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Multi-Objective Fuzzy Transshipment Problem

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Abstract. In this paper, fuzziness in the preemptive goal programming formulation of a multi-objective unbalanced transshipment problem with budgetary constraints in which the demand and budget are specified imprecisely.

Keywords: Fuzzy transshipment problem, Preemptive Optimization, Fuzzy demand, Fuzzy Goal.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

The transportation model is a special class of the linear programming problem. It deals with the situation in which a commodity is shipped from sources to destinations and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ respectively. The objective is to determine the amounts shipped from each source to each destination that minimizes the total shipped cost while satisfying both the supply limits and the demand requirements.

Orden [5] has extended this problem to include the case when transshipment is also allowed. In general, the real life problems are modeled with multi-objectives, which are measured on different scales and at the same time in conflict.

The paper organized as follows: Section 2 deals with the formulation of the multi-objective transshipment problem, transit point, preemptive optimization and the procedure of Transshipment Models with Transit point. Section 4 deals with afuzzy programming approach to solve the multi-objective transshipment problem. In section 6 numerical examples are illustrated.

2. Preliminaries

2.1. Formulation of the general transshipment problem

The transportation problem assumes that direct routes exist from each source to each destination. However, there are situations in which units may be shipped from one source to another or to other destinations before reaching their final destinations. This is called a transshipment problem. The purpose of transshipment the distinction between a source and destination is dropped so that a transportation problem with m source and n destinations gives rise to a transshipment problem with m + n source and m + n destinations. The basic feasible solution to such a problem will involve [(m + n) + (m +

n)-1] or 2m + 2n - 1 basic variables and if we omit the variables appearing in the (m + n) diagonal cells, we are left with m + n - 1 basic variables.
Thus the transshipment problem may be written as:
Minimize $F^k(x) = \sum_{i=1}^{m+n}\sum_{j=1,j\neq i}^{m+n} c^k{}_{ij}x_{ij}$
Subject to $\sum_{j=1,j\neq i}^{m+n} x_{ij} - \sum_{j=1,j\neq i}^{m+n} x_{ji} = a_i,$ i = 1,2,3,…,m
$\sum_{i=1,i\neq j}^{m+n} x_{ij} - \sum_{i=1,i\neq j}^{m+n} x_{ji} = b_j,$ j = m+1,m+2,m+3,…,m+n
where $x_{ij} \geq 0$, i, j=1,2,3,…,m+n, j≠i
where $\sum_{i=1}^{m} a_i = \sum_{j=1}^{m} b_j$ then the problem is balance otherwise unbalanced.

The above formulation is a transshipment model, where $F^k(x) = \{F^1(x), F^2(x), \dots, F^k(x)\}$ is a vector of k objective functions and the subscript on the both $F^k(x)$ and C_{ij}^k are used to identify the number of objective functions (k=1,2,3,…,q). Wthout oss of generality it will be assumed in the paper that $a_i > 0\ \forall\ i, b_j > 0\ \forall j, C_{ij}^k \geq 0\ \forall\ (i,j)\ and\ \sum_i a_i \neq \sum_j b_j$.

The transshipment model is reduced to transportation form as:
Minimize $F^k(x) = \sum_{i=1}^{m+n}\sum_{j=1,j\neq i}^{m+n} c^k{}_{ij}x_{ij}$
Subject to $\sum_{j=1}^{m+n} x_{ij} = a_i + T,$ i=1,2,3,…,m
$\sum_{j=1}^{m+n} x_{ij} = T,$ i=m+1, m+2, m+3,…, m+n
$\sum_{i=1}^{m+n} x_{ij} = T,$ j=1,2,3,…,m
$\sum_{i=1}^{m+n} x_{ij} = b_j + T,$ j=m+1, m+2, m+3,…, m+n
where $x_{ij} \geq 0$, i, j = 1,2,3,…,m + n, j ≠ i,
the above mathematical model represents a standard balanced transportation problem with (m+n) origins and (m+n) destinations. T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of good a transshipped at any point cannot exceed the amount produced or received and hence we take T= $\sum_{i=1}^{m} a_i\ or\ \sum_{j=1}^{m} b_j$.

2.2. Transshipment model with transit point
In this model ‘m’ origin and ‘n’ destination and ‘p’ transit points are included. In this model the total number of origins is m+p and the total number of destinations is p+n.

We now describe how the optimal solution to a transshipment problem can be found by solving a transportation problem. Given a transshipment problem, we create a balanced transportation problem by the following procedure (assume that total supply exceeds total demand):

Construct a transportation tableau as follows: A row in the tableau will be needed for each supply point and transshipment point, and a column will be needed for each demand point and transshipment point. Each supply point will have a supply equal to its original supply, and each demand point will have a demand equal to its original demand.
Let T = total available supply. Then each transshipment point will have a supply equal to (point’s original supply) + T and a demand equal to (point’s original demand) + T. This ensures that any transshipment point that is a net supplier will have a net outflow equal to the point’s original supply, and, similarly, a net demander will have a net inflow equal to the point’s original demand. Although we don’t know how much will be shipped through

each transshipment point, we can be sure that the total amount will not exceed T. This explains why we add T to the supply and demand at each transshipment point. By adding the same amounts to the supply and demand, we ensure that the net outflow at each transshipment point will be correct, and we also maintain a balanced transportation tableau.

2.3. Preemptive optimization

Preemptive optimization or lexicographic optimization performs multi-objective optimization by considering objectives one at a time. The most important is optimized; then the second most important that the first achieves its optimal value and so on.

3. A fuzzy programming approach for solving MOTrP

In 1970, Bellman and Zadec introduced three basic concepts: fuzzy goal(G) fuzzy constraints(C)and fuzzy decision(D) and explored the applications of these concepts to decision making under fuzziness.

Their fuzzy decision is defined as follows

$$D=G\cap C$$

The decision variables, supply constraints, fuzzy demand goals and multi-objective fuzzy budget goal are identified as follows.

3.1. Decision variables

Decision variables for the model are defined as x_{ij}, i=1,2,..m and j=1,2,…n where $x_{ij}\geq 0$ for all i,j.

3.2. System of supply constraints

$$\sum_{j=1}^{n} x_{ij} \leq a_i$$

where $a_i>0$, i=1,2,…m is the amount of goals available at i^{th} origin.

3.3. Fuzzy demand goals

$$\sum_{i=1}^{m} x_{ij} \geq b_j$$

where $b_j>0$, j=1,2,…n be the amount of goods required at the j^{th} destination.

3.4. Fuzzy budget goal

The fuzzy budget goal as

$$\sum_{k=1}^{n}\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij} \leq B$$

where B is the aspiration level of the budget.

3.5. Aspiration level

The aspiration level criterion does not yield an optimal decision in the sense of maximizing profit or minimizing cost. Rather it is a means of detailing acceptable courses of action. Consider , for the example, the situation where a person advertises a used car for sale. On receiving an offer the seller must decide, within a reasonable time span, whether it is acceptable or not. In this respect, the seller sets a price limit below which the car will not be sold. This is the aspiration level.

3.6. Solving method

In this proposed fuzzy model, firstly we convert the model to a linear programming model by using linear membership functions and max-min operator. The solution of linear programming model gives an efficient solution. The solution procedure is similar to the approach used by Zimmermann.

The membership functions of the fuzzy demand goal are defined as

$$\mu_{A_j}(x) = \begin{cases} 1 \quad if \displaystyle\sum_{i=1}^{m} x_{ij} \geq b_j \\ \dfrac{\sum x_{ij} - b_j}{b_j - b_j^*} \; if \; b_j^* < \displaystyle\sum_{i=1}^{m} x_{ij} < b_j \\ 0 \; if \displaystyle\sum_{i=1}^{m} x_i \leq b_j \end{cases}$$

where b_j^*, j=,2,…n is the lower tolerance limit of j[th] demand goal.

The membership function corresponding to the fuzzy budget goal is defined as

$$\mu_{A_{j+1}}^k(x) = \begin{cases} 1 \; if \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij} \geq B \\ \dfrac{B^* - \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij}}{B^* - B} \quad if \; B < \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} < B^* \\ 0 \;\; if \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij} \geq B^* \end{cases}$$

where k=,,…q and B^* is the upper tolerance limit of the budget goal. The intersection operator defines the overall decision function 'D'

$$D = \bigcap_{j=1}^{n} A_j \cap A_{j+1}^k$$

The membership function of the solution set is then

μ_p= min $\{\mu_{Aj}, \mu_{Aj+1}^k\}$,j=1,2,…n, where (j+1)[th] membership function corresponds to the fuzzy budget goal. And the maximizing decision

Maximin $\{\mu_{Aj}, \mu_{Aj+1}^k\}$

x≥0

As is well known, the problem is equivalent to solving the following L.P

Maximize λ

Subject to λ≤ $\mu_{Aj}(x)$, j=1,2,…n

λ≤ $\mu_{Aj+1}(x)$

where λ= min $\{\mu_{Aj}, \mu_{Aj+1}^k\}$=min$\left[\frac{\sum_{i=1}^{m} x_{ij} - b_j}{b_j - b_j^*}, \frac{B^* - \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij}}{B^* - B}\right]$

therefore, the goal-programming model of an unbalanced transportation problem with budgetary constraints can be stated as follows

Maximize λ

Subject to

$\lambda \le \mu_{Aj}(x)$,j=1,2,…n

λ≤ $\mu_{Aj+1}(x)$,k=1,2…q

λ,$x_{ij} \ge 0$

which is a linear programming model and can be solved by an appropriate linear programming algorithm.

4. Numerical example

Let us consider a multi-objective unbalanced transshipment problem with the following characteristics.

Supplies $a_1 = 8,\ a_2 = 16$

Demand $b_1 = 12,\ b_2 = 4,\ b_3 = 14$

Penalties

$$C^1 = \begin{bmatrix} 1 & 2 & 8 \\ 1 & 9 & 8 \end{bmatrix} C^2 = \begin{bmatrix} 6 & 4 & 3 \\ 5 & 8 & 9 \end{bmatrix}$$

Supply constraints

$$\sum_{j=1}^{3} x_{1j} \le 8, \qquad \sum_{j=1}^{3} x_{2j} \le 16$$

Fuzzy demand goal

$$\sum_{1=1}^{2} x_{i1} \le 12, \qquad \sum_{i=1}^{2} x_{i2} \le 4\ , \qquad \sum_{i=1}^{2} x_{i3} \le 14,$$

Then transit points are

Transit to Destinations is [4 6 4] and [8 4 3]

Transit to source is $\begin{bmatrix} 9 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 70 \end{bmatrix}$

Now the Transportation Problem is

$$C_{ij}^1 = \begin{bmatrix} 1 & 2 & 89 \\ 1 & 9 & 84 \\ 4 & 6 & 40 \end{bmatrix} C_{ij}^1 = \begin{bmatrix} 6 & 4 & 3\ 6 \\ 5 & 8 & 970 \\ 8 & 4 & 3\ 0 \end{bmatrix}$$

Here we take T=24

Fuzzy demand goal (after converting to transportation problem

$$\sum_{j=1}^{3} \sum_{i=1}^{4} C_{ij}^1 x_{ij} \le 84$$

$$\sum_{j=1}^{3} \sum_{i=1}^{4} C_{ij}^2 x_{ij} \le\ 116$$

where $C_{ij}{}'s$ are penalties taken from the above table. Let us take the first goal C^1, we get minimum cost using preemptive optimization.

Similarly we apply the same procedure to the goal C^2. We find minimum cost from these two goals.

For the four demand points, we have $b_1 = 12,\ b_2 = 4,\ b_3 = 14, b_4 = 24$ and the lower tolerance limit are $b_1^* = 6,\ b_2^* = 2,\ b_3^* = 7, b_4^* = 12$

$$\mu_{A_1}(x) = \begin{cases} 1 & if x_{11} + x_{21} + x_{31} \geq 12 \\ \dfrac{x_{11} + x_{21} + x_{31}}{12 - 6} & if\ 6 \leq x_{11} + x_{21} + x_{31} \leq 12 \\ 0 & if x_{11} + x_{21} + x_{31} \leq 6 \end{cases}$$

$$\mu_{A_2}(x) = \begin{cases} 1 & if x_{12} + x_{22} + x_{32} \geq 4 \\ \dfrac{x_{12} + x_{22} + x_{32}}{4 - 2} & if\ 2 \leq x_{12} + x_{22} + x_{32} \leq 4 \\ 0 & if x_{12} + x_{22} + x_{32} \leq 2 \end{cases}$$

$$\mu_{A_3}(x) = \begin{cases} 1 & if x_{13} + x_{23} + x_{33} \geq 14 \\ \dfrac{x_{13} + x_{23} + x_{33}}{14 - 7} & if\ 7 \leq x_{13} + x_{23} + x_{33} \leq 14 \\ 0 & if x_{13} + x_{23} + x_{33} \leq 7 \end{cases}$$

$$\mu_{A_4}(x) = \begin{cases} 1 & if x_{14} + x_{24} + x_{34} \geq 24 \\ \dfrac{x_{14} + x_{24} + x_{34}}{24 - 12} & if\ 12 \leq x_{14} + x_{24} + x_{34} \leq 24 \\ 0 & if x_{14} + x_{24} + x_{34} \leq 12 \end{cases}$$

$$\mu_{A_5}^1(x) = \begin{cases} 1 & if\ z \leq 84 \\ \dfrac{85 - z}{85 - 84} & if\ 84 \leq z \leq 85 \\ 0 & if z \geq 85 \end{cases}$$

$$\mu_{A_5}^{11}(x) = \begin{cases} 1 & if\ z \leq 116 \\ \dfrac{117 - z}{117 - 116} & if\ 116 \leq z \leq 117 \\ 0 & if z \geq 117 \end{cases}$$

where

$$z = x_{11} + 2x_{12} + 8x_{13} + 9x_{14} + x_{21} + 9x_{22} + 8x_{23} + 4x_{24} + 4x_{31} + 6x_{32} + 4x_{33} + 0x_{34}$$

$$z = 6x_{11} + 4x_{12} + 3x_{13} + 6x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 70x_{24} + 8x_{31} + 4x_{32} + 3x_{33} + 0x_{34}$$

Maximize λ

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 8$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 16$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 24$$
$$\lambda \leq \ \mu_{A_1}(x)$$
$$\lambda \leq \frac{x_{11} + x_{21} + x_{31}}{6} \leq 6$$
$$x_{11} + x_{21} + x_{31} - 6\lambda \ \geq 6$$

Similarly

$$x_{12} + x_{22} + x_{32} - 2\lambda \geq 2$$
$$x_{13} + x_{23} + x_{33} - 7\lambda \geq 7$$
$$x_{14} + x_{24} + x_{34} - 12\lambda \geq 12$$
$$x_{11} + 2x_{12} + 8x_{13} + 9x_{14} + x_{21} + 9x_{22} + 8x_{23} + 4x_{24} + 4x_{31} + 6x_{32} + 4x_{33} + 0x_{34} = 84$$

$$6x_{11} + 4x_{12} + 3x_{13} + 6x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 70x_{24} + 8x_{31} + 4x_{32} + 3x_{33} + 0x_{34} = 116$$

Output

The above example is solved by using TORA Computer Software Package. The Optimal of the first objective function is$x_{11} = 8,\ x_{21} = 4.51,\ x_{22} = 3.64,\ x_{23} = 5.52, x_{34} = 24, \lambda = 1.25$ and the Transportation cost is z = 85.25.

The second objective function is $x_{11} = 6.94, x_{13} = 1.06, x_{21} = 4.18, x_{23} = 5.59, x_{34} = 24, \lambda = 1.09$ and the Transportation cost is z = 117.09

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