

## **$(2, (c_1, c_2))$ -Pseudo Regular Intuitionistic Fuzzy Graphs**

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**Abstract.** In this paper, pseudo degree, pseudo total degree,  $d_2$ - pseudo degree of a vertex and  $d_2$ - pseudo total degree of a vertex in an intuitionistic fuzzy graphs are defined. Also  $(2, (c_1, c_2))$ -Pseudo regularity and  $(2, (c_1, c_2))$ -Pseudo total regularity of an intuitionistic fuzzy graphs are defined. A relation between  $(2, (c_1, c_2))$ -Pseudo regularity and  $(2, (c_1, c_2))$ -Pseudo total regularity on an intuitionistic fuzzy graph is studied.  $(2, (c_1, c_2))$ -Pseudo regularity on Peterson graph, a Ladder graph  $L_n$  ( $n > 1$ ) and a cycle  $C_n$  are studied with some specific membership functions.

**Keywords:** intuitionistic fuzzy graph, regular intuitionistic fuzzy graph, totally regular intuitionistic fuzzy graph,  $d_2$  - degree of a vertex in an intuitionistic fuzzy graph

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### **1. Introduction**

In 1965, Zadeh [8] introduced the concept of fuzzy subset of a set as method of representing the Phenomena of uncertainty in real life situation. Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one Karunambigai and Parvathi and Buvanewari introduced constant intuitionistic fuzzy graphs [4]. Narayanan and Maheswari introduced  $(2, (c_1, c_2))$  -Regular intuitionistic fuzzy graphs [6]. These motivates us to introduce pseudo degree, pseudo total degree,  $d_2$ - pseudo degree,  $d_2$ -pseudo degree of a vertex in an intuitionistic fuzzy graph and discussed some properties. Throughout this paper, the vertices take the membership values  $A = (\mu_1, \gamma_1)$  and edges take the membership values  $B = (\mu_2, \gamma_2)$ .

### **2. Preliminaries**

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

**Definition 2.1.** [3] A fuzzy graph  $G : (\sigma, \mu)$  is a pair of functions  $(\sigma, \mu)$ , where  $\sigma : V \rightarrow [0,1]$  is a fuzzy subset of a non empty set  $V$  and  $\mu : V \times V \rightarrow [0, 1]$  is a symmetric

fuzzy relation on  $\sigma$  such that for all  $u, v$  in  $V$ , the relation  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$  is satisfied. A fuzzy graph  $G$  is called complete fuzzy graph if the relation  $\mu(u, v) = \sigma(u) \wedge \sigma(v)$  is satisfied.

**Definition 2.2.** [7] Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The 2- degree of a vertex  $v$  in  $G$  is defined as the sum of degrees of the vertices adjacent to  $v$  and is denoted by  $t_G(v)$ . That is,  $t_G(v) = \sum d_G(u)$ , where  $d_G(u)$  is the degree of the vertex  $u$  which is adjacent with the vertex  $v$ .

**Definition 2.3.** [7] Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . A pseudo (average) degree of a vertex  $v$  in a fuzzy graph  $G$  is denoted by  $d_a(v)$  and is defined by  $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$ , where  $d_G^*(v)$  is the number of edges incident at  $v$ .

**Definition 2.4.** [2] An intuitionistic fuzzy graph with an underlying set  $V$  is defined to be a pair  $G = (V, E)$  where

- (i)  $V = \{v_1, v_2, v_3, \dots, v_n\}$  such that  $\mu_1: V \rightarrow [0, 1]$  and  $\gamma_1: V \rightarrow [0, 1]$  denote the degree of membership and non membership of the element  $v_i \in V$ , ( $i = 1, 2, 3, \dots, n$ ), such that  $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$
- (ii)  $E \subseteq V \times V$  where  $\mu_2: V \times V \rightarrow [0, 1]$  and  $\gamma_2: V \times V \rightarrow [0, 1]$  are such that  $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$  and  $\gamma_2(v_i, v_j) \leq \max\{\gamma_1(v_i), \gamma_1(v_j)\}$  and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ , for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ).

**Definition 2.5.** [2] If  $v_i, v_j \in V \subseteq G$ , the  $\mu$ -strength of connectedness between two vertices  $v_i$  and  $v_j$  is defined as  $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) : k = 1, 2, \dots, n\}$  and  $\gamma$ -strength of connectedness between two vertices  $v_i$  and  $v_j$  is defined as  $\gamma_2^\infty(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) : k = 1, 2, \dots, n\}$ . If  $u$  and  $v$  are connected by means of paths of length  $k$  then  $\mu_2^k(u, v)$  is defined as  $\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \dots \wedge \mu_2(v_{k-1}, v) : (u, v_1, v_2, \dots, v_{k-1}, v) \in V\}$  and  $\gamma_2^k(u, v)$  is defined as  $\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \dots \vee \gamma_2(v_{k-1}, v) : (u, v_1, v_2, \dots, v_{k-1}, v) \in V\}$ .

**Definition 2.6.** [4] Let  $G: (A, B)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then the degree of a vertex  $v_i \in G$  is defined by  $d(v_i) = (d\mu_1(v_i), d\gamma_1(v_i))$ , where  $d\mu_1(v_i) = \sum \mu_2(v_i, v_j)$  and  $d\gamma_1(v_i) = \sum \gamma_2(v_i, v_j)$ , for  $(v_i, v_j) \in E$  and  $\mu_2(v_i, v_j) = 0$  and  $\gamma_2(v_i, v_j) = 0$  for  $(v_i, v_j) \notin E$ .

**Definition 2.7.** [4] Let  $G : (A, B)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then the total degree of a vertex  $v_i \in G$  is defined by  $td(v_i) = (td\mu_1(v_i), td\gamma_1(v_i))$ , where  $td\mu_1(v_i) = d\mu_1(v_i) + \mu_1(v_i)$  and  $td\gamma_1(v_i) = d\gamma_1(v_i) + \gamma_1(v_i)$ .

**Definition 2.8.** [6] Let  $G : (A, B)$  be an intuitionistic fuzzy graph. The  $\mu$   $d_2$ - degree of a vertex  $u \in G$  is defined as  $d_{(2)}\mu_1(u) = \sum \mu_{(2)}^2(u, v)$  where  $\mu_{(2)}^2(u, v) = \sup\{\mu_2(u, u_1) \wedge \mu_2(u_1, v) : u, u_1, v \text{ is the shortest path connecting } u \text{ and } v \text{ of length } 2\}$ . The  $\gamma$   $d_2$ - degree of a vertex  $u \in G$  is defined as  $d_{(2)}\gamma_1(u) = \sum \gamma_{(2)}^2(u, v)$  where  $\gamma_{(2)}^2(u, v) = \inf\{\gamma_2(u, u_1) \vee \gamma_2(u_1, v) : u, u_1, v \text{ is the shortest path connecting } u \text{ and } v \text{ of length } 2\}$ . The  $d_2$ - degree of a vertex  $u$  is defined as  $d_{(2)}(u) = (d_{(2)}\mu_1(u), d_{(2)}\gamma_1(u))$ .

The minimum  $d_{(2)}$ - degree of  $G$  is  $\delta_{(2)}(G) = \wedge\{d_{(2)}(u) : u \in V\}$

The maximum  $d_{(2)}$ -degree of  $G$  is  $\Delta_{(2)}(G) = \vee\{d_{(2)}(u) : u \in V\}$ .

**3. d<sub>2</sub>- pseudo degree of a vertex in an intuitionistic fuzzy graph**

In this section, d<sub>2</sub> - pseudo degree of a vertex in an intuitionistic fuzzy graph is introduced.

**Definition 3.1.** Let G : (A,B) be an intuitionistic fuzzy graph. The membership pseudo degree of a vertex u ∈ G is defined as d<sub>(a)μ</sub>(u) =  $\frac{t_\mu}{d_i}$  where t<sub>μ</sub> is the sum of membership degrees of vertices incident with vertex u . The non-membership pseudo degree of a vertex u ∈ G is defined as d<sub>(a)γ</sub>(u) =  $\frac{t_\gamma}{d_i}$  where t<sub>γ</sub> is the sum of non-membership degrees of vertices incident with vertex u and d<sub>i</sub> is the total number of edges incident with the vertex u . The pseudo degree of a vertex u ∈ G is defined as d<sub>(a)</sub>(u) = (d<sub>(a)μ</sub>(u), d<sub>(a)γ</sub>(u)) .

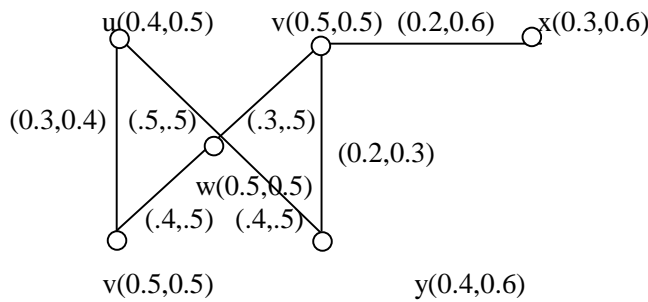
**Definition 3.2.** Let G : (A,B) be an intuitionistic fuzzy graph. The pseudo total degree of a vertex u ∈ G is defined as td<sub>(a)</sub>(u) = (td<sub>(a)μ</sub>(u), td<sub>(a)γ</sub>(u)) where td<sub>(a)μ</sub>(u) = d<sub>(a)μ</sub>(u) + μ<sub>1</sub>(u) and td<sub>(a)γ</sub>(u) = d<sub>(a)γ</sub>(u) + γ<sub>1</sub>(u) . It can also be defined as td<sub>(a)</sub>(u) = d<sub>(a)</sub>(u) + A(u) .

**Definition 3.3.** Let G : (A,B) be an intuitionistic fuzzy graph. The membership d<sub>2</sub>-pseudo degree of a vertex u ∈ G is defined as d<sub>(a)(2)μ</sub>(u) =  $\frac{\sum d_{(2)}\mu_1(u)}{d_i}$ . The non-membership d<sub>2</sub>-pseudo degree of a vertex u ∈ G is defined as d<sub>(a)(2)γ</sub>(u) =  $\frac{\sum d_{(2)}\gamma_1(u)}{d_i}$  where d<sub>i</sub> is the number of edges incident with the vertex u . The d<sub>2</sub>- pseudo degree of a vertex u is defined as d<sub>(a)(2)</sub>(u) = (d<sub>(a)(2)μ</sub>(u), d<sub>(a)(2)γ</sub>(u)) .

The minimum d<sub>2</sub>- pseudo degree of G is δ<sub>(a)(2)</sub>(G) =  $\wedge \{d_{(a)(2)}(v) : v \in V\}$ .

The maximum d<sub>2</sub> - pseudo degree of G is Δ<sub>(a)(2)</sub>(G) =  $\vee \{d_{(a)(2)}(v) : v \in V\}$

**Example 3.4.** Consider an intuitionistic fuzzy graph G = (A,B) on G\* : (V,E).



$$d_{(2)}(u) = (0.6, 1.0), d_{(2)}(v) = (0.8, 1.0), d_{(2)}(w) = (0.2, 0.6), d_{(2)}(x) = (0.7, 1.0), d_{(2)}(y) = (0.9, 1.6), d_{(2)}(z) = (0.4, 1.2)$$

$$d_{(a)(2)}(u) = \frac{d_{(2)}(v)+d_{(2)}(w)}{2} = \frac{(0.8,1)+(0.2,0.6)}{2} = \frac{(1,0.7)}{2} = (0.5, 0.35)$$

$$d_{(a)(2)}(v) = \frac{d_{(2)}(u)+d_{(2)}(w)}{2} = \frac{(0.6,1)+(0.2,0.6)}{2} = \frac{(0.8,0.7)}{2} = (0.4, 0.35)$$

similarly,  $d_{(a)(2)}(w) = (0.75, 1.15), d_{(a)(2)}(x) = (0.5, 1.1), d_{(a)(2)}(y) = (0.45, 0.8), d_{(a)(2)}(w) = (0.7, 1)$ .

**4.  $(2, (c_1, c_2))$ -pseudo regular and  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graphs**

In this section,  $(2, (c_1, c_2))$ -pseudo regular and  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graphs are introduced and the relation between them is established.

**Definition 4.1.** Let  $G : (A, B)$  be an intuitionistic fuzzy graph. If  $d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$  then  $G$  is said to be  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

**Definition 4.2.** Let  $G : (A, B)$  be an intuitionistic fuzzy graph. Then the  $d_2$ -pseudo total degree of a vertex  $u \in V$  is defined as  $td_{(a)(2)}(u) = (td_{(a)(2)}\mu(u), td_{(a)(2)}\gamma(u))$ , where  $td_{(a)(2)}\mu(u) = d_{(a)(2)}\mu(u) + \mu_1(u)$  and  $td_{(a)(2)}\gamma(u) = d_{(a)(2)}\gamma(u) + \gamma_1(u)$ . Also, it can be defined as  $td_{(a)(2)}(u) = d_{(a)(2)}(u) + A(u)$  where  $A(u) = (\mu_1(u), \gamma_1(u))$ .

**Definition 4.3.** Let  $G : (A, B)$  be an intuitionistic fuzzy graph. If each vertex of  $G$  has same  $d_2$ -pseudo total degree, then  $G$  is said to be  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph.

**Remark 4.4.** A  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph need not be  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

**Remark 4.5.** A  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph need not be  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph.

**Remark 4.6.** A  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph is also  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph.

**Theorem 4.7.** Let  $G : (A, B)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ .

Then  $A(u) = (k_1, k_2)$ , for all  $u \in V$  if and only if the following conditions are equivalent.

1.  $G : (A, B)$  is  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.
2.  $G : (A, B)$  is  $(2, (c_1 + k_1, c_2 + k_2))$ -pseudo totally regular intuitionistic fuzzy graph.

**Proof.** Suppose  $A(u) = (k_1, k_2)$ , for all  $u \in V$

Assume that  $G$  is a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

Then  $d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$ . So  $td_{(a)(2)}(u) = d_{(a)(2)}(u) + A(u) = (c_1, c_2) + (k_1, k_2) = (c_1 + k_1, c_2 + k_2)$ . Hence  $G$  is a  $(2, (c_1 + k_1, c_2 + k_2))$ -pseudo totally regular intuitionistic fuzzy graph.

Thus (i)  $\Rightarrow$  (ii) is proved.

Now suppose  $G$  is  $(2, (c_1 + k_1, c_2 + k_2))$ -pseudo totally regular intuitionistic fuzzy graph.

$td_{(a)(2)}(u) = (c_1 + k_1, c_2 + k_2)$ , for all  $u \in V \Rightarrow d_{(a)(2)}(u) + A(u) = (c_1 + k_1, c_2 + k_2)$ , for all  $u \in V$ .

$d_{(a)(2)}(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2)$ , for all  $u \in V \Rightarrow d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$ .

Hence  $G$  is  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph. Thus (i) and (ii) are equivalent.

Conversely assume (i) and (ii) are equivalent. Let  $G$  be a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph and  $(2, (c_1 + k_1, c_2 + k_2))$ -pseudo totally regular intuitionistic fuzzy graph. So,  $td_{(a)(2)}(u) = (c_1 + k_1, c_2 + k_2)$  and  $d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$ .  $d_{(a)(2)}(u) + A(u) = (c_1 + k_1, c_2 + k_2)$  and  $d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$ .

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$d_{(a)(2)}(u)+A(u) = (c_1, c_2)+(k_1, k_2)$  and  $d_{(a)(2)}(u) = (c_1, c_2)$ , for all  $u \in V$ .  
 $A(u) = (k_1, k_2)$ , for all  $u \in V$ . Hence  $A(u) = (k_1, k_2)$ .

**5. (2, (c<sub>1</sub>, c<sub>2</sub>))-pseudo regularity on peterson graph with specific membership functions**

In this section (2,( c<sub>1</sub>, c<sub>2</sub>))- pseudo regularity on peterson graph is discussed with some specific membership function.

**Theorem 5.1.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is peterson graph. If  $B$  is a constant function, then  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) -pseudo regular intuitionistic fuzzy graph.

**Proof.** Consider peterson graph on  $G^*(V,E)$ .

Let  $\mu_2(e_i) = k_1, \gamma_2(e_i) = k_2$ . Then  $d_{(2)}(u) = (6k_1, 6k_2)$ , for all  $u \in V$ .

$$d_{(a)(2)}(u) = \sum \frac{d_{(2)}(u)}{d_i} = \frac{(6k_1,6k_2)+(6k_1,6k_2)+(6k_1,6k_2)}{3} = (6k_1, 6k_2)$$

$$d_{(a)(2)}(u) = (c_1, c_2) \text{ where } c_1 = 6k_1, c_2 = 6k_2,$$

Hence  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) -pseudo regular intuitionistic fuzzy graph.

**Remark 5.2.** The converse of above theorem 5.1 need not be true.

**Theorem 5.3.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is peterson graph. If the edges on the cycle takes membership values (k<sub>1</sub>, k<sub>2</sub>) and the line joining the two cycle takes membership values (r<sub>1</sub>, r<sub>2</sub>), then  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) -pseudo regular intuitionistic fuzzy graph.

**Proof.** Consider Peterson graph on  $G^*(V,E)$ . Let the edges on the cycle takes membership values (k<sub>1</sub>, k<sub>2</sub>) and the line joining the two cycle takes membership values (r<sub>1</sub>, r<sub>2</sub>).

Then,  $d_{(2)}(u) = (2k_1 + 4r_1, 2k_2 + 4r_2)$

$$d_{(a)(2)}(u) = \sum \frac{d_{(2)}(u)}{d_i} = \frac{(2k_1+4r_1,2k_2+4r_2)+(2k_1+4r_1,2k_2+4r_2)+(2k_1+4r_1,2k_2+4r_2)}{3} = (2k_1 + 4r_1,$$

$$2k_2 + 4r_2)$$

$$d_{(a)(2)}(u) = (2k_1 + 4r_1, 2k_2 + 4r_2) = (c_1, c_2)$$

where  $c_1 = (2k_1 + 4r_1), c_2 = (2k_2 + 4r_2)$ .

Hence  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) -pseudo regular intuitionistic fuzzy graph.

**6. (2, (c<sub>1</sub>, c<sub>2</sub>))-pseudo regularity on ladder graph with specific membership functions**

In this section, (2,( c<sub>1</sub>, c<sub>2</sub>))- pseudo regularity on ladder graph is discussed with some specific membership function.

**Theorem 6.1.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is a ladder graph on 4 vertices. If  $B$  is a constant function, then  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) -pseudo regular intuitionistic fuzzy graph.

**Proof.** Suppose  $B$  is a constant function say  $B(uv) = (k_1, k_2)$ , for all  $uv \in E$ . Then

$d_{(2)}(u) = (2k_1, 2k_2)$  for end vertices and  $d_{(2)}(u) = (3k_1, 3k_2)$  for internal vertices  $\Rightarrow d_{(a)(2)}(u) = (k_1, k_2)$ , for all  $u \in V$  Hence  $G$  is (2, (c<sub>1</sub>, c<sub>2</sub>)) –pseudo regular intuitionistic fuzzy graph.

**Remark 6.2.** The converse of theorem 6.1 need not be true.

**Theorem 6.3.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph on  $G^*(V,E)$ , a ladder graph  $L_n(n > 1)$ . If alternate edges have same membership values, then  $G$  is  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph where  $c_1 = \min\{\mu_2(uv)\}$  and  $c_2 = \max\{\gamma_2(uv)\}$ .

**Remark 6.4.** If  $A$  is constant function, then the theorem 6.1 and 6.3 hold good for  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph.

### 7. $(2, (c_1, c_2))$ -pseudo regularity on a cycle with specific membership functions

In this section  $(2, (c_1, c_2))$ -pseudo regularity on a cycle is discussed with some specific membership function.

**Theorem 7.1.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is the cycle of length  $\geq 5$ . If  $\mu_2$  and  $\gamma_2$  are constant functions, then  $G$  is a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph where  $(c_1, c_2) = (2\mu_2, 2\gamma_2)$ .

**Remark 7.2.** The converse of the theorem 7.1 need not be true.

**Theorem 7.3.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is an even cycle of length  $\geq 6$ . If alternate edges have same membership and non-membership values then  $G$  is a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

**Proof.** If alternate edges have same membership and non-membership values then

$$\mu_2(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \quad \gamma_2(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

Here we have 4 possible cases

1.  $k_1 > k_2$  and  $k_3 > k_4$  2.  $k_1 > k_2$  and  $k_3 < k_4$  3.  $k_1 < k_2$  and  $k_3 > k_4$  4.  $k_1 < k_2$  and  $k_3 < k_4$

In all cases  $d_{(a)(2)}(u)$  is constant for all  $u \in V$ .

Hence  $G$  is a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph where  $d_{(a)(2)}(u) = (c_1, c_2)$ .

**Remark 7.4.** If all the vertices take same membership and non-membership values then the above theorem holds good for  $(2, (c_1, c_2))$ -pseudo totally regular intuitionistic fuzzy graph.

**Remark 7.5.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is an odd cycle of length  $> 5$ . Even if the alternate edges have same membership and same non-membership values, then  $G$  need not be a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

**Theorem 7.6.** Let  $G : (A,B)$  be an intuitionistic fuzzy graph such that  $G^*(V,E)$  is any cycle of length  $> 4$ . Let  $k_2 \geq k_1$  and  $k_3 \geq k_4$ . Let

$$\mu_2(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \quad \gamma_2(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

then  $G$  is a  $(2, (c_1, c_2))$ -pseudo regular intuitionistic fuzzy graph.

**Proof.** Case (i)  $G^*$  be an even cycle.

$$d_{(2)}(v_i) = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4) = (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3)$$

$$d_{(a)(2)}(v_i) = (2k_1, 2k_3)$$

(2, (c<sub>1</sub>, c<sub>2</sub>))-Pseudo Regular Intuitionistic Fuzzy Graphs

$d_{(a)(2)}(v_i) = (c_1, c_2)$  where  $c_1 = 2k_1, c_2 = 2k_3$

Hence  $G$  is  $(2, (c_1, c_2))$  -pseudo regular intuitionistic fuzzy graph.

Case (ii)  $G^*$  be an odd cycle. Let  $e_1, e_2, \dots, e_{2n+1}$  be edges of  $G^*$

$$d_{(2)}(v_1) = (\mu_2(e_1) \wedge \mu_2(e_2), \gamma_2(e_1) \vee \gamma_2(e_2)) + \mu_2(e_{2n}) \wedge \mu_2(e_{2n+1}), \gamma_2(e_{2n}) \vee \gamma_2(e_{2n+1})) \\ = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4) = (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3)$$

$$d_{(a)(2)}(v_1) = (2k_1, 2k_3)$$

$d_{(a)(2)}(v_1) = (c_1, c_2)$  where  $c_1 = 2k_1, c_2 = 2k_3$

$$d_{(2)}(v_2) = (\mu_2(e_2) \wedge \mu_2(e_3), \gamma_2(e_2) \vee \gamma_2(e_3)) + (\mu_2(e_1) \wedge \mu_2(e_{2n+1}), \gamma_2(e_1) \vee \gamma_2(e_{2n+1})) \\ = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4) = (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3)$$

$$d_{(a)(2)}(v_2) = (2k_1, 2k_3)$$

Proceeding like this we get  $d_{(a)(2)}(v_n) = (c_1, c_2)$  where  $c_1 = 2k_1, c_2 = 2k_3$ . Hence  $d_{(a)(2)}(v_i) = (c_1, c_2)$  for all  $i$ . So  $G$  is  $(2, (c_1, c_2))$  -pseudo regular intuitionistic fuzzy graph.

**Remark 7.7.** The above theorem 7.6 holds good for  $(2, (c_1, c_2))$  -pseudo totally regular intuitionistic fuzzy graph if all the vertices take same membership and same non-membership values.

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REFERENCES

1. K.T.Atanassov, *Intuitionistic fuzzy sets: Theory, applications*, Studies in fuzziness and soft computing, Heidelberg, New York, Physica-Verl., 1999.
2. K.T.Atanassov, P.R.Yager and V.Atanassov, *Intuitionistic fuzzy graph* interpretations of multi-person multi-criteria decision making, EUSFLAT Conf., 2003,177-182.
3. J.N.Moderson and P.S.Nair, *Fuzzy Graphs and Fuzzy Hypergraphs* Physica-Verlag, Heidelberg (2000).
4. M.G.Karunambigai, R.Parvathi and P.Buvaneswari, Constant intuitionistic fuzzy graphs, *Notes on Intuitionistic Fuzzy Sets*, 17 (2011) 37-47.
5. M.G.Karunambigai, S.Sivasankar and K.Palanivel, Some properties of regular intuitionistic fuzzy graphs, *International Journal of Mathematics and Computation*, 26 (4) (2015) .
6. M.Pal and H.Rashmanlou, Irregular interval-valued fuzzy graphs, *Annals of Pure and Applied Mathematics*, 3(1) (2013) 56-66.
7. R.S.Narayanan and S.Maheswari, On  $(2, (c_1, c_2))$ -regular intuitionistic fuzzy graphs, *IJAR CET*, 4 (12) (2015) 4352-4358.
8. S.Maheswari and C.Sekar, Pseudo regular fuzzy graphs, *Annals of Pure and Applied Mathematics*, 11(2) (2016) 73-84.
9. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.