in \beta^* O(X)$ for every $V \in \beta^* O(Y)$.
(iii) M-β^* -open if $f(V) \in \beta^* O(Y)$ for every $V \in \beta^* O(X)$.
(iv) M-β^* -closed if $f(V) \subset \beta^* C(Y)$ for every $V \in \beta^* C(X)$.
(v) Strongly β^* -irresolute if $f^{-1}(V) \in \beta^* O(X)$ for every open set Vin Y.
(vi) StronglyM-β^* -open if $f(V) \in \beta^* O(Y)$ for every open set Vin X.
(vii) Strongly M-β^* -closed if $f(V) \in \beta^* C(Y)$ for every closed set Vin X.

Lemma 3.3. A function $f: X \to Y$is aβ^*-irresolute if and only if$\beta^* Cl\left(f^{-1}(B)\right) \subset f^{-1}(\beta^*\left(Cl(B)\right) \subset f^{-1}(Cl(B))$, for each $B \subset Y$.

Proof: Follows from the Definition 3.5.

Theorem 3.8. Let $f: (X, \tau) \to (Y, \sigma)$be a β^* -irresolute function. If K is β^* -connected in X, then $f(K)$ is β^* -connected in Y.

Proof: By using Definition 3.20.and Lemma 3.3, it is direct consequence of Theorem 3.6.

3.1. Strongly β^*-connectedness in compact spaces

Definition 3.1.1. A space X is strongly β^* -connected if and only if it is not a disjoint union of countably many but more than one β^* -closed set i.e. if E_i are nonempty disjoint closed setsof$X$, then $X \neq E_1 \cup E_2 \cup ....$ Otherwise$X$ is said to be strongly β^* -disconnected. Note the similarity between Definition 4.1.and that of β^* -connectedness. If X is β^* -connected, and E_1andE_2 are any two nonempty disjoint closed sets of$X$, then $X \neq E_1 \cup E_2$.

Lemma 3.1.1. For any surjectiveβ^* -irresolute function $f: X \to Y$. The image $f(X)$ is strongly β^* -connected if X is strongly β^* -connected.

Proof: Suppose $f(X)$ is strongly β^* -disconnected, by Definition 4.1. it is a disjoint union of countably many but more than one β^* -closed sets. Since f is β^* -irresolute, then the inverse image of β^* -closed sets are still β^* -closed, X is also a disjoint union of β^* -closed sets. Therefore, $f(X)$ is strongly β^* -connected.

Theorem 3.1.1. A space X is strongly β^* -connected if there exists a constant surjective β^*-irresolute function$f : X \rightarrow D$, where D denote to a discrete space of$X$.
Proof: Let X be strongly β^*-connected and $f : X \rightarrow D$ be a surjectiveβ^*-irresolute function, then by Lemma 3.1.1., $f(X)$ is strongly β^*-connected. The only strongly β^*-connected subset of D are the one-point spaces. Hence $f$is constant. Conversely, suppose X is a disjoint union of countably many but more than one β^*-closed sets, $X = \cup_{\text{i}} E_{\text{i}}$. Then define $f: X \rightarrow D$ by taking $f(x) = i$whenever$x \in E_{\text{i}}$. This $f$is a surjectiveβ^*-irresolute and not constant. So $X$is strongly β^*-connected. Strongly β^*-connectedness is a stronger notion of β^*-connectedness. In other words, given aβ^*-connected space, we can make it strongly β^*-connected by adding some conditions. But what conditions should be added is the difficulty. Our starting point is β^*-connected spaces, thus aβ^*-continuum may be useful. The concept of a β^*-continuum is defined on a β^*-connected set.

Definition 3.1.2. A compact β^* -connected set is called a β^* continuum.

Definition 3.1.3. A space X is called:
(i)$\beta^* T_1$if for each $x, y \in X, x \neq y$, there exist two disjoint β^*-open sets U and V such that$x \in U, y \notin U$, and $x \notin V, y \in V$.
(ii) $\beta^* T_2$iffor each $x, y \in X, x \neq y$, there exist two disjoint β^*-open sets U and V such that $x \in U, y \in V$and $U \cap V = \phi$.
(iii) β^*-normal for any pair of disjoint β^*-closed sets F_1 and F_2, there exist disjointβ^*-opensets$U$ and V such that $F_1 \subset U$ and $F_2 \subset V$ such that $U \cap V = \phi$.

Lemma 3.1.2. If A is any β^* -continuum in a $\beta^* T_2$ space X and B is any β^* -open set such that $A \cap B \neq \phi \neq A \cap (X - B)$, then every componentof $(\text{A} \cap \beta^* Cl(B)) \cap \beta^* - \text{bd}(B) \neq \phi$.
Proof: It is obvious by Definitions 2.2., 3.1.2.and 3.1.3.

Theorem 3.1.2. Let$X$ be a compact $\beta^* T_2$-space.Then X is β^*-connected if and only if X is strongly β^*-connected.
Proof: It is clear that if X is strongly β^*-connected, then X is β^*-connected. Now, supposethat X is a compact $\beta^* T_2$ β^*-connected space and it is strongly β^*-disconnected, then$X$ is aunion of a countably many but more than one disjoint β^*-closed sets. Then$X = \cup K_{\text{i}}$, where $K_i$are β^*-closed disjoint sets. Since a compact $\beta^* T_2$-space isβ^*-normal, then X, by Definition 3.1.3., is a β^*-normal space. So there exist a β^*-open sets U such that $K_2 \subset U$ and$\beta^* Cl(U) \cap K_1 = \phi$. Let X_1 be acomponent of $\beta^* Cl(U)$ which intersects K_2. Then X_1 is compact and β^*-connected. Now by Lemma 3.1.2.,$X_1 \cap \beta^* - \text{bd}(U) \neq \phi$i.e. X_1 contains a point $p \in \beta^* - \text{bd}(U)$ such that $p \notin U$ and $p \notin K_1$. Hence $X_1 \cap K_{\text{i}} \neq \phi$for some $i > 2$. Let K_{n2} be the first K_i for $i > 2$ which intersectsX_1, and let V be a β^*-open set satisfying $K_{n2} \subset V$,and$\beta^* Cl(V) \cap K_2 = \phi$.Then let X_2 be a component of $X_1 \cap \beta^* Cl(V)$ which contains a point of K_{n2}. Again we have $X_2 \cap \beta^* - \text{bd}(V) \neq \phi$, and X_2 contains some

point $p \in \beta^{*}$ – bd (V) such that $p \notin V,\ p \notin K_1 \cup K_2$. Hence $X_2 \cap K_{\mathrm{i}} \neq \phi$ for some $i > n_2$, and $X_2 \cap K_{\mathrm{i}} = \phi$ for$i < n_2$. Let K_{n3} be the first K_i for $i > n_2$, which intersects X_2, then by methods similar to the above we can find a compact β^{*} -connected X_3 such that $X_3 \subset X_2 \subset X_1$, and X_3 intersects some K_i with $i > n_3$ but $X_3 \cap K_i = \phi$ for $i < n_3$. In this manner, we obtain a sequence of sub continuous of $X : X_1 X_2 X_3 \ldots$, such that for each$j, X_j \cap K_i = \phi$ for$i < n_j$ and $n_j \to \infty$as$j \to \infty$. We know that $\cap_{\mathrm{i}} X_i \neq \phi$. Also,$(\cap_{\mathrm{i}} X_i) \cap K_j = \phi$for all j, so that $(\cap_{\mathrm{i}} X_{\mathrm{i}}) \cap (\cup_{\mathrm{i}} K_j) = \phi$or $(\cap_{\mathrm{i}} X_i) \cap X = \phi$.But $(\cap_{\mathrm{i}} X_i) \subset X$, which contradicts the fact that $\cap_{\mathrm{i}} X_i \neq \phi$. Therefore X is stronglyβ^{*}connected.

Theorem 3.1.3. Let$X$ be a locally compact $\beta^{*}T_2$-space. If X is locally β^{*}-connected, then$X$ is locally strongly β^{*}-connected.
Proof: Let O be a β^{*}-open β^{*}-nbd of a point $x \in X$. Then there exists a compact β^{*}-nbd$V$ of $x$lying inside O. Let C be a β^{*}-connected component of Vcontaining x.Since V is a β^{*}-nbd of x and X is locally β^{*}-connected, C is β^{*}-nbd of$x$. Since C isβ^{*}-closed in V and$V$is compact, then C is compact. So C is a compact β^{*}-connectedβ^{*}-nbd of x lying inside OBy Theorem 3.1.2., C is strongly β^{*}-connected.

Theorem 3.1.4. Let X be a locally compact $\beta^{*}T_2$ -space. If X is locally β^{*}-connected andβ^{*}-connected, then$X$ is strongly β^{*} -connected.
Proof: This follows from Theorems 3.1.2 and 3.1.3.

Lemma 3.1.3. For a space X the following statements are equivalent:
(i) $X$is a $\beta^{*}T_1$ -space.
(ii) For any point $x \in X$, the singleton set $\{x\}$ is β^{*} -closed.

Corollary 3.1.1. strongly β^{*}-connected $\beta^{*}T_1$-space having more than one point isuncountable.
Proof: By Lemma 3.1.3., a one-point set in a $\beta^{*}T_1$ -space is β^{*} -closed. Thus by Definition 3.1.1., a $\beta^{*}T_1$ –space cannot have countably many but more than one point.

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