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Two New Algebraic Properties Defined Over Intuitionistic Fuzzy Sets

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Abstract. The concept of intuitionistic fuzzy sets as modified fuzzy sets introduced by Atanassov has been researched in many existing literatures. In this paper, two new algebraic properties defined over intuitionistic fuzzy sets were proposed, proved and verified by numerical example. We also introduced the concept of complementarity of two intuitionistic fuzzy sets and gave a proposition with a proof based on the concept.

Keywords: Algebraic properties, fuzzy sets, intuitionistic fuzzy sets, operations

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

In [1, 2], intuitionistic fuzzy set (IFS) was introduced as an extension or modification of fuzzy set earlier proposed in [18]. Intuitionistic fuzzy set attracts much attention due to its significant in tackling vagueness or the representation of imperfect knowledge. Volumes of research have been carried out involving the fundamentals and theory of IFSs in [3-17, 19]. In this research article, we review the algebraic properties of IFSs. Two new algebraic properties defined on IFSs will be proposed, proved, and exemplified numerically. Also, we introduce the concept of complementarity in IFSs with a related proposition.

2. Concept of intuitionistic fuzzy sets

Definition 1. [18] Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where$\mu_A(x)$: $X \longrightarrow [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2. [10] Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x) : X \longrightarrow [0,1]$define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e.,$\pi_A(x): X \longrightarrow [0,1]$and $0 \leq \pi_A \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Note:
1. Every fuzzy set is intuitionistic fuzzy set, but the reverse is not true.
2. $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$.

Definition 3. Let A, B be two IFSs in X, we define the following operations;
$A' = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$,
$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$,
$A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle : x \in X\}$,
$A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle : x \in X\}$,
$A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle : x \in X\}$.

Definition 4. Let A be an IFS in X, then B is called the sub-IFS of A denoted by $B \subseteq A$ if $\mu_B(x) \leq \mu_A(x)$ and $\nu_B(x) \geq \nu_A(x)$.

3. Some algebraic properties of intuitionistic fuzzy sets [2, 14]
Let A, B and C be IFSs in X, then the following properties are valid;
Complementary law: $(A')' = A$
Idempotent laws: $(i) A \cup A = A$ $(ii) A \cap A = A$
Commutative laws: (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$ $(iii) A \oplus B = B \oplus A$
$(iv) A \otimes B = B \otimes A$
Associative laws:(i) $(A \cup B) \cup C = A \cup (B \cup C)$$(ii)$ $(A \cap B) \cap C = A \cap (B \cap C)$
$(iii) A \oplus (B \oplus C) = (A \oplus B) \oplus C$ $(iv) A \otimes (B \otimes C) = (A \otimes B) \otimes C$
Distributive laws:
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$(ii) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$(iii) A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$(iv) A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
$(v) A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$ $(vi) A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$
De Morgan's laws:
(i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$
$(iii)(A \oplus B)' = A' \otimes B'$
$(iv)(A \otimes B)' = A' \oplus B'$
Absorption laws: (i) $A \cap (A \cup B) = A$ (ii) $A \cup (A \cap B) = A$.

Note: Distributive laws hold for both right and left distributions. The proofs follow from the basic operations.

4. New algebraic properties of intuitionistic fuzzy sets
Theorem: Let A and B be two sub-IFSs of C in a nonempty set X s.t. $A = B'$and $B = A'$, then

1. $(A' \cup B) \cap (A \cup B') = (A' \cap B') \cup (A \cap B)$

2. $(A' \cap B) \cup (A \cap B') = (A' \cup B') \cap (A \cup B)$

Proof:
The result follows if LHS equals RHS. Since $A = B'$and $B = A'$, then $A' \cup B = B \cup B = B$ i.e. idempotent. Also, $A \cup B' = A \cup A = A$, for the same reason. Then $(A' \cup B) \cap (A \cup B') = B \cap A = A \cap B$ i.e. commutative (LHS).

Again, $A' \cap B' = B \cap A = A \cap B$. Then $(A' \cap B') \cup (A \cap B)$= $(A \cap B) \cup (A \cap B) = A \cap B$ i.e. idempotent (RHS). The result follows.

For the second part, we get $(A' \cap B) \cup (A \cap B') = (B \cap B) \cup (A \cap A) = B \cup A = A \cup B$ (LHS).

Again, $(A' \cup B') \cap (A \cup B) = (B \cup A) \cap (A \cup B) = A \cup B$ (RHS). The result follows.

Example 1.

Let $A = \{(0.5,0.3), (0.6,0.2)\}$, $B = \{(0.3,0.5), (0.2,0.6)\}$ for $X = \{x_1, x_2\}$ and $A, B \in X$.
Then $A' = \{(0.3,0.5), (0.2,0.6)\}$ and $B' = \{(0.5,0.3), (0.6,0.2)\}$
Case 1: $(A' \cup B) \cap (A \cup B') = (A' \cap B') \cup (A \cap B)$
$A' \cup B$=$\{(0.3,0.5), (0.2,0.6)\}$, $A \cup B' = \{(0.5,0.3), (0.6,0.2)\}$
$(A' \cup B) \cap (A \cup B')$=$\{(0.3,0.5), (0.2,0.6)\}$ (LHS)
$A' \cap B'$=$\{(0.3,0.5), (0.2,0.6)\}$, $A \cap B = \{(0.3,0.5), (0.2,0.6)\}$
$(A' \cap B') \cup (A \cap B)$=$\{(0.3,0.5), (0.2,0.6)\}$ (RHS)
Case 2: $(A' \cap B) \cup (A \cap B') = (A' \cup B') \cap (A \cup B)$
$A' \cap B = \{(0.3,0.5), (0.2,0.6)\}$, $A \cap B' = \{(0.5,0.3), (0.6,0.2)\}$
$(A' \cap B) \cup (A \cap B') = \{(0.5,0.3), (0.6,0.2)\}$ (LHS)
$A' \cup B' = \{(0.5,0.3), (0.6,0.2)\}$, $A \cup B = \{(0.5,0.3), (0.6,0.2)\}$
$(A' \cup B') \cap (A \cup B)$=$\{(0.5,0.3), (0.6,0.2)\}$ (RHS)
Note, in both cases, LHS=RHS.

Definition 5. Two sub-IFSs B, C of A are complementary if $B \cap C = \{0\}$ and $B \oplus C = A$.

Proposition 1. Let B, C be two sub-IFSs of A. then B, C are complementary sub-IFSs if and only if each $\alpha \in A$ can be written as $\alpha = \beta + \gamma$ with $\beta \in B$ and $\gamma \in C$ for α, β, $\gamma \in [0,1]$.
Proof: Suppose first that B, C are complementary sub-IFSs and let $\alpha \in A$. Then $B \oplus C = A$, so we can find $\beta \in B$ and $\gamma \in C$ with $\alpha = \beta + \gamma$. If we also have $\alpha = \beta' + \gamma'$ with $\beta' \in B$and $\gamma' \in C$, then we have $\beta - \beta' = \gamma' - \gamma$. The left-hand side lies in B and the right-hand side lies in C, and so both sides (being equal) must lies in $B \cap C = \{0\}$. Hence both sides are zero, which means $\beta = \beta'$and $\gamma' = \gamma$, so the expression is unique.

Conversely, suppose that every $\alpha \in A$ can be written uniquely as $\alpha = \beta + \gamma$ with $\beta \in B$ and $\gamma \in C$.Then certainly $B \oplus C = A$. If α is non-zero in $B \cap C$, then in fact α has two distinct expressions $\beta + \gamma$ with $\beta \in B$ and $\gamma \in C$, one with $\beta = \alpha$, $\gamma = 0$ and the other with $\beta = 0, \gamma = \alpha$. Hence $B \cap C = \{0\}$, and B and C are complementary.

REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, 1983.
2. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 20 (1) (1986) 87-96.
3. K.T.Atanassov, Review and new results on intuitionistic fuzzysets, Preprint IM-MFAIS-1- 88, Sofia, 1988.
4. K.T.Atanassov, More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 33 (1) (1989) 37- 46.
5. K.T.Atanassov, Temporal intuitionistic fuzzy sets, *C.R. Acad. Bulgare. Sci.,* 44 (7) (1991) 5-7.
6. K.T.Atanassov, Remark on the intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 51 (1) (1992) 117-118.
7. K.T.Atanassov, New operations defined over intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 61(2) (1994) 137-142.
8. K.T.Atanassov, *Intuitionistic fuzzy sets: theory and application*, Springer-Verlag, 1999.
9. K.T.Atanassov, Intuitionistic fuzzy sets past, present, and future, CLBME-Bulgarian Academy of Science, Sofia, 2003.
10. K.T.Atanassov, *On intuitionistic fuzzy sets*, Springer-Verlag, 2012.
11. L.C.Atanassova, Remark on the cardinality of intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 75 (1995) 399-400.
12. H.Bustince, J.Kacprzyk and V.Mohedano, Intuitionistic fuzzy generators, applications tointuitionistic fuzzy complementation, *Fuzzy Sets and Systems*, 144 (2000) 485-504.
13. S.K.De, R.Biswas and A.R.Roy, Some operations on intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 114 (2000)477-484.
14. P.A.Ejegwa, S.O.Akowe, P.M.Otene and J.M. Ikyule, Anoverview on intuitionistic fuzzy sets, *Int. J. of Scientific & Tech. Research*, 3 (3) (2014) 142-145.
15. L.Huawen, Axiomatic construction for intuitionistic fuzzy sets, *The Journal of Fuzzy Mathematics*, 8 (3) (2000) 645-650.
16. L.Huawen, Difference operation defined over the intuitionistic fuzzy sets, School of Mathematics and System Sciences,Shandong University, Jinan, Shandong 250100, China.
17. A.M.Ibrahim and P.A.Ejegwa, Remark on some operations in intuitionistic fuzzy sets, *Int. Journal of Science and Technology*, 2 (1) (2013) 94-96.
18. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.
19. W.Zeng and H.Li, Note on "Some operations on intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, 157 (2006) 990-991.