Intern. J. Fuzzy Mathematical Archive
Vol. 10, No. 2, 2016, 155-160
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 6 June 2016
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

Fuzzy Strongly G∗-Super Closed Sets and Fuzzy δg∗-Super Closed Sets

M.K. Mishra* and *M.Shukla

Department of MCA, EGS Pillay Engineering College, Nagapattinam-611002, India
Department of Science, Arignar Anna College, Karaikal-609602, India
Email: drmkm1969@rediffmail.com

Received 21 May 2016; accepted 27 May 2016

Abstract. In this paper, we studied and introduce a new class of fuzzy sets called fuzzy strongly g *- super closed sets is introduced and explore some of its characterization.

Keywords: Fuzzy topological spaces, Fuzzy generalized super closed sets, Fuzzy g ∗-super closed sets, Fuzzy strongly g ∗- super closed sets

AMS Mathematics Subject Classification (2010): 46S40, 47S40

1. Introduction and preliminaries

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$ of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \le B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Definition 1.1. A subset A of a fuzzy topological space (X,τ) is called

1. fuzzy super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. fuzzy super interior $sint(A) =\{x\in X:cl(U)\le A\neq\phi\}$
3. fuzzy super closed if $scl(A) \le A$.
4. fuzzy super open if 1-A is fuzzy super closed $sint(A)=A$

Let X be a fuzzy topological space and A be a fuzzy set of X. The fuzzy interior (respectively fuzzy closure) of a fuzzy set A in X will be denoted by int(A) (resp.cl(A)).

Definition 1.2. A subset A of a fuzzy topological space X is called

1. fuzzy pre super open set if A $\leq$ int(cl (A)) and a pre super closed set if cl(int (A))$\leq$ A.
2. fuzzy semi super open set if A $\leq$clint (A) and a semi super closed set if int(cl (A))$\leq$A.
3. fuzzy regular super open set if A = int(cl (A)) and a regular super closed set if A = cl(int(A)).
4. fuzzy π- super open set if A is a finite union of regular super open sets.
5. fuzzy regular semi super open[4]if there is a regular super open U such that U $\leq$A $\leq$ cl(U).

Definition 1.3. A fuzzy set A of (X, Γ) is called,

1. semi super open (in short, fs-open) if A $\leq$ cl (int (A)) and a fuzzy semi super closed (fs- super closed) if int (cl (A)) $\leq$ A.
2. fuzzy pre- super open (fp-open) if A $\leq$ int [cl (A)] and a fuzzy pre super closed (fp- super closed) if cl (int (A)) $\leq$ A.
3. fuzzy α- super open (fα- super open) if A $\leq$ int [cl (int (A))] and a fuzzy α- super closed (fα- super closed) if cl (int [cl (A)]) $\leq$ A.
4. fuzzy semi pre- super open (fsp- super open) if A $\leq$ cl (int [cl (A)]) and a fuzzy semi pre- super closed (fsp- super closed) if int [cl (int (A))] $\leq$ A.
5. fuzzy θ- super open (fθ- super open) if A = int θ (A) and a fuzzy θ- super closed (fθ- super closed) if A = cl θ (A) where cl θ (A) = $\cap$ {cl (μ) : A $\leq$ μ, $\mu \in \tau$ }.
6. fuzzy generalized super closed (fg- super closed) if cl (A) $\leq$ H, whenever A $\leq$ H and H is fuzzy super open set in X.
7. fuzzy generalized semi super closed (gfs- super closed) if scl (A) $\leq$ H, whenever A $\leq$ H and H is fs- super open set in X. This set is also called generalized fuzzy weakly semi super closed set.
8. fuzzy generalized semi super closed (fgs- super closed) if scl (A) $\leq$ H, whenever A $\leq$ H and H is fuzzy super open set in X.
9. fuzzy generalized pre- super closed (fgp- super closed) if pcl (A) $\leq$ H, whenever A $\leq$ H and H is fuzzy super open set in X.
10. fuzzy α-generalized super closed (fαg- super closed) if αcl (A) $\leq$ H, whenever A $\leq$ H and H is fuzzy super open set in X.
11. fuzzy generalized α super -closed (fgα- super closed) if αcl (A) $\leq$ H, whenever H is fuzzy super open set in X.
12. fuzzy generalized semi pre- super closed (fsp- super closed) if spcl (A) $\leq$ H,whenever A $\leq$ H and H is fuzzy super open set in X.
13. fuzzy semi pre-generalized super closed (fspg- super closed) if spcl (A) $\leq$ H, whenever A $\leq$ H and H is fs- super open in X.
14. fuzzy θ-generalized super closed (fθg- super closed) if cl (θ (A)) $\leq$ H, whenever A $\leq$ H and H is fuzzy super open in X.
15. fuzzy g $*$- super closed (fg $*$- super closed) if cl (A) $\leq$ H, whenever A $\leq$ H and H is fg- super open in X.

Definition 1.4. A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all y ∈ X except one, say x ∈ X. If its value at x is p ($0 < p \leq 1$), we denote this fuzzy point by x_p, where the point x is called its support.

2. δg ∗ - super closed sets

Definition 2.1. A fuzzy point x_a is said to be a fuzzy δ-cluster point of a fuzzy set A in a fts X if every fuzzy regular super open quasi neighborhood of H of x_a is quasi coincident with A.

Definition 2.2. The union of all fuzzy δ-cluster points of A is called the fuzzy δ- super closure of A, is denoted by δcl (A).

Definition 2.3. A subset A of a topological space (X, Γ) is called a fuzzy g- super closed set if fcl (A) ≤ H, whenever A ≤ H and H is super open in (X, Γ).

Definition 2.4. The complement of a fg- super closed set is called a fg- super open set.

Definition 2.5. A subset A of a space X is called fδg-closed if fclδ (A) ≤H, whenever A ≤ H and H is a fuzzy super open set.

Definition 2.6. A fuzzy set A in fts (X, τ) is called fuzzy δg ∗- super closed if and only if fclδ (A) ≤ B, whenever A ≤ B and B is fuzzy g- super open in X.

Theorem 2.7. Every fuzzy δ- super closed set is a fuzzy δg ∗-super closed set in (X, Γ).
Proof. Let A be a fuzzy δ- super closed set in a fts X and B be a fuzzy g- super open set in X such that A ≤ B. Since A is a fuzzy δ- super closed, fcl δ (A) = A. Therefore fcl δ (A) = A ≤ B. Hence A is a fuzzy δg ∗- super closed set.

Theorem 2.8. If A is fuzzy δ- super open and fuzzy δg ∗- super closed in (X, τ), then A is fuzzy δ- super closed in (X, Γ).
Proof. Let A be fuzzy δ- super open and fuzzy δg ∗- super closed in X. Suppose A ≤ A, then fcl δ (A) ≤ A. But A ≤ fcl δ (A), which implies that fcl δ (A) = A. Hence A is fuzzy δ- super closed.

Theorem 2.9. Let (X, Γ) be a fts and A be a fuzzy set of X. Then A is fuzzy δg ∗- super closed if and only if A –q B implies fcl δ (A) –q B for every fuzzy δg ∗-closed set B of (X, Γ).
Proof. Suppose A is a fuzzy δg ∗- super closed set of X. Let B be a fuzzy g super closed set in X such that A −q B. Then A ≤ 1 − B and 1 − B is a fuzzy g- super open set of X. Therefore fcl δ (A) ≤ 1 − B, as A is fuzzy δg ∗- super closed. Hence fcl δ (A) –q B. Conversely, let D be a fuzzy g- super open set in X such that A ≤ D. Then A –q (1 − D) and 1 − D is a fuzzy g- super closed set in X. By hypothesis, fcl δ (A) –q (1 − D) which implies, cl δ (A) ≤ D. Hence A is fuzzy δg ∗- super closed.

Theorem 2.10. If A is a fuzzy $\delta g *$-closed set in (X, τ) and $A \leq B \leq fcl\ \delta\ (A)$, then B is a fuzzy $\delta g *$- super closed set in (X, Γ).
Proof. Let A be a fuzzy $\delta g *$-closed set in (X, τ). Given $A \leq B \leq fcl\ \delta\ (A)$. Suppose $B \leq H$ where H is fuzzy g- super open set. Since $A \leq B \leq H$ and A is a fuzzy $\delta g *$- super closed set, we get $fcl\ \delta\ (A) \leq H$. As $B \leq fcl\ \delta\ (A)$, $f\ cl\delta\ (B) \leq f\ cl\delta\ (f\ cl\delta\ (A)) = f\ cl\delta\ (A)$ we get $f\ cl\delta\ (B) \leq H$. Hence B is a fuzzy $\delta g *$- super closed set in (X, τ).

Theorem 2.11. If A is a fuzzy $\delta g *$-open set in (X, τ) and $fint\delta\ (A) \leq B \leq A$, then B is a fuzzy $\delta g *$- super open set in (X, Γ).
Proof. Let A be fuzzy $\delta g *$- super open set and B be any fuzzy set in X such that $fint\delta\ (A) \leq B \leq A$. Then $1 - A$ is a fuzzy $\delta g *$-closed set and $1 - A \leq 1 - B \leq fcl\delta\ (1 - A)$, as $1 - f\ int\delta\ (A) = fcl\delta\ (1 - A)$. Therefore $1 - B$ is a fuzzy $\delta g *$- super closed. Hence B is fuzzy $\delta g *$- super open.

3. Fuzzy strongly g $*$- super closed sets in fuzzy topological spaces

Definition 3.1. Let (X, τ) be a fuzzy topological space. A fuzzy set A of (X, Γ) is called fuzzy strongly g $*$- super closed if $cl\ (int\ (A)) \leq H$, whenever $A \leq H$ and H is fg- super open in X.

Theorem 3.2. Every fuzzy closed set is a fuzzy strongly g $*$- super closed set in the fuzzy topological space (X, Γ).
Proof. Let A be fuzzy super closed set in X and H be a fg- super open set in X such that $A \leq H$. Since A is fuzzy super closed, $cl\ (A) = A$. Therefore $cl\ (A) \leq H$.Now, $cl\ (int\ (A)) \leq cl(A) \leq H$. Hence A is fuzzy strongly g*-super-closed set in X.

Theorem 3.1. Every fuzzy g *- super closed set is a fuzzy strongly g *- super closed set in (X, Γ).
Proof. Suppose that A is fg *- super closed in X. Let H be a fg- super open set in X such that $A \leq H$. Then $cl\ (A) \leq H$, since A is fg *- super closed. Now,$cl\ (int\ (A)) \leq cl\ (A) \leq H$, hence A is fuzzy strongly g *- super closed set inX. However the converse of the Theorem 4.5 need not be true in general.

Theorem 3.2. Let A be a fuzzy strongly g *- super closed set in $(X,\Gamma\tau)$ and x_p be a fuzzy point of (X, Γ) such that x pqcl (int(A)) then cl (int(x p)) qA.
Proof. Let A be a fuzzy strongly g *- super closed set in (X,Γ) and x p be a fuzzy point of (X, τ) such that x pqcl (int (A)). Suppose $cl\ (int\ (x_p))-qA$,then $cl\ (int\ (x_p))\ q1 - A$ and hence $A \leq 1 - cl\ (int\ (x_p))$. Now, $1 -cl\ (int\ (x_p))$ is fuzzy super open. Moreover, since A is fuzzy strongly g *- super closed, $cl\ (int\ (A)) \leq 1 - cl\ (int\ (x_p)) \leq 1 - x_p$. Hence $x_p-qcl\ (int\ (A))$, which is a contradiction.

Theorem 3.3. If A is a fuzzy strongly g *- super closed set in (X, Γ) and $A \leq B \leq cl\ (int\ (A))$, then B is fuzzy strongly g *- super closed in (X, τ).
Proof. Let A be a fuzzy strongly g *- super closed set in (X, Γ) and $B \leq H$ where H is a fuzzy g- super open set in X. Then $A \leq H$. Since A is a fuzzy strongly g *- super closed set, it follows that $cl\ (int\ (A)) \leq H$. Now, $B \leq cl\ (int\ (A))$ implies $cl\ (int\ (B)) \leq cl\ (int\ (cl\ (int\ (A)))) = cl\ (int\ (A))$.We get, $cl\ (int\ (B)) \leq H$. Hence, B is a fuzzy strongly g *- super

closed set in (X,Γτ).

Definition 3.2. A fuzzy set A of (X, τ) is called fuzzy strongly g *- super open set in X if and only if 1 − A is fuzzy strongly g *- super closed in X. In other words, A is fuzzy strongly g *- super open if and only if H ≤cl (int (A)), whenever H ≤ A of H is fg- super closed in X.

Theorem 3.4. Let (Y, τY) be a subspace of a fuzzy topological space (X,Γ) and A be a fuzzy set of Y. If A is fuzzy strongly g *- super closed in X, then A is a fuzzy strongly g *- super closed in Y.

Proof. Let Y be a subspace of X and H be a fg- super open set in Y such that A ≤ H. We have to prove that cly (int y (A)) ≤ H. Since H is fg-super open in Y, we have H = G ∩ Y where G is fg- super open in X. Hence A ≤ H = G ∩ Y implies A ≤ G and A is fuzzy strongly g *- super open in X. We get cl (int (A)) ≤ G. Therefore cl (int (A)) ∩ Y ≤ G ∩ Y = H. Thus cl (int (A)) ≤ H, whenever A ≤ H and H is fuzzy g-super open in Y. Hence A is fuzzy strongly g *- super open in Y.

Theorem 3.5. If a fuzzy set A of a fuzzy topological space X is both fuzzy super open and fuzzy strongly g *- super closed, then it is fuzzy super closed.

Proof. Suppose that a fuzzy set A of X is both fuzzy super open and fuzzy strongly g *- super closed. Now, A ≥ cl (int (A)) ≥ cl (A). That is A ≥ cl (A), since A ≤ cl (A). So we get A = cl (A). Hence A is fuzzy super closed in X.

Theorem 3.6. If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g *- super closed and fuzzy semi super open, then it is fg *- super closed.

Proof. Suppose a fuzzy set A of X is both fuzzy strongly g *- super closed and fuzzy semi open in X. Let H be a fg-open set such that A ≤ H. Since A is fuzzy strongly g *- super closed, therefore cl (int (A)) ≤ H. Also since A is fs- super open, A ≤ cl (int (A)). We have cl (A) ≤ cl (int (A)) ≤ H. Hence A is fg *- super closed in X.

REFERENCES

1. A.S.Bin Shahana, On fuzzy strong semi continuity and fuzzy pre continuity, *Fuzzy Sets and Systems*, 44 (1991) 303-308.
2. K.K.Axad, On fuzzy semi continuity fuzzy almost continuity and fuzzy weakly continuity, *J.Math. Anal. Appl.*, 82 (1981) 14-32.
3. A.S.Bin Sahana, Mapping in fuzzy topological spaces, *Fuzzy Sets and Systems,* 61 (1994) 209-213.
4. A.S.Bin Sahana, On fuzzy strongly semi continuity and fuzzy pre continuity, *Fuzzy Sets and Systems,* 44 (1991) 303-308.
5. C.L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24 (1968) 182-190.
6. D.Andrijevic, Semi preopen sets, *Mat. Vesnik*, 38 (1986) 24-32.
7. G.J.Klir and B.Yuan, Fuzzy sets and fuzzy logic theory and applications Prentice Hall of India, New Delhi, 2003.
8. K.K.Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl.*, 82 (1981) 14-32.
9. L.A. Zadeh, Fuzzy Sets, *Inform and Control*, 8(1965) 338-35.

10. Y.M.Lin and K.M.Lou, Fuzzy topology, World Scientific Publication Singapore (1997)
11. N.Livine, Generalized closed sets in topology, *Rand. Circ Mat. Palermo,* 19(2) (1970) 571-599.
12. N.Livine, Semi Open Sets and semi continuity in topological spaces, *Amer. Math. Monthly,* 70 (1963) 36-41.
13. R.Lowen, A comparison of different compactness notion on fuzzy topological spaces, *J. Math. Anal. Appl.,* 64 (1978) 446-454.
14. R.Lowen, Fuzzy topological Spaces and fuzzy compactness, *J. Math. Anal. Appl.,* 56 (1976) 621-633.
15. A.S.Mashour, M.F.Abd.Monsef and S.N.EI.,Deeb, On pre continuous and weak precontinious mappings, *Proc. Math and Phys. Soc, Egypt*, 53 (1982) 47-53.
16. S.Nanda, On fuzzy topological spaces, *Fuzzy Sets and Systems,* 19(2) (1986) 193-197.
17. S.Nanda, Strongly compact fuzzy topological spaces, *Fuzzy Sets and Systems,* 42 (1990) 259-262.
18. P.M.Pu and Y.M.Liu, Fuzzy topology I. Neighbourhood structure of a fuzzy point and moore smith convergence, *J.Math. Anal. Appl.*, 76 (1980) 571-599.
19. N.Palaniappan and K.C.Rao, Regular generalized closed sets, *Kyungpook Math. J.*, 33(2) (1932) 11-219.
20. R.Prasad, S.S.Thakur and R.K.Saraf, Fuzzy α-irresolute mapping, *J. Fuzzy Math*, 2(2) (1994) 335-339.
21. P.M.Pu and Y.M.Lin, Fuzzy topology II. Product quotient spaces, *J.Math. Anal. Appl.*, 77 (1980) 20-27
22. P.M.Pu and Y.M.Lin, Fuzzy topology, I. Neighborhood structure of a fuzzy point Moore Smith convergence, *J.Math.Anal.Appl.*, 76 (1980) 571-599.
23. S.Ganguli and S.Saha, On separation axioms and separations of connected sets in fuzzy topological spaces, *Bull. Cal. Math.*, 79 (1987) 215-225.
24. T.H.Yalvac, Semi interior, semi closure of a fuzzy set, *J. Math. Anal. Appl.*, 132 (1988) 356-364.
25. C.K.Wong, On fuzzy points and local properties of fuzzy topology, *J. Math Anal. Appl.*, 46 (1974) 316-328.