

Fuzzy Markov Decision Model in the Analysis of Agricultural System for Enhancing Agricultural Production

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Abstract. In this paper, a fuzzy Markov approach for assessing the age specific finding of nutrient requirement of various parts of crop and fertilizer requirement is presented. A fuzzy Markov chain model is dynamic that can be viewed as a hybrid of possibility theory and fuzzy expert system. For this purpose, the expert agricultural system is designed for enhancing agricultural production. In order to develop decision support system and to make the expert system more efficient, the sophisticated mathematical algorithm such as fuzzy Markov approach is designed. By using the fuzzy Markov approach, age specific crop rate is calculated. Finally, the calculated values are indexed. The index for the age specific crop rate varies as the product of proportion of possibility of becoming requirement of nutrients and fertilizers at a given age in two age / time steps.

Keywords: Fuzzy Set, Complement of Fuzzy Set, Fuzzy Relation, Possibility Distribution Function, Fuzzy Markov Chain, Age specific crop rate.

AMS Mathematics Subject Classification (2010): 60J20

1. Introduction

Agriculture worldwide faces daunting challenges because of increasing population growth and changing food consumption patterns, natural resource scarcity, environmental degradation, climate change, and global economic restructuring. Yet, at the same time, there are unprecedented hopeful changes and opportunities for the future, including a remarkable emergence of innovations in farming practices and systems and technological advances that have generated promising results for improving agricultural sustainability and an increase in consciousness and concern by consumers about the sources of their food and how it is produced.

Application of a Fuzzy systems approach to agriculture is not limited to the farm level. The collective and potentially synergistic effects of agricultural systems at a landscape or community scale have gained recognition. However, the scientific foundation for and data needed to develop a landscape approach for improving sustainability of agriculture is sparse. Research suggests that the distribution of farm

B. Usha Rani, A. Hari Ganesh and S. Jayakumar

types and farming activities across a landscape could be designed to achieve greater productivity, resistance, and resilience and improve the sustainability of local and regional agricultural systems that support personal and community well-being.

The term Agricultural System can comprise a diverse range of activities. Agricultural systems that move toward greater sustainability generally strive for several fundamental qualities. One of those qualities is to work with natural ecological and biogeochemical processes and cycles to maximize synergistic interactions and the beneficial use of internal resources, and to minimize dependence on external inputs. Another quality is to close nutrient, energy, and other resource cycles to the maximum extent feasible to reduce undesirable losses to the environment and additional waste disposal activities. Third, farmers, conventional or alternative, who work toward improved sustainability tend to understand and work with the social, cultural, and economic goals of people and institutions throughout the farm and food chain, which encourages synergistic relationships in the social and economic realm and increases the likelihood of desired outcomes emerging from investment of time and resources.

The main aim of this article is to introduce the fuzzy mathematical model by using fuzzy Markov chain in order to analyse the agricultural system particularly to identify nutrient requirement of various parts of crop at the different age group also the model identifies fertilizer requirement level at the different age group for crop growth and yield.

The following chapters will illustrate as follows. In section 2, the definition of possibility distribution function is briefly outlined, followed by fuzzy expert system is designed for the analyses of agricultural system in section 3. In section 4, we introduce an analytic technique in order to analyse the agricultural system by using fuzzy Markov chain.

2. Basic concepts

In this section, we introduce the definition of possibility distribution function, fuzzy set and fuzzy Markov chain. Based on these definitions, the fuzzy expert system is introduced in order to analyse the agricultural system particularly to identify nutrient and fertilizer requirement at different age group.

2.1. Definition (Possibility Distribution Function)

Let \tilde{F} be a fuzzy set in a universe of discourse U , which is characterized by its membership function $\mu_{\tilde{F}}(u)$ which is interpreted as the compatibility of $u \in U$ with the concept labeled \tilde{F} . Let X be a variable taking values in U and \tilde{F} act as a fuzzy restriction, $\tilde{R}(X)$, associated with X . Then the proposition 'X is \tilde{F} ', which translates into $\tilde{R}(X) = \tilde{F}$ associates a fuzzy event, $\tilde{\pi}$, with X which is postulated to be equal to $\tilde{R}(X)$. The possibility distribution function, $\tilde{\pi}_{\tilde{R}}(u)$ characterizing the fuzzy event $\tilde{\pi}$ is defined to be numerically equal to the membership function $\mu_{\tilde{F}}(u)$ of \tilde{F} , that is $\tilde{\pi} \cong \mu_{\tilde{F}}$, that is, $\tilde{\pi} \cong \pi_{\tilde{F}}(u)$. The symbol \cong will always stand for 'denotes' or 'is equal to be'.

Fuzzy Markov Decision Model in the Analysis of Agricultural System for Enhancing
Agricultural Production

2.2. Definition (finite fuzzy sets)

A (finite) fuzzy set (a fuzzy event, a fuzzy relation or a fuzzy restriction,) $\tilde{\pi}$, on S , is characterized by possibility distribution function $\tilde{\pi}_{\tilde{R}}$,

$$\tilde{\pi}_{\tilde{R}} : S \rightarrow [0, 1]$$

that takes on a finite member of possible fuzzy sets will be denoted by

$\tilde{\pi} = \{\tilde{\pi}_n, n = 0, 1, 2, \dots\}$. The set of all fuzzy set on S is denoted by $\mathcal{F}(S)$.

Let $\{\tilde{\pi}_n, n = 0, 1, 2, \dots\}$ be a sequence of random fuzzy sets (or a discrete fuzzy parameter stochastic process). Let the possible outcomes of $\tilde{\pi}_n$ be \tilde{i} ($\tilde{i}, 0, 1, 2, \dots$), where the number of outcomes may be finite (say, m) or denumerable. The possible values of $\tilde{\pi}_n$ constitute a set $S = \{0, 1, 2, \dots\}$, and that the process has the fuzzy state space S . Unless otherwise stated, by fuzzy state space of a fuzzy Markov Chain, we shall imply discrete fuzzy state space (having a finite or a countably infinite number of elements); it could be $\mathcal{F}(S) = \{0, 1, 2, \dots\}$.

2.3. Definition (fuzzy Markov chain)

The fuzzy stochastic process $\{\tilde{\pi}_n, n = 0, 1, 2, \dots\}$ is called a Fuzzy Markov Chain, if, for $\tilde{i}, \tilde{j}, \tilde{i}_0, \tilde{i}_1, \dots, \tilde{i}_{n-1} \in \mathcal{F}(S)$.

$$\begin{aligned} \pi_{\tilde{R}}\{x_{n+1} = \tilde{j} / x_n = \tilde{i}, x_{n-1} = \tilde{i}_{n-1}, \dots, x_0 = \tilde{i}_0\} \\ = \pi_{\tilde{R}}\{x_{n+1} = \tilde{j} / x_n = \tilde{i}\} \\ = \pi_{\tilde{i}\tilde{j}} \quad (\text{say}) \end{aligned}$$

whenever the first member is defined.

3. Fuzzy expert system in the analysis of agricultural system

Expert system in Agriculture consists of agricultural expert and fuzzy system. We apply the fuzzy expert system in Agricultural System, particularly to find the fertilizer requirement of soil for different parts growth of crop at different time.

The following classifications are related to medical diagnosis:

1. Nutrient and observation
2. The agricultural knowledge
3. Nutrient requirement
4. Fertilizer requirement

$\tilde{N} = (\tilde{n}_1, \dots, \tilde{n}_m)$; set of nutrients

$\tilde{P} = (\tilde{p}_1, \dots, \tilde{p}_n)$; set of parts of crop

$\tilde{T} = (\tilde{t}_1, \dots, \tilde{t}_q)$; set of age groups

$\tilde{F} = (\tilde{f}_1, \dots, \tilde{f}_r)$; set of fertilizers

B. Usha Rani, A. Hari Ganesh and S. Jayakumar

All $\tilde{N}, \tilde{P}, \tilde{T}, \tilde{F}$ are fuzzy sets characterized by their respective membership values. The membership values of these relationships $[\tilde{T}, \tilde{N}]$, $[\tilde{N}, \tilde{P}]$ and $[\tilde{P}, \tilde{F}]$ converted into fuzzy relation $[\tilde{R}_1, \tilde{R}_2, \tilde{R}_3]$ are mentioned in two aspects.

- (i) Observance of \tilde{n}_i by \tilde{p}_i (injected - \tilde{I}_1, \tilde{I}_2)
- (ii) Non - Observance of \tilde{n}_i by \tilde{p}_i (uninjected - \tilde{U}_1, \tilde{U}_2)

This leads to the definition of fuzzy relation. Assume two fuzzy sets such as observance ($x_i = 1$) and non-observance ($x_i = 0$). The transition of possibility distribution functions of fuzzy Markov process in a 2×2 matrix is considered;

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}$$

All elements in matrix Π are non-negative numbers, and $\sum_{j=0}^{\infty} \pi_{ij} = 1, i = 0, 1$. for

observed and re-observed, we get $\pi_{10} = 0, \pi_{11} = 1$ for $\pi_{01}(t) = 1 - \pi_{00} \cdot \pi_{00}(t)$ given by

$$\pi_{00}(t) = \pi_{\alpha\tilde{R}}\{x_t | x_{t-1}\} = \pi_{\alpha\tilde{R}}(x_t) / \pi_{\alpha\tilde{R}}(x_{t-1})$$

Thus the n-step transition process can be written as

$$\pi_{00}^{(n)}(t) = \prod_{k=t}^{t+n-1} \pi_{00}(k) = \pi_{\alpha\tilde{R}}(x_{t+n-1}) / \pi_{\alpha\tilde{R}}(x_{t-1})$$

From the original data of the age distribution, multistep transition process is calculated. Here t is the state of age group, and we get the new age specific member rate (ASMR) between the age group t to t + n is given by

$$(ASMR)_t = 100 \pi_{\alpha\tilde{R}}(x_t) \pi_{01}^{(n)}(t) = \frac{100 \pi_{\alpha\tilde{R}}(x_t) (\pi_{\alpha\tilde{R}}(x_{t-1}) - \pi_{\alpha\tilde{R}}(x_{t+n-1}))}{\pi_{\alpha\tilde{R}}(x_{t-1})}$$

The membership functions of these (injected - \tilde{I}_1, \tilde{I}_2 and uninjected - \tilde{U}_1, \tilde{U}_2) two fuzzy states are defined to be

$$\mu_{\tilde{I}}(x) = \max_{n \in N} \{ \min(\mu_{\tilde{I}_1}(x), \mu_{\tilde{I}_2}(x)) \}, \quad x \in X \times X$$

$$\mu_{\tilde{U}}(y) = \max_{n \in N} \{ \min(\mu_{\tilde{U}_1}(y), \mu_{\tilde{U}_2}(y)) \}, \quad y \in Y \times Y$$

The \tilde{n}_i, \tilde{p}_i observance relationships are acquired empirically from agricultural experts using the membership values. Other relationships such as crop age / nutrient, nutrient / part of crop are also defined as fuzzy sets. Possibility interpretations of relations (max - min) are used. Given an crop age - group / part of crop relationships yield fuzzy indications that are basis for establishing observance and non - observance of nutrient by the various parts of crop for crop growth. Finally, two different relations are calculated by means of fuzzy relation (fuzzy composition).

$$\tilde{I} = \tilde{I}_1 \circ \tilde{I}_2 = \{x, \max_{n \in N} \{ \min(\mu_{\tilde{I}_1}(x), \mu_{\tilde{I}_2}(x)) \} | x \in X \times X\}$$

$$\tilde{U} = \tilde{U}_1 \circ \tilde{U}_2 = \{(y, \max_{n \in N} \{ \min(\mu_{\tilde{U}_1}(y), \mu_{\tilde{U}_2}(y)) \}) | y \in Y \times Y\}$$

Fuzzy Markov Decision Model in the Analysis of Agricultural System for Enhancing
Agricultural Production

To calculate the crop age specific member rate for the above fuzzy sets, we use the following formulas.

$$R_t(\tilde{I}) = \frac{100\pi_{\alpha\tilde{I}}(x_t)(\pi_{\alpha\tilde{I}}(x_{t-1}) - \pi_{\alpha\tilde{I}}(x_{t+n-1}))}{\pi_{\alpha\tilde{I}}(x_{t-1})}$$

$$R_t(\tilde{U}) = \frac{100\pi_{\alpha\tilde{U}}(y_t)(\pi_{\alpha\tilde{U}}(y_{t-1}) - \pi_{\alpha\tilde{U}}(y_{t+n-1}))}{\pi_{\alpha\tilde{U}}(y_{t-1})}$$

4. Application on agricultural system for nutrient and fertilizer requirement for crop growth

Proper nutrition is essential for satisfactory crop growth and production. The use of soil tests can help to determine the status of available nutrients to evaluate the suitability of land for various agricultural crops. There are at least seventeen elements are considered essential nutrients for plant growth, and 14 of these elements come from the soil. If there is a deficiency or superfluency of any essential nutrients in soil, plants growth and yield may be affected. So the land with suitable nutrient level has to be identified for cultivation of various crops.

In the pathology of type of lands, we consider the following set N of common Nutrients:

$$\tilde{N} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}\}$$

n ₁ – Nitrogen (N) (K)	n ₂ – Phosphorous (P)	n ₃ – Potassium
n ₄ – Calcium (Ca)	n ₅ – Magnesium (Mg)	n ₆ – Zinc (Zn)
n ₇ – Copper (Cu) (Mn)	n ₈ – Iron (Fe)	n ₉ – Manganese
n ₁₀ – Nickel (Ni)		

We consider the following set \tilde{P} of parts of crop

$$\tilde{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$$

p1 – Leaf
 p2 – leaf Sheath
 p3 – Stem
 p4 – Crown Roots
 p5 – Internode
 p6 – Nodal Roots
 p7 – Tiller
 p8 – Node

We consider the following set of \tilde{F} fertilizers for the growth of crop for these various parts.

$$\tilde{F} = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}\}$$

f₁ – Urea
 f₂ – Ammonium Nitrate
 f₃ – Ammonium Sulphate
 f₄ – Calcium Nitrate

B. Usha Rani, A. Hari Ganesh and S. Jayakumar

- f₅ – Single SuperPhosphate
- f₆ – Concentrated SuperPhosphate
- f₇ – Potassium Sulphate
- f₈ – Potassium Chloride
- f₉ – Mixed Fertilizer (without Cu)
- f₁₀ – Mixed Fertilizer (with Cu)

We consider the following \tilde{T} of common crop age group $\tilde{T} = \{t_0, t_1, \dots, t_5\}$. The linguistic values of these relationships $[\tilde{T}, \tilde{N}]$, $[\tilde{N}, \tilde{P}]$ and $[\tilde{P}, \tilde{F}]$ are converted into fuzzy values and three matrices have been formed. The three matrices are:

- Crop Age, Nutrient observation confirmation relationship matrix (\tilde{R}_1)
- Nutrient observation, various parts of crop confirmation relationship matrix (\tilde{R}_2)
- Various parts of Crop, Fertilizer confirmation relationship matrix (\tilde{R}_3)

The above matrices are given as Table 1, Table 2 and Table 3, respectively. Then the computation of the max – min composition $\tilde{R}_4 = \tilde{R}_1 \circ \tilde{R}_2$ is assumed to describe the state of the crop (age – wise) in terms of nutrient observation by the various parts as a fuzzy subset \tilde{R}_4 of $\tilde{T} \times \tilde{P}$ characterized by its membership function,

$$\mu_{\tilde{R}_4}(t, p) = \max_{n \in N} \{ \min(\mu_{\tilde{R}_1}(t, n), \mu_{\tilde{R}_2}(n, p)) \}, \quad (t, p) \in \tilde{T} \times \tilde{P}$$

The max-min composition of \tilde{R}_4 and \tilde{R}_3 is defined as \tilde{R}_5 follows:

$$\tilde{R}_5 = \tilde{R}_4 \circ \tilde{R}_3 = \left\{ \left((t, f), \max_{f \in \tilde{F}} \{ \min(\mu_{\tilde{R}_4}(t, p), \mu_{\tilde{R}_3}(p, f)) \} \right) \mid (t, f) \in \tilde{T} \times \tilde{F} \right\}$$

on $\mu_{\tilde{R}_4 \circ \tilde{R}_3} = \mu_{\tilde{R}_5}(t, f)$ is again the membership function of a fuzzy relation of fuzzy sets.

A nutrient requirement is perfect at the age of the crop if $\mu_{\tilde{R}_6, j} = 1$ that is the fuzzy membership grade $\mu_{\tilde{R}_6, j} = 1$ implies the confirmation of perfect requirement of nutrient (n_i) for the part of the crop (p_i) at the (t_i th) age. For testing this model, the nutrient requirements of many crops (different age group) are taken linguistically in the presence of agricultural experts. The model largely depends upon the database. The accuracy and precision of the output will depend mainly on the database of fuzzy agricultural relationship. If the size of the database is increased, the present model can find the suitable level of fertilizer at the different age of the crop more accurately. We get better decision by multistep transmission of Markovian chain method and we get demographic information by the following calculation. The possibility distribution function of the complement of fuzzy set \tilde{R}_4 , $\pi_{\alpha \tilde{R}_4}(t, p)$ is denoted by $\pi_{\alpha \tilde{R}_4}(t, p) = 1 - \pi_{\tilde{R}_4}(t, p)$. The grades of possibility of complement of fuzzy set (\tilde{R}_4) is given in Table 4. Similarly, the possibility distribution function of the compliment of the fuzzy set $\tilde{R}_5 = \tilde{R}_4 \circ \tilde{R}_3$, $\mu_{\alpha \tilde{R}_5}(t, f)$ is denoted by $\pi_{\alpha \tilde{R}_5}(t, f) = 1 - \pi_{\tilde{R}_5}(t, f)$. The grades of possibility of complement of fuzzy set (\tilde{R}_5) is given in Table 5. From the above

Fuzzy Markov Decision Model in the Analysis of Agricultural System for Enhancing Agricultural Production

information, the Markovian risk rate is set up in Table 6 as a measure of nutrient requirement of various parts of crop (MCGROCP – Measure of common growth rate of crop part). The varies as the product of proportion of possible growth at a given age t, which is possibility of becoming required / injected in two age / time steps, $\pi_{\tilde{U}\tilde{I}}^{(2)}(t, p)$. Its calculated formula, is

$$MCGROCP_t(\tilde{T}, \tilde{P}) = 100\pi_{\alpha\tilde{R}_4}(t, p)\pi_{\tilde{U}\tilde{I}}^{(2)}(t, p), \text{ where,}$$

$$\pi_{\tilde{U}\tilde{I}}^{(2)}(t, p) = (\pi_{\alpha\tilde{R}_4}(t-1, p) - \pi_{\alpha\tilde{R}_4}(t+1, p)) / \pi_{\alpha\tilde{R}_4}(t-1, p)$$

$$t = 0, 1, 2, \dots, 7; p = p_1, p_2, \dots, p_8.$$

As such from the table 7, the Markovian rate of fertilizer requirement (MFRR – Measure of Fertilizer requirement rate) formula is,

$$MFRR_t(\tilde{T}, \tilde{F}) = 100\pi_{\alpha\tilde{R}_5}(t, f)\pi_{\tilde{U}\tilde{I}}^{(2)}(t, f) \text{ where}$$

$$\pi_{\tilde{U}\tilde{I}}^{(2)}(t, f) = (\pi_{\alpha\tilde{R}_5}(t-1, f) - \pi_{\alpha\tilde{R}_5}(t+1, f)) / \pi_{\alpha\tilde{R}_5}(t-1, f)$$

$$t = 0, 1, 2, \dots, 7; f = f_1, f_2, \dots, f_{10}.$$

Thus, the fuzzy Markov chain model can give precise and reliable information for different age – specific prevalence and its related factors.

Table 1: Crop Age, Nutrient observation confirmation relationship matrix (\tilde{R}_1)

Nutrient Age	n ₁	n ₂	n ₃	n ₄	n ₅	n ₆	n ₇	n ₈	n ₉	n ₁₀
Extremely Low (t ₀)	0	0	0	0	0	0	0	0	0	0
Very Low (t ₁)	.1	.1	.2	.3	.1	.3	0	0	.1	.2
Low (t ₂)	.1	.1	.3	.3	.2	.4	.2	.1	.1	.3
Normal (t ₃)	.3	.1	.3	.3	.3	.4	.2	.1	.2	.4
High (t ₄)	.4	.2	.4	.4	.3	.5	.3	.3	.3	.4
Very High (t ₅)	.6	.3	.5	.5	.4	.6	.4	.5	.4	.5
Extremely High (t ₆)	.6	.4	.6	.6	.5	.6	.5	.5	.5	.6

Table 2: Nutrient observation, various parts of crop confirmation relationship matrix (\tilde{R}_2)

Part of Crop Nutrient	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈
n ₁	.9	.7	.8	.7	.7	.9	1	1
n ₂	.7	0	.7	0	0	0	.8	.8
n ₃	0	0	.5	0	0	.5	1	1
n ₄	.8	.9	.9	0	0	0	0	0
n ₅	0	0	0	0	0	1	0	0
n ₆	0	0	0	0	1	1	0	0
n ₇	1	0	1	0	0	0	.7	1
n ₈	.8	0	.8	0	0	0	.7	.7
n ₉	0	0	0	1	0	0	0	0
n ₁₀	0	0	.8	0	0	1	0	.8

Table 3: Various parts of crop, fertilizer requirement confirmation relationship matrix \tilde{R}_3

Fertilizer Part of Crop	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆	f ₇	f ₈	f ₉	f ₁₀
P ₁	.9	.8	1	.8	.7	0	0	0	0	0
P ₂	.7	.9	0	0	0	0	0	0	0	0
P ₃	.8	.9	1	.8	.7	.8	0	0	.5	0
P ₄	.7	0	0	0	0	0	0	0	0	1
P ₅	.7	0	0	0	0	0	1	0	0	0
P ₆	.9	0	0	0	0	1	1	1	.5	0
P ₇	1	0	.7	.7	.8	0	0	0	1	0
P ₈	1	0	1	.7	.8	.8	0	0	1	0

Table 4: The grades of possibility of complement of fuzzy set (\tilde{R}_4)

Part of Crop Age	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈
Extremely Low (t ₀)	1	1	1	1	1	1	1	1
Very Low (t ₁)	.7	.7	.7	.9	.7	.7	.8	.8
Low (t ₂)	.7	.7	.7	.9	.6	.6	.7	.7
Normal (t ₃)	.7	.7	.6	.9	.6	.6	.7	.6
High (t ₄)	.6	.6	.6	.6	.5	.5	.6	.6
Very High (t ₅)	.4	.4	.4	.4	.4	.4	.4	.4
Extremely High (t ₆)	.4	.4	.4	.4	.4	.4	.4	.4

Table 5: The grades of possibility of complement of fuzzy set (\tilde{R}_5)

Fertilizer Age	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆	f ₇	f ₈	f ₉	f ₁₀
Extremely Low (t ₀)	1	1	1	1	1	1	1	1	1	1
Very Low (t ₁)	.7	.7	.7	.7	.7	.7	.7	.7	.7	.9
Low (t ₂)	.6	.7	.7	.7	.7	.6	.6	.6	.6	.9
Normal (t ₃)	.6	.6	.6	.6	.6	.6	.6	.6	.6	.9
High (t ₄)	.5	.6	.6	.6	.6	.5	.5	.5	.5	.6
Very High (t ₅)	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4
Extremely High (t ₆)	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4

Table 6: Index for measure of common growth rate of crop part

Part of Crop Age	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈
Very Low (t ₁)	21.0	21.0	21.0	09.0	28.0	28.0	24.0	24.0
Low (t ₂)	00.0	00.0	10.0	00.0	08.6	08.6	08.8	17.5
Normal (t ₃)	10.0	10.0	08.6	30.0	10.0	10.0	10.0	08.6
High (t ₄)	17.1	17.1	10.0	27.0	08.3	08.3	17.1	10.0
Very High (t ₅)	08.0	08.0	08.0	08.0	08.0	08.0	08.0	08.0
Extremely High (t ₆)	08.0	08.0	08.0	08.0	08.0	08.0	08.0	08.0

Table 7: Index of measure of fertilizer requirement rate

Fuzzy Markov Decision Model in the Analysis of Agricultural System for Enhancing
Agricultural Production

Fertilizer Age	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆	f ₇	f ₈	f ₉	f ₁₀
Very Low (t ₁)	28.00	21.00	21.00	21.00	21.00	28.00	26.00	28.00	28.00	09.00
Low (t ₂)	08.57	10.00	10.00	10.00	10.00	08.57	08.57	08.57	08.57	00.00
Normal (t ₃)	10.00	08.57	08.57	08.57	08.57	10.00	10.00	10.00	10.00	30.00
High (t ₄)	08.33	10.00	10.00	10.00	10.00	08.33	08.33	08.33	08.33	26.67
Very High (t ₅)	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00
Extremely High (t ₆)	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00	08.00

Table 6 and Table 7 show that there is wide variation as the product of proportion of becoming requirement of nutrients and fertilizers at a given age t_i in two age / time steps.

- From Table 6, it seems that nutrients injection and growth of all the parts of the crop occurs in all the ages.
- The highest nutrients observations and growth occurs in t_1 , t_3 and t_4 of age groups.
- The maximum requirement of fertilizers is also occurs in t_1 , t_3 and t_4 of age groups.

5. Conclusion

Thus in this paper, we make use of fuzzy Expert Decision Support System using Fuzzy Markov Chain for finding age specific requirement of fertilizers and nutrients of crop and its parts respectively. The same technique may also be extended for other agricultural crops with suitable modifications in the parameters involved.

REFERENCES

1. Fertilizer use by crop in Egypt First version, Published Food and Agriculture Organization of the United Nations, Rome, (2005).
2. S.Jayakumar and B.Arunraj, On fuzzy modelling for computing an index for medical diagnosis system, *Bio-Science Research Bulletin*, 25(2) (2009) 69 -77.
3. P.T. Whitacre, Toward Sustainable Agricultural Systems in the 21st Century, the National Academies Press, Washington, (2010).
4. H.J.Zimmermann, Fuzzy set theory and its applications, 2nd revised edn., Kulwer Academic Publishers (1996).