Intern. J. Fuzzy Mathematical Archive
Vol. 5, No. 2, 2014, 95-102
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 26 December 2014
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

Decompositions of rg-Continuity via Idealization

***K. Indirani*[1], *V.Rajendran*[2], *M. Meharin*[3] and *O. Ravi*[4]**

[1]Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu India. e-mail: indirani009@ymail.com

[2]Department of Mathematics, KSG College, Coimbatore, Tamil Nadu, India e-mail: mathsraj05@yahoo.co.in

[3]School of Youth Empowerment, Madurai Kamaraj University, Madurai, Tamil Nadu India. e-mail: smms252829@gmail.com

[4]Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District Tamil Nadu, India. e-mail: siingam@yahoo.com

Received 21 November 2014; accepted 20 December 2014

Abstract. In this paper, we introduce the notions of rαg-I-open sets, gpr-I-open sets, C_r-I-sets and C_r^*-I-sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of rg-continuity.

Keywords: rαg-I-open sets, gpr-I-open sets, C_r-I-sets and C_r^*-I-sets

AMS Mathematics Subject Classification (2010): 14P25

1. Introduction and preliminaries

In 1970, Levine [11] initiated the study of so called g-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandranet.al in 1991 [2]. In 1993, Palaniappan and Rao [17] introduced the notions of regular generalized closed (rg-closed) sets and rg-continuity in topological spaces. In 2000, Sundaram and Rajamani [20] obtained three different decompositions of rg-continuity by providing two types of weaker forms of continuity, namely C_r -continuity and C_r^* -continuity. Recently, Noiriet. al. [15] introduced the notions of αg-I-open sets, gp-I-open sets, gs-I-open sets, C(S)-I-sets, C^*-I-sets and S*-I-sets to obtain three different decompositions of g-continuity via idealization. Recently Ravi et. al. [18] obtained three different decompositions of rg-continuity. In this paper, we introduce the notions of rαg-I-open sets, gpr-I-open sets, C_r-I-sets and C_r^* -I-sets to obtain further decompositions of rg-continuity.

Let (X, τ) be a topological space. An ideal is defined as a nonempty collection Iof subsets of X satisfying the following two conditions:

(i) If A ∈ I and B ⊆A, then B ∈I

ii) If A ∈ I and B ∈ I, then A ∪B∈I.

For a subset A ⊆X, A*(I) = {x∈X/U∩A∉I for each neighborhood U of x} is called the local function of A with respect to I and τ [9]. We simply write A* instead of A*(I) in case there is no chance for confusion. X* is often a proper subset of X. For every ideal topological space (X, τ, I) there exists a topology τ*(I), finer than τ, generated by β(I,τ) =

{U\I:U∈τand I∈I }, but in general β(I,τ) is notal ways a topology [22]. Also, cl*(A) = A∪A*defines a Kuratowski closure operator for τ*(I) [22]. Additionally, cl*(A) ⊆cl (A) for any subset A of X [7]. Throughout this paper, X denotes the ideal topological space (X,τ,I) and also cl(A) and int(A)denote the closure of A and the interior of A in (X; τ), respectively.

Definition 1.1. A subset A of (X,τ) is said to be
(1) α-open [13] if A ⊆int(cl(int(A))),
(2) preopen [12] if A ⊆int(cl(A)),
(3) regular open [19] if A = int(cl(A)),
(4) rg-open [17] iff F ⊆int (A) whenever F ⊆A and F is regular closed in (X,τ),
(5) gpr-open [4] iff F⊆pint(A) whenever F ⊆A and F is regular closed in (X,τ),
(6) rαg-open [18] iff F⊆αint(A) whenever F ⊆A and F is regular closed in (X,τ),
(7) a t-set [21] if int(A) = int(cl(A)),
(8) anα^*-set [6] if int(A) = int(cl(int(A))),
(9) a Cr-set [18] if A = U ∩V, where U is rg-open and V is a t-set in (X, τ),
(10) aC_r^* -set [18] if A = U ∩ V, where U is rg-open and V is an α^*-set in (X, τ).

The complements of the above mentioned open sets are called their respective closed sets. The pre interior pint(A) (resp. α-interior, αint(A)) of A is the union of all preopen sets (resp. α-open sets) contained in A. The α-closure αcl(A) of A is the intersection of all α-closed sets containing A.

Lemma 1.2. [1] If A is a subset of X, then
(1) pint(A) = A ∩ int(cl(A)),
(2) αint(A) = A ∩ int(cl(int(A))) and αcl(A) = A∪ cl(int(cl(A))).

Definition 1.3. A subset A of an ideal topological space (X,τ, I) is said to be
(1) pre-I-open [3] if A ⊆int(cl*(A)),
(2) α-I-open [5] if A ⊆int(cl*(int(A))),
(3) a t-I-set [5] if int(cl*(A)) = int(A),
(4) an α^*-I-set [5] if int(cl*(int(A))) = int(A).

Also, we have α-I-int(A) = A ∩int(cl*(int(A))) [15] and p-I-int(A) = A ∩int(cl* (A)) [15], where α-I-int(A) denotes the α-I interior of A in (X, τ, I) which is the union of all α-I-open sets of (X,τ,I) contained in A. p-I-int(A) has similar meaning
.

Remark 1.4. The following hold in a topological space.
(1) Every rg-open set is gpr-open but not conversely [4]
2) Every rg-open set is rαg-open but not conversely [18]

2. rαg-I-open sets and gpr-I-open sets

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is called
(1) rαg-I-open if F⊆α -I-int(A) whenever F ⊆A and F is regular closed in X.
(2) gpr-I-open if F⊆ p-I-int(A) whenever F⊆ A and F is regular closed in X.

Proposition 2.2. For a subset of an ideal topological space, the following hold:
(1) Every rαg-I-open set is rαg-open.

(2) Every gpr-I-open set is gpr-open.

(3) Every rαg-open set is gpr-open.

Proof: (1) Let A be an rαg-I-open set. Let F⊆A and F is regular closed in X. Then, F⊆α -I-int(A) = A ∩(int(cl*(int(A)))) ⊆A∩int(cl(int(A))) = αint(A). This shows that A is rαg-open.

(2) Let A be gpr-I-open set. Let F ⊆A and F is regular closed in X. Then, F⊆ p-I-int(A) = A ∩int(cl*(A))⊆ A∩int(cl(A)) = pint(A). This shows that A isgpr-open.

(3) It follows from the definitions.

Remark 2.3. The converses of Proposition 2.2 are not true, in general.

Example 2.4. Let X = {a, b, c, d},τ = {∅, X, {a}, {b}, {a,b},{b,c},{a,b,c}} and I ={∅, {b}}. Then {a, b, d} is rαg-open but not an rαg-I-open set.

Example 2.5. In Example 2.4, {a, b, d} is gpr-open but not a gpr-I-open set.

Example 2.6. Let X= f{a, b, c, d} and τ= {∅, {d}, {a,c},{a,c,d},X}. Then {b, c, d} is gpr-open set but not an rαg-open.

Proposition 2.7. For a subset of an ideal topological space, the following hold:

(1) Every rαg-I-open set is gpr-I-open.

(2) Every rg-open set is gpr-I-open.

(3) Every rg-open set is rαg-I-open.

Proof: (1) Let A be rαg-I-open. Then, for any regular closed set F with F⊆A, we have F ⊆α-I-int(A) = A ∩int(cl*(int(A)))⊆ A ∩int(cl*(A)) = p-I-int(A) which implies that A is gpr-I-open.

(2) Let A be an rg-open set. Then, for any regular closed set F with F ⊆A, we have F⊆int(A) ⊆int((int(A))*) ∪int(A) = int((int(A))*)∪int(int(A))⊆int((int(A))*∪int(A)) = int(cl* (int(A))). That is, F⊆ A∩int(cl*(int(A))) =α-I-int(A) = A∩int(cl*(int(A))) ⊆A∩int(cl* (A)) = p-I-int(A) which implies that A is gpr-I-open.

(3) Let A be an rg-open set. Then, for any regular closed set F with F⊆ A, we have F ⊆int(A) ⊆int((int(A))*) ∪int(A) = int((int(A))*) ∪int(int(A))⊆int((int(A))*∪int(A)) = int(cl* (int(A))). That is, F ⊆A ∩int(cl*(int(A)))= α-I-int(A) which implies that A is rαg-I-open.

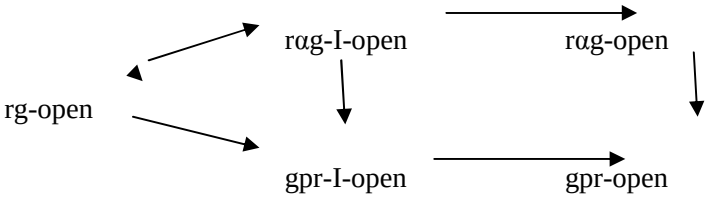
Remark 2.8. The converses of Proposition 2.7 are not true, in general.

Example 2.9. Let X = {a, b, c, d, e}, τ= {∅,X, {a},{e},{a,e},{c,d},{a,c,d},{c,d,e},{a,c,d,e},{b,c,d,e}} and I = {∅,{a},{e},{a,e}}. Then {a, b, d, e} is gpr-I-open but not an rαg-I-open set.

Example 2.10. Let X = {a, b, c, d},τ = {∅,X,{a},{b},{a,b},{b,c},{a,b,c}} and I = {∅,{d}}. Then {a, b, d} is gpr-I-open but not arg-open set.

Example 2.11. In Example 2.10, {a, b, d} is rαg-I-open but not arg-open set.

Remark 2.12. By Remark 1.4, Propositions 2.2 and 2.7, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.



3. C_r-I-sets and C_r^*-I-sets

Definition 3.1. A subset A of an ideal topological space (X, τ, I) is called

(1) aC_r-I-set if A = U ∩ V, where U is rg-open and V is a t-I-set,

(2) aC_r^* -I-set if A = U ∩V, where U is rg-open and V is an α*-I-set.

We have the following proposition:

Proposition 3.2. For a subset of an ideal topological space, the following hold:

(1) Every t-I-set is an α*-I-set [5] and a C_r-I-set.

(2) Every α*-I-set is a C_r^* -I-set.

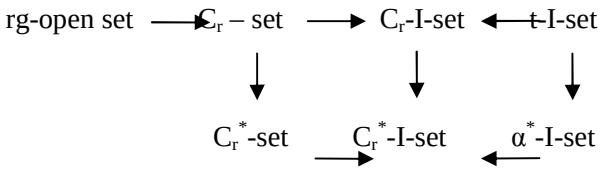
(3) Every C_r -I-set is a C_r^*-I-set.

(4) Every rg-open set is a C_r -I-set.

(5) Every C_r -set is a C_r -I-set and a C_r^* -I-set.

(6) Every C_r^* -set is a C_r^* -I-set.

From Proposition 3.2, we have the following diagram.



Remark 3.3. The converses of implications in Diagram II need not be true as the following examples show.

Example 3.4. In Example 2.4, {a, b, d} is C_r -I-set but not a C_r -set.

Example 3.5. In Example 2.4, {a, b, c} is C_r -I-set but not a t-I-set.

Example 3.6. In Example 2.4, {a, d} is C_r-set but not arg-open set.

Example 3.7. In Example 2.6, {a, b, d} is C_r^*-set but not a C_r-set.

Example 3.8. In Example 2.9, {a, b, c, e} is C_r^*-I-set but not a C_r -I-set.

Example 3.9. In Example 2.4, {a, b, d} is C_r^*-I-set but not a C_r^* -set.

Example 3.10. In Example 2.4, {a, b, c} is C_r^*-I-set but not an α*-I-set.

Example 3.11. Let X = {a, b, c},τ = {∅,X,{a,b}} and I = {∅,{a}}. Then {b} is α*-I-set but not a t-I-set.

Remark 3.12. Examples 3.13 and 3.14 show that C_r-I-sets and C_r^*-sets are independent of each other.

Example 3.13. In Example 2.9, {a, c, d} is C_r^*-set but not a C_r -I-set.

Example 3.14. In Example 2.4, {a, b, d} is C_r -I-set but not a C_r^* -set.

Proposition 3.15. A subset A of X is rg-open if and only if it is both gpr-I-open and a C_r-I-set in X.
Proof: Necessity is trivial. We prove the sufficiency. Assume that A is gpr-I-open and a C_r-I-set in X. Let F⊆ A and F is regular closed in X. Since A is a C_r-I-set in X, A = U ∩V, where U is rg-open and V is a t-I-set. Since A is gpr-I-open, F⊆ p-I-int(A) = A ∩int(cl*(A)) = (U ∩ V) ∩int(cl*(U ∩ V))⊆ (U ∩ V) ∩int(cl*(U)∩ cl* (V)) = (U ∩ V) ∩int(cl*(U)) ∩int(cl*(V)). This implies F⊆int(cl*(V)) = int(V) since V is a t-I-set. Since F is regular closed, U is rg-open and F⊆ U, we have F⊆int(U). Therefore, F⊆int(U)∩int(V) = int(U ∩ V) = int(A). Hence A is rg-open in X.

Corollary 3.16. A subset A of X is rg-open if and only if it is both rαg-I-open and a C_r-I-set in X.
Proof: This is an immediate consequence of Proposition 3.15.

Proposition 3.17. A subset A of X is rg-open if and only if it is both rαg-I-open and a C_r^* -I-set in X.
Proof: Necessity is trivial. We prove the sufficiency. Assume that A is rαg-I-open and a C_r^* -I-set in X. Let F ⊆A and F is regular closed in X. Since A is a C_r^* -I-set in X, A = U ∩V , where U is rg-open and V is an α*-I-set. Now since F is regular closed, F⊆U and U is rg-open, F⊆int(U). Since A is rαg-I-open, F⊆α-I-int(A) = A ∩int(cl*(int(A))) = (U ∩ V) ∩int(cl*(int(U∩ V))) = (U ∩ V) ∩int(cl*(int(U)∩int(V)))⊆ (U ∩V) ∩int(cl*(int(U)) ∩ cl* (int(V))) = (U ∩V) ∩int(cl*(int(U))) ∩int(cl*(int(V))) = (U ∩ V) ∩int(cl*(int(U))) ∩int(V), since V is an α*-I-set. This implies F⊆int(V). Therefore, F⊆int(U) ∩int(V) = int(U ∩ V) = int(A). Hence
A is rg-open in X.

Remark 3.18. (1) The concepts of gpr-I-open sets and C_r-I-sets are independent of each other.
(2) The concepts of rαg-I-open sets and C_r-I-sets are independent of each other.
(3) The concepts of rαg-I-open sets and C_r^*-I-sets are independent of each other.

Example 3.19. In Example 2.10,
(1) {b, c, d} is C_r-I-set but not a gpr-I-open.

(2) {a, b, d} is gpr-I-open but not a C_r-I-set.

Example 3.20. In Example 2.10,
(1) {b, c, d} is C_r-I-set but not an rαg-I-open set.
(2) {a, b, d} is rαg-I-open set but not a C_r-I-set.

Example 3.21. In Example 2.10,
(1) {b, c, d} is C_r^*-I-set but not an rαg-I-open set.
(2) {a, b, d} is rαg-I-open set but not a C_r^*-I-set.

4. Decompositions of rg-continuity

Definition 4.1. A mapping f : (X, τ) → (Y,σ) is said to be rg-continuous [17] (resp. gpr-continuous [4], rαg-continuous [18], C_r-continuous [18] and C_r^*-continuous [18]) if f^{-1}(V) is rg-open (resp. gpr-open, rαg-open, C_r-set and C_r^*-set) in (X, τ) for every open set V in (Y, σ).

Definition 4.2. A mapping f : (X,τ , I) → (Y,σ) is said to be rαg-I-continuous (resp. gpr-I-continuous, C_r-I-continuous and C_r^*-I-continuous) if for every V□σ , f^{-1}(V) is rαg-I-open (resp. gpr-I-open, a C_r-I-set and a C_r^*-I-set) in (X, τ, I).

From Propositions 3.15 and 3.17 and Corollary 3.16 we have the following decompositions of rg-continuity.

Theorem 4.3. Let (X,τ,I) be an ideal topological space. For a mapping f : (X,τ ,I) → (Y,σ), the following properties are equivalent:
(1) f is rg-continuous;
(2) f is gpr-I-continuous and C_r-I-continuous;
(3) f is rαg-I-continuous and C_r-I-continuous;
(4) f is rαg-I-continuous and C_r^*-I-continuous.

Remark 4.4. (1) The concepts of gpr-I-continuity and C_r-I-continuity are in- dependent of each other.
(2) The concepts of rαg-I-continuity and C_r-I-continuity are independent of each other.
(3) The concepts of rαg-I-continuity and C_r^*-I-continuity are independent of each other.

Example 4.5. (1) Let X = Y = {a, b, c, d},τ = {∅,X,{a},{b},{a,b},{b,c},{a,b,c}}, I = {∅,{b}} and σ= {∅,Y,{a,d}}. Let f : (X, τ, I) → (Y,σ) be the identity function. Then f is C_r-I-continuous but not gpr-I-continuous.

(2) Let X=Y={a, b, c, d},τ = {∅,X,{d},{a,c},{a,c,d}}, I = {∅,{a},{d},{a,d}} and σ= {∅,Y,{b,c,d}}. Let f : (X, τ, I) → (Y,σ) be the identity function. Then f is gpr-I-continuous but not C_r-I-continuous.

Example 4.6. (1) Let X=Y={a, b, c, d}, τ = {∅,X,{a},{b},{a,b},{b,c},{a,b,c}}, I = {∅,{c}} and σ= {∅,Y,{b,c,d}}. Let f : (X, τ, I) → (Y,σ) be the identity function. Then f is C_r-I-continuous but not rαg-I-continuous.

(2) Let X=Y={a, b, c, d},τ = {∅,X,{a},{b},{a,b},{b,c},{a,b,c}}, I = {∅,{d}} and σ= {∅,Y,{a,b,d}}. Let f : (X, τ, I) → (Y,σ) be the identity function. Then f is rαg-I-continuous but not C_r-I-continuous.

Example 4.7. Let X, Y,τ , I and f be as in Example 4.6 (2). Let σ= {∅,Y,{b,c,d}}. Then f is C_r^* -I-continuous but not rαg-I-continuous.

In Example 4.6 (2), f is rαg-I-continuous but not C_r^*-I-continuous.

Corollary 4.8. [18] Let (X,τ , I) be an ideal topological space and I = {∅}. For a mapping f : (X, τ, I) → (Y,σ), the following properties are equivalent:

(1) f is rg-continuous;
(2) f is gpr-continuous and C_r-continuous;
(3) f is rαg-continuous and C_r-continuous;
(4) f is rαg-continuous and C_r^*-continuous.

Proof: Since I= {∅}, we have A* = cl(A) and cl*(A) = A*∅ A = cl(A) for any subset A of X [[5], Proposition 2.4(a)]. Therefore, we obtain (1) A is rαg-I-open (resp. gpr-I-open) if and only if it is rαg-open (resp. gpr-open) and (2) A is a C_r-I-set (resp. a C_r^*-I-set) if and only if it is a C_r-set (resp. a C_r^*-set). The proof follows from Theorem 4.3 immediately.

REFERENCES

1. D.Andrijevic, Semi-preopen sets, *Mat. Vesnik*, 38(1) (1986) 24-32.
2. K.Balachandran, P. Sundaram and H.Maki, On generalized continuous maps in topological spaces, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.,* 12 (1991) 5-13.
3. J.Dontchev, On pre-I-open sets and a decomposition of I-continuity, *Banyan Math. J.,* 2 (1996).
4. Y.Gnanambal, On generalized preregular closed sets in topological spaces, *Indian J. Pure Appl. Math.,* 28(3) (1997) 351-360.
5. E.Hatir and T.Noiri, On decompositions of continuity via idealization, *Acta Math. Hungar.,* 96(4) (2002) 341-349.
6. E.Hatir, T.Noiri and S.Yuksel, A decomposition of continuity, *Acta Math. Hungar.,* 70 (1996) 145-150.
7. E.Hayashi, Topologies defined by local properties, *Math. Ann.,* 156 (1964) 205-215.
8. D.Jankovic and T.R.Hamlett, New topologies from old via ideals, *Amer. Math. Monthly,* 97(4) (1990) 295-310.
9. K.Kuratowski, Topology, Vol. 1, Academic Press, New York, 1966.
10. N.Levine, A decomposition of continuity in topological spaces, *Amer. Math. Monthly,* 68 (1961) 44-46.
11. N.Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo,* 19(2) (1970) 89-96.
12. A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, On precontinuous and weak recontinuous mappings, *Proc. Math. Phys. Soc. Egypt.,* 53(1982) 47-53.
13. O.Njastad, On some classes of nearly open sets, *Pacific J. Math.,* 15 (1965) 961-970.

14. T.Noiri, Almost g-closed functions and separation axioms, *Acta Math. Hungar.*, 82(3) (1999) 193-205.
15. T.Noiri, M.Rajamani and V.Inthumathi, On decompositions of g-continuity via idealization, *Bull. Cal. Math. Soc.*, 99(4) (2007) 415-424.
16. T.Noiri, M.Rajamani and V.Inthumathi, Some decompositions of regular generalized continuity via idealization, *Kochi J. Math.*, 5 (2010) 87-95.
17. N.Palaniappan and K.C.Rao, Regular generalized closed sets, *Kyungpook Math. J.*, 33 (1993) 211-219.
18. O.Ravi, A.Pandi, I.Rajasekaran and A.HamariChoudhi, Further decompositions of rg-continuity, submitted.
19. M.H.Stone, Applications of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41 (1937) 375-481.
20. P.Sundaram and M.Rajamani, Some decompositions of regular generalized continuous maps in topological spaces, *Far East J. Math. Sci.*, Special Vol., Part II, (2000) 179-188.
21. J.Tong, On decomposition of continuity in topological spaces, *Acta Math. Hungar.*, 54 (1989) 51-55.
22. R.Vaidyanathaswamy, The localization theory in set topology, *Proc. Indian Acad. Sci., Sect A*, 20 (1944) 51-61.