

On the Correspondence between Intuitionistic Fuzzy Sets and Democracy

P.A.Ejegwa and J.T.Alabaa

Department of Mathematics, Statistics and Computer Science, University of Agriculture
P.M.B. 2373, Makurdi, Nigeria

Email: ocholohi@gmail.com and ejegwa.augustine@uam.edu.ng

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Abstract. Intuitionistic fuzzy set has proven paramount in fuzzy mathematics due to its ability of tackling uncertainty involve in decision making. In this research, we proposed a correspondence between intuitionistic fuzzy sets and democracy called intuitionistic fuzzy set –democracy model. Delegates’ votes are obtained assuming primary election is conducted for a particular position with five aspirants within a political party. The delegates’ votes are converted into intuitionistic fuzzy values, and from which declaration is made based on the intuitionistic fuzzy values.

Keywords: Correspondence, delegates, democracy, fuzzy sets, intuitionistic fuzzy sets, model, votes

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1. Introduction

Atanassov [1, 2] introduced intuitionistic fuzzy sets (IFSs) as an extension of fuzzy sets earlier proposed by Zadeh in [8]. Intuitionistic fuzzy sets attracted much attention due to its significance in tackling vagueness or uncertainty involve in decision making. Intuitionistic fuzzy set is a tool in modelling real life problems like medical diagnosis, sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc. since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object [7]. Atanassov [3, 4] carried out rigorous research on the theory and applications of intuitionistic fuzzy sets, from which many properties of IFSs are stated and proved respectively. The cardinal aim of this work is to give a connective model of intuitionistic fuzzy sets and democracy.

Definition 1. [8] Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{(x, \mu_A(x)): x \in X\}$, where $\mu_A(x): X \rightarrow [0, 1]$ is the membership function of the fuzzy set A .

Definition 2. [2] Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \rightarrow [0, 1]$ define respectively, the degree of membership and degree of non-membership of the

element $x \in X$ to the set A , which is a subset of X , and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, we have a parameter, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A . $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0, 1]$ i.e., $\pi_A(x): X \rightarrow [0, 1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x \in X$. This parameter expresses the lack of knowledge of whether x belongs to IFS A or not.

Let us quickly note the following important points about IFSs;

1. IFS is an extension of Zadeh's fuzzy set
2. Every fuzzy set is an IFS, but the reverse is not necessarily true
3. $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$

2. Intuitionistic fuzzy sets – democracy model

The idea of intuitionistic fuzzy sets in electoral system was introduced in [2] and elaborated by Ejegwa, *et al.*[5] but here, we present a step by step model of it using the principle proposed by Renčová [6]. Let $C = \{c_1, c_2, c_3, c_4, c_5\}$ be the set of all candidates, X be the set of all delegates (i.e. hundred each from five provinces), $F(x)$ be the number of delegates that voted for, $A(x)$ be the number of delegates that voted against, and $U(x)$ be the number of delegates that remained undecided or cast invalid votes for every $x \in X$. If n is the total number of delegates from each province (i.e. $n = F(x) + A(x) + U(x)$), we get

$$\mu(x) = \frac{F(x)}{n} \tag{1}$$

$$\nu(x) = \frac{A(x)}{n} \tag{2}$$

Similarly, $\pi(x) = 1 - \frac{F(x)}{n} - \frac{A(x)}{n}$,

$$\text{Implies, } \pi(x) = \frac{n - F(x) - A(x)}{n} \tag{3}$$

Adding (1) and (2), we get $\mu(x) + \nu(x) = \frac{F(x)}{n} + \frac{A(x)}{n}$, but $\mu_A(x) + \nu_A(x) \leq 1$ from def. 2, $\frac{F(x)}{n} + \frac{A(x)}{n} \leq 1$, and yields

$$F(x) + A(x) \leq n \tag{4}$$

and

$$U(x) = n - F(x) - A(x) \tag{5}$$

We assume after the polling process, we obtained the results in Table 1, and converted the results into intuitionistic fuzzy values which include membership degree $\mu(x)$, non-

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membership degree $\nu(x)$, and hesitation margin $\pi(x)$ as in Table 2 using the set of equations, 1, 2, and 3 respectively.

Table 1:

	$F(x)$	$A(x)$	$U(x)$	n
c_1	50	40	10	100
c_2	60	35	05	100
c_3	40	50	10	100
c_4	35	60	05	100
c_5	55	44	01	100
Total	240	229	31	500

Table 2:

	Choices
c_1	$\langle 0.50, 0.40, 0.10 \rangle$
c_2	$\langle 0.60, 0.35, 0.05 \rangle$
c_3	$\langle 0.40, 0.50, 0.10 \rangle$
c_4	$\langle 0.35, 0.60, 0.05 \rangle$
c_5	$\langle 0.55, 0.44, 0.01 \rangle$

From Table 2 above, $\mu(x) = 0.5$, $\nu(x) = 0.4$, and $\pi(x) = 0.1$ for c_1 , likewise for c_2 , c_3 , c_4 , and c_5 respectively.

3. Declaration and conclusion

From table 2 above, the second candidate won the primary election with membership degree of 0.6. The interpretation suggests that, intuitionistic fuzzy set is a convenient and appropriate tool in democracy because it involves three parameters (i.e. membership function, non-membership function, and hesitation margin) which are integral components in decision making.

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