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**g Closed Sets in Topological Spaces

***N.Gomathi*[1], *N.Meenakumari*[2] and *T.Indira*[3]**

[1]Department of Mathematics, Srimad Andavan Arts and Science College (Autonomous)
Trichy-620005, e-mail: gomathi198907@gmail.com
Corresponding Author

[2, 3]Department of Mathematics, Seethalakshmi Ramasamy College (Autonomous)
Trichy-620002, e-mail: [2]meemamega25@gmail.com, [3]drtindirachandru@gmail.com

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Abstract. The aim of this paper is to introduce a new class of sets called **g closed sets and investigate some of the basic properties of this class of sets.

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1. Introduction

Levine [5] introduced generalized closed sets and semi open sets. Abd Monsef, Deeb and Mahmoud introduced β sets and Njastad introduced α sets and Mashour, Abd El-Monsef and Deeb introduced pre-open sets. Andregvic called β sets as semi pre-open sets. Veerakumar [12] introduced g* closed sets. The aim of this paper is to introduce a new type of closed sets namely **g closed sets and investigate some of the basic properties of this class of sets.

2. Preliminaries

Definition 2.1. A subset A of topological space (X, τ) is called

(1) a **generalized closed** [5] (briefly g-closed) set, if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ); the complement of a g-closed set is called a **g-open** set.
(2) a **generalized semi-closed** (briefly gs- closed) set, if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) ;the complement of a gs-closed set is called a **gs-open** set.
(3) a **semi-generalized closed**(briefly sg- closed) set , if $scl(A) \subseteq U$ and $A \subseteq U$ and U is semi-open in (X, τ); the complement of a sg-closed set A is called a **sg-open** set.
(4) a **Ψ-closed** set [9], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ) the complement of a Ψ -closed set is called a **Ψ-open** set.
(5) a α **-generalized closed** [5] (briefly α g- closed) set, if $\alpha\, cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ); the complement of an αg-closed set is called a α **g-open** set.

(6) a **generalized α - closed** [4] (briefly gα - closed) set, if α cl(A)$\subseteq$U whenever A$\subseteq$U and U is α -open in (X, τ); the complement of a gα -closed set is called a **gα -open** set.
(7) a **generalized pre-closed** (briefly gp-closed) set, if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, τ); the complement of a gp-closed set is called a **gp-open** set.
(8) a **generalized semi-pre closed** [2] (briefly gsp- closed) set, if Spcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, τ); the complement of a gsp-closed set is called a **gsp-open** set.
(9) a **generalized*-closed**(briefly g*-closed) set, if cl(A$\subseteq$U whenever A$\subseteq$U and U is g open in (X, τ); the complement of a g*-closed set is called a **g*-open** set.
(10) a **generalized pre*-closed** (briefly gp*-closed) set, if cl (A)$\subseteq$U whenever A$\subseteq$U and U is gp-open in (X, τ); the complement of a gp*-closed setis called a **gp*-open** set.
(11) a **generalized $^{\#}$ closed** [7] (briefly g# closed) set, if cl(A)$\subseteq$U whenever A$\subseteq$U and U is αg open in (X, τ); the complement of a g$^{\#}$ closed set is called **g$^{\#}$ open** set.
(12) a **regular generalized –star closed** (briefly rg*closed) set, if rcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in (X, τ); the complement of a rg* closed set is **rg* open** set.
(13) a ***generalized closed**(briefly *g closed) set, if cl(A)$\subseteq$U whenever A$\subseteq$U and U is ĝ –open in (X, τ); the complement of a *g-closed set is called a ***g-open** set.
(14) a **strongly g* closed**(briefly sg* closed) set, if cl(int(A))$\subseteq$U whenever A$\subseteq$U and U is g open in (X, τ); the complement of a.
(15) a **regular weakly generalized closed (**briefly rwg closed)set, if cl(int(A))$\subseteq$U whenever A$\subseteq$U and U is regular open in (X, τ); the complement of a rwg closed set in called **rwg open** set.

3. **g closed sets

Definition 3.1. If a subset A of a topological space (X,τ) is called **g closed set, if cl(A)$\subseteq$U whenever A$\subseteq$U and U is g* open in (X, τ); the complement of a **g closed set is called **g open set.

Example 3.1. X={a,b,c}

τ ={X,ϕ,{a},{b},{a,b}}

τ (Closed sets)={ X,ϕ,{c},{b,c},{a,c}}

g closed sets={ X,ϕ,{c},{b,c},{a,c}}

g* closed sets={ X,ϕ,{c},{b,c},{a,c}}

g* open sets={ X,ϕ,{a},{b},{a,b }

**g closed sets={ X,ϕ,{c},{b,c},{a,c}}

**g open sets={X,ϕ,{a,b},{a},{b}}.

Theorem 3.1. Every closed set is **g closed set.
Proof: Let A be a closed set.
TPT: A is **g closed set**,** (i.e.) **TPT:** if cl(A)$\subseteq$U , whenever A$\subseteq$U and U is g* open
Since A is closed, A= cl(A).
Let A$\subseteq$U, U is g* open.
Now cl(A)=A$\subseteq$U
⇨ A is **g closed.

Example 3.2. The Converse of the above theorem is need not be true as seen from the following example X={a,b,c}.
τ={X,φ,{a},{a,c}} τ(Closed sets)={ X,φ,{b},{b,c}}
**g closed sets ={ X,φ,{b},{a,b}{b,c}}
Here A={a,b} be a **g closed set but not a closed set.

Theorem 3.2. Every g* closed set is **g closed set.
Proof: Let A be g* closed set
TPT: A is **g closed, (i.e.) **TPT** if cl(A)$\subseteq$U whenever A$\subseteq$U and U is g* open
By the definition of g* closed set, if cl(A)$\subseteq$ U whenever A$\subseteq$U and U is g open
Let A$\subseteq$U where U is g* open.
Since U is g*open=> U is g open.
Thus cl(A)$\subseteq$ U , U is g* open.
⇨ A is **g closed.

Example 3.3. The converse of the above theorem is need not be true as seen from the following example X={a,b,c}.
τ ={X,ϕ,{a},{a,c}} τ (Closed sets)={X,ϕ,{b},{b,c}}
g* closed sets={ X,φ,{c},{a,c}{b,c}}**g closed sets ={ X,φ,{b},{a,b}{b,c}}
Here A={a,b} is a **g closed set , but not a g* closed set

Remark 3.1. **g closedness is independent of g*ψ closedness. It can be seen from the following examples.

Example 3.4. X={a,b,c, d}.
τ={{X,φ,{a},{b},{a,b},{a,b,c}} τ(Closed sets)={{X,φ,{d},{c,d},{a,c,d},{b,c,d}}
**g closed sets={{X,φ,{d},{c,d},{a,c,d},{b,c,d},{a,d},{b,d},{a,b,d}}
Here A={b} be a g*ψ closed set, but not **g closed set

Example 3.5. X={a,b,c}.
τ={{X,φ,{a},{b,c}} τ(Closed sets)={{X,φ,,{a},{b,c,}}.
**g closed sets={{X,φ,{a},{b},{c},{a,b},{b,c},{a,c}}.
Here A={a,b} is a **g closed set, but not g*ψ set.

Remark 3.2. Similarly we can prove **g closedness is independent of g# closedness.

Theorem 3.3. If A is strongly g* closed and A is open then A is **g closed set.
Proof: Let $A \subseteq U$ and U is g* open.
Since A is strongly g* closed set.
cl(int((A)) $\subseteq$ U whenever $A \subseteq U$ and U is g open in X.
Since A is open, int(A)=A , also g* open=>g open.
Thus cl(A) $\subseteq$ U, U is g* open.

- ⇨ A is **g closed.

3.1. Properties of **g closed sets
Theorem 3.1.1. If A and B are**g closed sets then $A \cup B$ is **g closed set.
Proof: Let U be a g* open in X such that $A \cup B \subseteq U$.
Since A and B are **g closed sets.
Then, cl(A) $\subseteq$ U , $A \subseteq U$ and U is g*open.
Similarly cl(B) U , $B \subseteq U$ and U is g*open.

- ⇨ cl($A \cup B$)=cl(A) $\cup$ cl(B) $\subseteq$ U
- ⇨ cl($A \cup B$) $\subseteq$ U
- ⇨ $A \cup B$ is **g closed.

Example 3.1.1. X={a,b,c}
τ={ X, φ,{a},{b},{a,b}}
**g closed sets ={ X, φ,{c},{b,c},{a,c}}
Let A={c} and B={b,c}.
Then $A \cup B$ ={b,c} which is **g closed.

Remark 3.1.1. The intersection of two **g closed sets need not be **g closed set.

Theorem 3.1.2. If a subset A of X is **g closed set if cl(A) – A does not contain any non-empty g*closed set.
Proof: Let A be a **g closed set.
Suppose cl(A) – A contains a non-empty g*closed set namely F, (ie) $F \subset$ cl(A) – A
$F \subset$ cl(A) & $F \subset A^c \rightarrow$(1), (i.e.) $A \subset F^c$
=>cl(A) $\subset F^c$
=>$F \subset$ cl(A^c) $\rightarrow$ (2)
From (1) and (2)
$F \subset$ cl(A) $\cap$ cl(A^c)

- ⇨ F= φ

This is a contradiction to our assumption.
Hence the theorem.
The converse is not true.

Theorem 3.1.3. If A is a **g closed set of X such that $A \subseteq B \subseteq$ cl(A) then B is also a **g closed set.
Proof: Let U be a g*open set of X such that $B \subseteq U$.
Now $B \subseteq$ cl(A)

- ⇨ cl(B) $\subseteq$ cl(cl(A))
- ⇨ cl(A) $\subseteq$ U , since A is **g closed.

cl(B) $\subseteq$ U
=>B is **g closed set.

Theorem 3.1.4. If A is g* open and **g closed set then A is closed set.
Proof: Let A be both g*open and **g closed.
TPT: A is closed
Since A is **g closed, cl(A) $\subseteq$ U, U is g*open
Since A is g*open , we can take A=U

- ⇨ Cl(A) $\subseteq$ A ,but A $\subseteq$ cl(A)
- ⇨ A=cl(A)
- ⇨ A is closed.

Theorem 3.1.5. If a subset A of a topological space (X, τ) is both open and **g closed then A is closed set.
Proof: Let A be both open and **g closed
TPT : A is closed
cl(A) $\subseteq$ U where U is g*open
We know that, open =>g*open

- ⇨ A is g*open

Now take U=A
cl(A) $\subseteq$ A
Also A $\subseteq$ cl(A)

- ⇨ cl(A)=A
- ⇨ A is closed.

Theorem 3.1.6. If a subset A of a topological space(X, τ) is both open and regular closed then A is **g closed set.
Proof: Let A be both open and regular closed.
TPT: A be a **g closed
A=cl(int(A))
Let A $\subseteq$ U whenever A $\subseteq$ U and U is g*open.
Since A is open, int(A)=A.
Since A is regular closed, cl(int(A))=A.

- ⇨ cl(int(A))=A
- ⇨ cl(A) $\subseteq$ U [since int(A)=A]
- ⇨ A is **g closed.

Theorem 3.1.7. If a subset A of a topological space (X, τ) is both open and **g closed then A is both regular open and regular closed set.
Proof: Let A be both open and **g closed.
Then by the above theorem (4.7)
A is closed, (ie) cl(A)=A

Also as A is open int(A)=A -->(*)
Sub A=cl(A) in eq (*)

- ⇨ int(cl(A))=A
- ⇨ A is regular open

Since A is Closed, cl(A)=A
cl(int(A))=A [since int(A)=A]

- ⇨ A is regular closed.

Theorem 3.1.8. If a subset A of a topological space (X, τ) is both open and **g closed then A is rg closed set.
Proof: If A is both open and **g closed then by the above theorem (3.1.5).
A is closed, (ie) cl(A)=A
Also by the above theorem (3.1.7)
A is both regular open and regular closed
Let $A \subseteq U$ where U is regular open
cl(A) $\subseteq$ U=A

- ⇨ A is rg closed.

Theorem 3.1.9. Let A be a **g closed and suppose that F is closed then $A \cap F$ is **g closed.
Proof: Let A be a **g closed.
TPT : $A \cap F$ is **g closed.
Let F be such that $A \cap F \subseteq U$ where U is g*open.
Since F is closed $A \cap F$ is closed in A.

- ⇨ $cl(A \cap F)= A \cap F \subseteq U$
- ⇨ $cl(A \cap F) \subseteq U$
- ⇨ $A \cap F$ is **g closed.

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