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Metric in Fuzzy Labeling Graph

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Abstract. In this paper the concept of metric is applied to fuzzy labeling graph. Some results related with μ-length, eccentricity, diameter and radius of fuzzy labeling graph G have been derived. It has been proved that, the center of G is a cut node of G and if G* is complete, then there exist only one center. And a necessary condition for a graph G to have more than one center is given. Some relation between diametrical nodes and eccentric nodes is also given.

Keywords: μ-length, eccentricity, diameter, radius, center, fuzzy labeling graph

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

In 1965, Zadeh [10] introduced the modern concept of uncertainty by fuzzy set through the publication of a seminal paper. A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set.

The fuzzy graph introduced by Rosenfield [6] using fuzzy relation, represents the relationship between the objects by preciously indicating the level of the relationship between the objects of the function set. Also he coined many fuzzy analogous graph theoretic concepts like bridge, cut vertex and tree. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. In the same year Yeh and Bang [9], published a paper which contains fuzzy relation, fuzzy graph and their application in cluster analysis. Bhattacharya [1] introduced the notion of centre in fuzzy graph. Sunitha and Vijayakumar introduced the notion of a self centered fuzzy graph in [7,8]. Nagoorgani and Chandrasekar [2] discussed multiple properties of fuzzy graphs in their book entitled "A First Look at Fuzzy Graph Theory". Ramakrishanan and Laxmi [4] introduced the concept of strong and super strong vertex in fuzzy graph. For other works on fuzzy graphs see [12-17].

In this paper section 1 deal with μ- distances related results and section 2 deals with some centre properties of fuzzy labeling graph. Section 3 contains the some relationship between diametrical nodes and eccentric nodes.

2. Preliminaries

We first introduce some notations and recall some basics about fuzzy graph and fuzzy labeling graph. Let U and V be two sets. Then ρ is said to be a *fuzzy relation* [10] from U into V if ρ is a fuzzy set of U×V.A *fuzzy graph* G = (σ, μ) is a pair of functions σ: V → [0, 1] and μ: V×V → [0, 1], where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$. A *path* P in a fuzzy graph is a sequence of distinct nodes $v_1, v_2, ..., v_n$ such that $\mu(v_i, v_{i+1}) > 0$; $1 \leq i \leq n$; here $n \geq 1$ is called the *length of the path* P. The consecutive pairs (v_i, v_{i+1}) are called the *edge of the path*. A path P is called *a cycle* if $v_1=v_n$ and $n \geq 3$. The *strength* of a path P is defined to be the weight of the weakest arc of the path. Let G :(σ, μ) be a fuzzy graph. The *strength of connectedness* between two vertices u and v is $\mu^{\infty}(u,v)=\sup\{\mu^k(u,v)/k=1,2,\ldots\}$ where $\mu^k(u,v)= \sup\{\mu(uu_1)$ □ $\mu(u_1u_2)$□…. □ $\mu(u_{k-1}v)$ / $u_1,\ldots,u_{k-1} \in V\}$. A *strongest path* joining any two nodes x, y has strength $\mu^{\infty}(x, y)$; sometimes this is referred as connectedness between the nodes. An arc of a fuzzy graph is called *strong* if its weight is at least as great as the strength of the connectedness of its end nodes when it is deleted. An edge is called a *fuzzy bridge* of G if its removal reduces the strength of connectedness between some pair of nodes in G. A node is a *fuzzy cut node* of G = (σ, μ) if removal of it reduces the strength of connectedness between some other pair of nodes. A node u of G is called the *end node* if it has at most one strong neighbor in G.

A graph G = (σ, μ) is said to be a *fuzzy labeling graph*, if σ: V → [0, 1] and μ: V×V → [0, 1] is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all u, v ∈ V. Let G = (σ, μ) be a fuzzy graph and let $v \in \sigma^*$. v is called a *super strong vertex* if $\mu^{\infty}(v,x)= \alpha$ for every $x (\neq v) \in \sigma^*$ and for some $\alpha \in (0,1]$.

An elegant definition of a metric in a fuzzy graph has been given in Rosenfeld (1975). If ρ is the path consisting of the vertices $x_0, x_1, \ldots, x_n$ in a fuzzy graph G= (σ, μ), the *μ- length* of ρ is defined by $l(\rho) = \sum_{i=1}^{n} \mu(x_{i-1}, x_i)^{-1}$ if n=0 then $l(\rho)$ is chosen to be 0. Now, for two vertices x, y in G, the *μ- distance* δ(x, y) is defined to be the minimum of the μ- length of all paths joining x and y. It has been shown that δ is metric in [5]. Now we introduce some basic definitions in metric. Suppose G= (σ, μ) is a fuzzy graph with V as the set of vertices. The *eccentricity* e(v) of a vertex v ∈V is defined to be the maximum of all the μ- distances δ(v, w) for all w in V. A *center* of a connected fuzzy graph is a vertex whose eccentricity is the minimum. The *radius* of a connected fuzzy graph is the minimum of all eccentricities of the vertices of the fuzzy graph. An *eccentric node* v, is a node v* such that e(v) = δ(v, v*). A node v is called a *diametrical node* if e(v) = d(G).

3. μ- distance in fuzzy labeling graph

Theorem 3.1. If G is a fuzzy labeling graph, such that μ (x, y) ∈ G is a bridge of G, then $\delta(x, y) = \frac{1}{\mu(x,y)}$.

Proof: Let $\mu_1(x, y)$ be a bridge of G and $\mu_2, \mu_3\ldots$ be the strength of the paths joining x and y. And let $\mu_{r_1}, \mu_{r_2} \ldots \mu_{r_n}$ be the weights of the remaining edges in G.

Now $\mu^{\infty}(x, y)$ = Max $\{\mu_1(x, y)$□μ_2□ ... □$\mu_n\}$

$\Rightarrow \mu_1(x, y) > \mu_2 > \ldots > \mu_n$.

$\Rightarrow l(\mu_1) > l(\mu_2) > \ldots > l(\mu_n)$.

$$\Rightarrow \frac{1}{\mu_1(x,y)} < \frac{1}{\mu_2} < \ldots < \frac{1}{\mu_n} \tag{3.1}$$

$\therefore \delta(x, y) = \text{Min}\{\frac{1}{\mu_1(x,y)}, \frac{1}{\mu_2} + \frac{1}{\mu_{r_1}} + \ldots + \frac{1}{\mu_{r_n}}, \frac{1}{\mu_3} + \frac{1}{\mu_{r_1}} + \cdots + \frac{1}{\mu_{r_n}}, \ldots, \frac{1}{\mu_n} + \frac{1}{\mu_{r_1}} + \cdots + \frac{1}{\mu_{r_n}}\}$

$= \frac{1}{\mu_1(x,y)}$ {by (3.1)}

Remark 3.2. Converse of the above proposition is not true.

Example 3.3. Consider a triangle (i.e) V = {u, v, w} such that $\mu(u, v) = 0.02$, $\mu(v, w) = 0.03$ and $\mu(w, u) = 0.04$. Here $\delta(u, v) = 50$, but (u, v) is not bridge of G.

Theorem 3.4. If G is a fuzzy labeling graph, such that G* is a cycle, then $\delta(x, y) = \frac{1}{\mu(x,\ y)}$ for all x, y ∈ V x V.

Proof: If (x, y) ∈ V x V, then the μ-distance between the nodes x and y has only two paths.

(i.e.) ρ_1 : x, y.

ρ_2 : x, u, v, w,…y.

$\therefore \delta(x, y) = \text{Min}\{ l(\rho_1), l(\rho_2)\}$

$= \text{Min}\{ \frac{1}{\mu(x,y)}, \sum_i \frac{1}{\mu_i' s}\}$

$= \frac{1}{\mu(x,y)}$

Observation 3.5. The maximum μ-length of a fuzzy labeling graph G is unique.

Proposition 3.6. If the maximum μ-length of a graph G exists between the nodes u and v, then there exist a strongest path between the nodes u and v.

Proof: It is proved by constructing a graph G such that G* is a cycle with 5 nodes v_1,v_2,v_3,v_4,v_5 and assume that $\mu_1(v_1,v_2) < \mu_2(v_2,v_3) < \mu_3(v_3,v_4) < \mu_4(v_4,v_5) < \mu_5(v_5,v_1)$. By observation 1.5 there exist a unique maximum μ-length of G between the nodes (v_2, v_5). Since between v_2 and v_5 there exist two disjoint path ρ_1 : v_2, v_3, v_4, v_5 and ρ_2 : v_2, v_1, v_5.

$\therefore \delta(v_2,v_5) = \text{Min}\{l(\rho_1) \text{ and } l(\rho_2)\}$

$= l(\rho_2)$

Since $\frac{1}{\mu_1(v_1,v_2)} > \frac{1}{\mu_2(v_2,v_3)} > \frac{1}{\mu_3(v_3,v_4)} > \frac{1}{\mu_4(v_4,v_5)} > \frac{1}{\mu_5(v_5,v_1)}$

$\Rightarrow \frac{1}{\mu_1(v_1,v_2)} + \frac{1}{\mu_5(v_5,v_1)} > \frac{1}{\mu_2(v_2,v_3)} + \frac{1}{\mu_3(v_3,v_4)} + \frac{1}{\mu_4(v_4,v_5)}$

Now the connectedness of G between v_2,v_5 is given by

$\mu^\infty(v_2,v_5) = \text{Max}\{\mu_1, \mu_2\} = \mu_2$

Therefore, there exist a strongest path between the nodes of v_2 and v_5. Hence the proof

Remark 3.7. Even σ and μ are bijective, the μ-length $\delta(x_i, x_{i+1})$ need not be unique. This is verified by the following example.

Example 3.8.

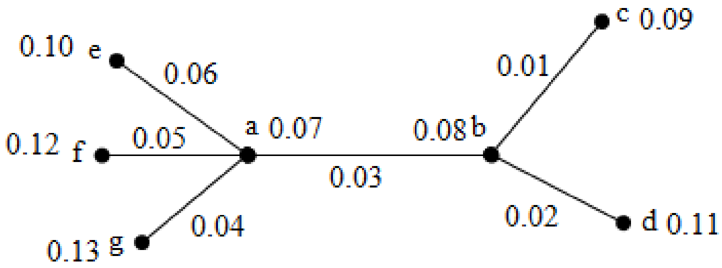

Figure 3.1: A fuzzy labeling graph

In fig: 3.1, δ (a, b) = 33.33, δ (a, c) = 133.33, δ (a, d) = 83.33, δ (a, e) = 16.67, δ (a, f) = 20, δ (a, g) = 25, δ (b, c) = 100, δ (b, d) = 50, δ (b, e) = 50, δ (b, f) = 53.33, δ (b, g) = 58.33, δ (c, d) = 150, δ (c, e) = 150, δ (c, f) = 153.33, δ (c, g) = 158.33, δ (d, e) = 100, δ (d, f) = 103.33, δ (d, g) = 108.33, δ (e, f) = 36.67, δ (e, g) = 41.67, δ (f, g) = 45. Here δ (b, d) = δ (b, e) and δ (c, d) = δ (c, e). But in example 1.3, every μ-length of G is unique.

Proposition 3.9. Let G be a fuzzy labeling graph such that G* is a cycle. If {e(x), e(y)} = diameter of G, then there exist a weakest arc of G between the nodes of x and y.

Proof: Let diameter G = {e(x), e(y)}, it is trivial that there exist two disjoint path between the nodes of x and y (i.e.) ρ_1, $\rho_2$by proposition 1.6. If ρ_1 is the strongest path, then $\rho_2$is a path which contains the weakest arc. Since, there exist only one weakest arc if G* is a cycle in a fuzzy labeling graph.

Theorem 3.10. Let G be a fuzzy labeling graph, if μ(x, y) ∈ V x V such that μ(x, y) < μ_i's then max {e(x), e(y)}= diameter of G.

Proof: Construct a graph G with four vertices x, y, u, v and assume that μ(x,y) < μ(y,v) < μ(v,u) < μ(u,x) < μ(x,v). Here μ(x, y) < μ_i's where i= 1,2,3.

Now δ (x,y) = Min {$l(\rho_1)$, $l(\rho_2)$, $l(\rho_3)$} = $l(\rho_2)$, where, ρ_1 : x, y ρ_2 : x, v, y ρ_3 : x, u, v, y.

$l(\rho_1) = \frac{1}{\mu(x,y)}$, $l(\rho_2) = \frac{1}{\mu(x,v)} + \frac{1}{\mu(v,y)}$

$l(\rho_3) = \frac{1}{\mu(x,u)} + \frac{1}{\mu(u,v)} + \frac{1}{\mu(v,y)}$

Consider ρ_1 and ρ_2

$\because \mu(x,y) < \mu(y,v) < \mu(v,x)$

$\Rightarrow \frac{1}{\mu(x,y)} > \frac{1}{\mu(y,v)} > \frac{1}{\mu(v,x)}$

$$\Rightarrow \frac{1}{\mu(x,y)} \geq \frac{1}{\mu(y,v)} + \frac{1}{\mu(v,x)} \tag{3.2}$$

Consider ρ_2 and ρ_3

$\because \mu(u,v) < \mu(x,u) < \mu(x,v)$ (common arcs were omitted on both sides)

$\Rightarrow \frac{1}{\mu(u,v)} > \frac{1}{\mu(x,u)} > \frac{1}{\mu(x,v)}$

$$\Rightarrow \frac{1}{\mu(x,u)} + \frac{1}{\mu(u,v)} \geq \frac{1}{\mu(x,v)} \tag{3.3}$$

From (3.2) and (3.3)

δ (x,y) = $l(\rho_2)$.
Similarly δ (u,v) = Min {$l(\rho_4)$, $l(\rho_5)$, $l(\rho_6)$} = $l(\rho_6)$.
δ (u,v) = Min {$l(\rho_7)$, $l(\rho_8)$, $l(\rho_9)$, $l(\rho_{10})$} = $l(\rho_7)$, where ρ_4 : u, v ρ_5 : v, x, u ρ_6 : v, y, x, u.

ρ_7 : y, v, u ρ_8 : y, x, u ρ_9 : y, v, x, u ρ_{10} : y, x, v, u.
and δ (x, v) = $\frac{1}{\mu(x,v)}$, δ (x, u) = $\frac{1}{\mu(x,u)}$, δ (y, v) = $\frac{1}{\mu(y,v)}$ ∵ (x,v), (x, u), (y, v) are bridges of G. Here e(x) = $l(\rho_2)$

$\because\ \mu(x,v) > \mu(u,x) > \mu(y,v)$
$\Rightarrow \frac{1}{\mu(x,v)} < \frac{1}{\mu(u,x)} < \frac{1}{\mu(y,v)}$
$\Rightarrow \frac{1}{\mu(u,x)} \leq \frac{1}{\mu(y,v)} + \frac{1}{\mu(v,x)}$
Similarly e(y) = $l(\rho_7)$ = e(u), e(v) = $\frac{1}{\mu(y,v)}$

Therefore Max {e(x), e(y)}= Max{$l(\rho_2)$, $l(\rho_7)$} = $l(\rho_7)$ = diameter of G
$\because \mu(x,v) > \mu(v,u) > \mu(y,v)$
$\Rightarrow \frac{1}{\mu(x,v)} < \frac{1}{\mu(v,u)} < \frac{1}{\mu(y,v)}$
$\Rightarrow \frac{1}{\mu(v,u)} \geq \frac{1}{\mu(x,v)} + \frac{1}{\mu(y,v)}$.

Remark 3.11. The converse of the above theorem is true.

Example 3.12.

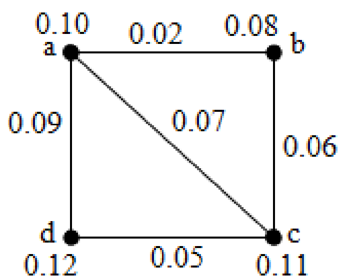

Figure 3.2. Connected FLG

In the above fig.3.2, δ (a, b) = 30.95, δ (a, c) = 14.29, δ (a, d) = 11.11, δ (b, c) = 16.67, δ (b, d) = 36.67, δ (c, d) = 20, e (a) = 30.95, e (b) = 36.67, e (c) = 20, e (d) = 36.67, d (G) = 36.67. Here μ(a, b) < μ_i's. Therefore Max {e (a), e (b)} = {30.95, 36.67}= 36.67.

Note 3.13. The above theorem is not true if G* is a cycle.

In fig. 3.3, e (a) = 95, e (b) = 126.79, e (c) = 132.5, e (d) = 130.96, e (e) = 126.79, e (f) = 132.5, e (g) = 130.96, e (h) = 107.5. Here μ(d, e) < μ_i's. But Max {e (d), e (e)} = {130.96, 126.79}= 130.96 ≠ d(G) = 132.5.

Example 3.14.

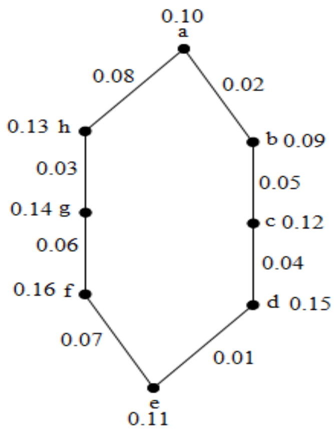

Figure 3.3:

4. Center in fuzzy labeling graph

Theorem 4.1. If x G be the center of G, then r (G) = ——— for all μ (x, y_i) > 0.

Proof: Consider a fuzzy labeling graph G such that G* is a cycle with three vertices x_1, x_2, x_3 and assume that $\mu (x_1, x_2) < \mu (x_2, x_3) < \mu (x_3, x_1)$

$\mu (x_1, x_2)$ is the least and x_3 is the center of G.

r (G) = e (x_3)

= Max [$\delta (x_3, x_1)$, $\delta (x_3, x_2)$]

= Max ——— ———

= ——— ——— ———

= ——— for all > 0, i = 1,2.

Note 4.2. The above condition is not necessary for a node to be the center.

Example 4.3.

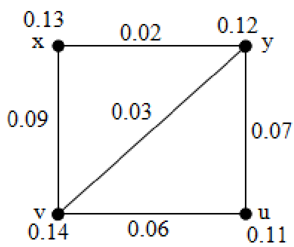

Figure 4.1:

In the above graph, Fig.4.1, e (x) = 42.06, e (y) = 42.06, e (u) = 27.78, e (v) = 30.95 and 'u' is the center with radius 27.78 $\neq$ ———

Theorem 4.4. The fuzzy labeling graph G has exactly only one center, if G* is complete.

Proof: Let G be a fuzzy labeling graph such that G* is complete with four vertices v_1,v_2,v_3,v_4 and assume that $\mu(v_1,v_2) < \mu(v_2,v_3) < \mu(v_3,v_4) < \mu(v_4,v_1) < \mu(v_1,v_3) < \mu(v_2,v_4)$.

Consider,

$$\delta(v_1,v_2) = \text{Min}\{l(\rho_1), l(\rho_2), l(\rho_3), l(\rho_4), l(\rho_5)\} = l(\rho_3)$$

where, $\rho_1 : v_1,v_2$ $\rho_2 : v_1,v_3,v_2$ $\rho_3 : v_1,v_4,v_2$ $\rho_4 : v_1,v_4,v_3,v_2$ $\rho_5 : v_1,v_3,v_4,v_2$

$l(\rho_1) = \frac{1}{\mu(v_1,v_2)}$, $l(\rho_2) = \frac{1}{\mu(v_1,v_3)} + \frac{1}{\mu(v_3,v_2)}$

$l(\rho_3) = \frac{1}{\mu(v_1,v_4)} + \frac{1}{\mu(v_4,v_2)}$

$l(\rho_4) = \frac{1}{\mu(v_1,v_4)} + \frac{1}{\mu(v_4,v_3)} + \frac{1}{\mu(v_3,v_2)}$

$l(\rho_5) = \frac{1}{\mu(v_1,v_3)} + \frac{1}{\mu(v_3,v_4)} + \frac{1}{\mu(v_4,v_2)}$

Consider ρ_1 and ρ_3

$\because \mu(v_1,v_2) < \mu(v_1,v_4) < \mu(v_4,v_2)$

$\Rightarrow \frac{1}{\mu(v_1,v_2)} > \frac{1}{\mu(v_1,v_4)} > \frac{1}{\mu(v_4,v_2)}$

$\Rightarrow \frac{1}{\mu(v_1,v_2)} \geq \frac{1}{\mu(v_1,v_4)} + \frac{1}{\mu(v_4,v_2)}$ (4.1)

Consider ρ_2 and ρ_3

$\because \mu(v_2,v_3) < \mu(v_1,v_3) < \mu(v_1,v_4) < \mu(v_2,v_4)$

$\Rightarrow \frac{1}{\mu(v_2,v_3)} > \frac{1}{\mu(v_1,v_3)} > \frac{1}{\mu(v_1,v_4)} > \frac{1}{\mu(v_2,v_4)}$

$\Rightarrow \frac{1}{\mu(v_2,v_3)} + \frac{1}{\mu(v_1,v_3)} \geq \frac{1}{\mu(v_1,v_4)} + \frac{1}{\mu(v_2,v_4)}$ (4.2)

Consider ρ_4 and ρ_3

$\because \mu(v_4,v_2) > \mu(v_4,v_3) > \mu(v_3,v_2)$ [$\because \mu(v_1,v_4)$ is common in both paths, which is omitted]

$\Rightarrow \frac{1}{\mu(v_4,v_2)} < \frac{1}{\mu(v_4,v_3)} < \frac{1}{\mu(v_3,v_2)}$

$\Rightarrow \frac{1}{\mu(v_4,v_2)} \leq \frac{1}{\mu(v_4,v_3)} + \frac{1}{\mu(v_3,v_2)}$ (4.3)

Consider ρ_3 and ρ_5

It is obvious that $l(\rho_5) > l(\rho_3)$ (4.4)

From (4.1), (4.2), (4.3) and (4.4)

$\delta(v_1,v_2) = l(\rho_3)$

By our assumption $\delta(v_1,v_3) = \frac{1}{\mu(v_1,v_3)}$, $\delta(v_1,v_4) = \frac{1}{\mu(v_1,v_4)}$, $\delta(v_2,v_3) = \frac{1}{\mu(v_2,v_3)}$,

$\delta(v_2,v_4) = \frac{1}{\mu(v_2,v_4)}$, $\delta(v_3,v_4) = \frac{1}{\mu(v_3,v_4)}$

Now the eccentricity $e(v_1) = l(\rho_3)$ $\because l(\rho_3) > \frac{1}{\mu(v_1,v_4)} > \frac{1}{\mu(v_1,v_3)}$

Similarly $e(v_2) = l(\rho_3)$

and $e(v_3) = \frac{1}{\mu(v_2,v_3)}$ $\because \frac{1}{\mu(v_2,v_3)} > \frac{1}{\mu(v_3,v_4)} > \frac{1}{\mu(v_1,v_3)}$

$e(v_4) = \frac{1}{\mu(v_3,v_4)}$ $\because \frac{1}{\mu(v_3,v_4)} > \frac{1}{\mu(v_1,v_4)} > \frac{l}{\mu(v_2,v_4)}$

$\therefore$ Radius of G = $e(v_4)$ $\because \frac{1}{\mu(v_3,v_4)} < \frac{1}{\mu(v_2,v_3)} < l(\rho_3)$

$\Rightarrow$Center of G = v_4

Here the proposition is proved.

Remark 4.5. The above result is not true, If G* is a cycle.
Example 4.6.

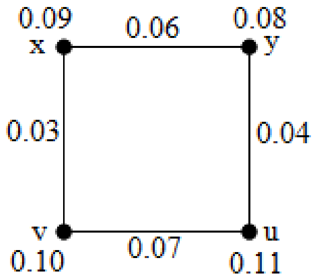

Figure 4.2:

The above graph G in fig.4.2 with four vertices has two center's 'y' and 'v'.

Theorem 4.7. If $x \in G$ is the center of G, then it is a cut node of G and the converse is not true.
Proof: By the proof of theorem 4.4, v_4 in the center of G and by the definition of cut node v_1 and v_4 are cut nodes of G. conversely, v_1 and v_4 are cut nodes of G, but v_1 is not a center of G.

Proposition 4.8. Removal of a center reduces the strength of connectedness between the nodes.
Proof: By theorem 4.7, center of G is a cut node of G. Therefore removal of a center reduces the strength of a connectedness between the nodes.

Proposition 4.9. If $x \in G$ is the center of a fuzzy labeling graph, then it G, then it is not super strong vertex of G.
Proof: If 'x' is the center of a fuzzy labeling graph G, then by theorem 4.7, 'x' is a cut node of G.
By proposition 3.11 [3] 'x' is not a super strong vertex of G.

5. Diametrical and eccentric nodes in FLG

Proposition 5.1. If each node of a fuzzy labeling graph G is eccentric, then it has two centers.

Remark 5.2. The above condition is not sufficient. In the following example, the graph G has two centers but each node is not eccentric.

Proposition 5.4. If G is a fuzzy labeling graph with exactly only one center x □ G, then x is not an eccentric node of G.

In Fig. 5.1 e (a) = 42.07= e (e), e (v) = 26.79 = e (x), e(w) = 30.96 and
a* = c, e, b* = d, c* = -, d*= b, e* = a

Example 5.3.

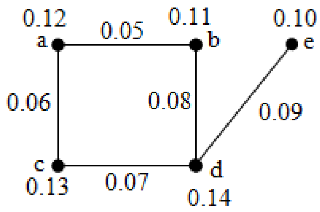

Figure 5.1:

Proposition 5.5. If G is a fuzzy labeling graph then the diametrical nodes of G are eccentric nodes of G.

Example 5.6.

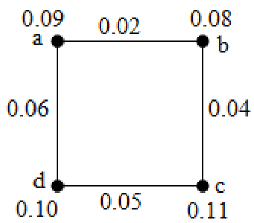

Figure 5.2:

Here, δ (a, b) = 50, δ (a, c) = 36.67, δ (a, d) = 16.67, δ (b, c) = 25, δ (b, d) = 45, δ (c, d) = 20, e (a) = 50 = e (b) , e (c) = 36.67, e (d) = 45, d (G) = 50. Center c (G) = ‘c’ and a* = c, b, b* = a, d. In fig. 7 G has exactly only one center ‘c’ which is not eccentric. The diametrical nodes of G are ‘a’ and ‘b’ which are the eccentric nodes of G.

Proposition 5.7. If G is a fuzzy labeling graph such that G* is a tree, then the diametrical nodes are the only eccentric nodes in G.

Example 5.8.

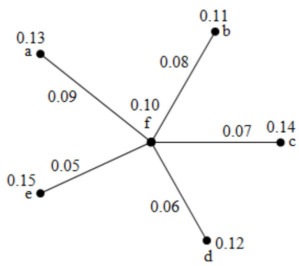

Figure 5.3:

In fig. 5.3, ‘f’ is the center of G and the diametrical nodes of G are‘d’ and ‘e’ which are the only eccentric nodes of G. (i.e) δ (a, f) = 11.11, δ (a, b) = 23.61, δ (a, c) = 25.37, δ (a, d) = 27.78, δ (a, e) = 31.11, δ (b, c) = 26.79, δ (b, d) = 29.17, δ (b, e) = 32.5, δ (b, f) = 12.5, δ (c, d) = 30.95, δ (c, e) = 34.29, δ (c, f) = 14.29, δ (d, e) = 36.67, δ (d, f) = 16.67, δ (e, f) = 20. And e(a) = 31.11, e(b) = 32.5, e(c) = 34.29, e(d) = 36.67= e(e), e(f) = 20. d(G) = 36.67. Therefore e* = a, b, c, d, f d* = e.

Proposition 5.9. If G is a fuzzy labeling graph such that G* is a path then the end nodes of G are the diametrical nodes of G.

Proof: Consider a path v_1-v_i. As G* is a path, the end nodes of G are-v_1 and v_i. Now it is trivial that $\delta(v_{1,}v_i)$ will have the maximum μ- length, since G* is a path. And since μ is symmetric $\delta(v_{1,}v_i) = \delta(v_{i,}v_1)$. By the definition of eccentricity e(v) and e (v_i) will have the maximum eccentricity and are same. Hence v and v_i are the diametrical nodes of G.

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