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Fuzzy Semi-Open Sets and Fuzzy Pre-Open Sets in Fuzzy Quad Topological Space

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Abstract. The aim of this paper is to introduce two new types of fuzzy open sets namely fuzzy q-semi-open sets and fuzzy q-pre-open sets in fuzzy q-topological spaces and also defined the fuzzy continuity namely fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity in fuzzy q-topological spaces.

Keywords: fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity.

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

Levine [11] introduced the idea of semi-open sets and semi-continuity in topological space and Mashhour et al. [3] introduced the concept of pre-open sets and pre continuity in a topological space. Maheshwari and Prasad [14] introduced semi-open sets in bitopological spaces. Jelic [8] generalized the idea of pre-open sets and pre continuity in bitopological space.

The study of tri-topological space was first initiated by Kovar [9]. Palaniammal [15] studied tri topological space and introduced semi and pre-open sets in tri topological space and he also introduced fuzzy tri topological space. Hameed and Moh. Abid [10] gives the definition of 123 open set in tri topological spaces. We [17] studied properties of tri semi-open sets and tri pre-open sets in tri topological space. Mukundan [5] introduced the concept on topological structures with four topologies, quad topology) and defined new types of open (closed) set. We have [18] introduced semi and pre-open sets in quad topological spaces.

In 1965, Zadeh [7] introduced the concept of fuzzy sets. In 1968 Change [4] introduced the concept of fuzzy topological spaces. Kandil [1] introduced fuzzy bitopological spaces in 1991, Fuzzy semi-open sets and fuzzy semi continuous mappings in fuzzy topological spaces was studied by Azad [6]. Bin [2] defined the concept of pre-open sets in fuzzy topological space. Thakur and Malviya [16] introduced semi-open sets, semi continuity in fuzzy bitopological spaces. Sampath Kumar [13] defined a (τ_i, τ_j) fuzzy pre-open set and characterized a fuzzy pair wise pre continuous mappings on a fuzzy bitopological space. We have [12] introducedfuzzy tri semi-open sets and fuzzy tri pre-open sets, fuzzy tri continuous function, fuzzy tri semi-continuous function and fuzzy tri pre-continuous functions and their basic properties.

In this paper, we introduce fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity and their fundamental properties in fuzzy q-topological space.

2. Preliminaries

Definition 2.1. [12] Consider two fuzzy tri topological spaces $(X, \tau_1, \tau_2, \tau_3)$, $(Y, \tau'_1, \tau'_2, \tau'_3)$. A fuzzy function $f : I^X \to I^Y$ is called a fuzzy tri continuous function if χ_λ is fuzzy tri open in X, for every tri open set χ_λ in Y.

Definition 2.2. [12] Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space then a fuzzy subset χ_λ of X is said to be fuzzy tri semi-open set if $\chi_\lambda \le cl\left(int\chi_\lambda\right)$ and complement of fuzzy tri semi-open set is fuzzy tri semi-closed. The collection of all fuzzy tri semi-open sets of X is denoted by $tri-FSO(X)$

Definition 2.3. [12] Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space then a fuzzy subset χ_λ of X is said to be fuzzy tri pre-open set if $\chi_\lambda \le tri-int\left(tri-cl\chi_\lambda\right)$ and complement of fuzzy tri pre-open set is fuzzy tri pre-closed. The collection of all fuzzy tri semi-open sets of X is denoted by $tri-FPO(X)$.

Definition 2.4. [5] Let X be a nonempty set and τ_1, τ_2, τ_3 and τ_4 are general topologies on X .Then a subset A of space X is said to be quad-open(q-open) set if $A \prec \tau_1 \vee \tau_2 \vee \tau_3 \vee \tau_4$ and its complement is said to be q-closed and set X with four topologies called q-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 2.5. [5] Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a quad topological space and let $A \subset X$. The intersection of all q-closed sets containing A is called the q-closure of A & denoted by $q-clA$.We will denote the q-interior (resp. q-closure) of any subset ,say of A by $q-\text{int}\,A$ $(q-clA)$,where $q-clA$ is the union of all q-open sets contained in A, and $q-clA$ is the intersection of all q-closed sets containing A.

3. Fuzzy q-semi-open sets and fuzzy q-pre-open sets in fuzzy q-topological space

Definition 3.1. Let X be a nonempty set τ_1, τ_2, τ_3 and τ_4 are fuzzy topologies on X. Then a fuzzy subset χ_λ of space X is said to be fuzzy q-open if $\chi_\lambda \prec \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be fuzzy q-closed and set X with four fuzzy topologies called fuzzy q-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 3.2. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space and let $\chi_\lambda \prec X$. The intersection of all fuzzy q-closed sets containing χ_λ is called the fuzzy q-closure of χ_λ and denoted by $Fqcl(\chi_\lambda)$. We will denote the fuzzy q-interior (resp. fuzzy q-closure) of any fuzzy subset, say of χ_λ by fuzzy $Fq\operatorname{int}(\chi_\lambda)(\mathrm{Fq}\,cl(\chi_\lambda))$, where $Fq\operatorname{int}(\chi_\lambda)$ is the union of all fuzzy q-open sets contained in χ_λ, and $Fqcl(\chi_\lambda)$ is the intersection of all fuzzy q-closed sets containing χ_λ.

Definition 3.3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space then a fuzzy subset χ_λ of X is said to be fuzzy q-semi-open set if

$$\chi_\lambda \leq Fqcl(Fqint\ \chi_\lambda).$$

Complement of fuzzy q-semi-open set is called fuzzy q-semi-closed set. The collection of all fuzzy q-semi-open sets of *X* are denoted by F*qSO*(*X*)

Example 3.4. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X

$$\tau_1 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}\},$$

$$\tau_2 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,d\}}\},$$

$$\tau_3 = \{\tilde{1}X, \tilde{0}X, \chi_{\{c,d\}}\},$$

$$\tau_4 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,c,d\}}\}.$$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.
Fuzzy q-open sets of

$$X = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \quad \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \quad \chi_{\{a,c,d\}}\}.$$

Fuzzy q-semi-open sets of *X* are denoted by

$$FqSO(X) = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \chi_{\{a,c,d\}}\}.$$

Definition 3.5. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space then a fuzzy subset χ_λ of X is said to be fuzzy q-pre-open set if $\chi_\lambda \leq Fqint(Fqcl\, \chi_\lambda)$. Complement of fuzzy q-pre-open set is called fuzzy q-pre-closed set. The collection of all fuzzy q-pre-open sets of X is denoted by F$qPO(X)$.

Example 3.6. Let $X = \{a,b,c,d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X

$$\tau_1 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}\},$$

$$\tau_2 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,d\}}\},$$

$$\tau_3 = \{\tilde{1}X, \tilde{0}X, \chi_{\{b,d\}}\},$$

$$\tau_4 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,c,d\}}\}.$$

Fuzzy open sets in fuzzy q-topological space are union of all four fuzzy topologies.
Then fuzzy q-open sets of

$$X = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{b,,d\}}, \chi_{\{a,c,d\}}\}.$$

Fuzzy q-pre-open sets of X denoted by

$$FqSO(X) = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{b,,d\}}, \chi_{\{a,c,d\}}\}.$$

Definition 3.7. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space. The intersection of all fuzzy q-semi-closed sets of X containing a fuzzy subset χ_λ of X is called fuzzy q-semi closure of χ_λ and is denoted by Fqsint(χ_λ).

Definition 3.8. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space. The intersection of all fuzzy q-pre-closed sets of X containing a fuzzy subset χ_λ of X is called fuzzy q-pre closure of χ_λ and is denoted by Fqpint(χ_λ).

Definition 3.9. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space. The intersection of all fuzzy q-semi-closed sets of X containing a fuzzy subset χ_λ of X is called fuzzy q-semi closure of χ_λ and is denoted by F$qscl$(χ_λ).

Definition 3.10. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space. The intersection of all fuzzy q-pre closed sets of X containing a fuzzy subset χ_λ of X is called fuzzy q-pre closure of χ_λ and is denoted by F$qpcl$(χ_λ) .

Theorem 3.11. χ_λ is fuzzy q-semi open if and only if χ_λ=Fqsint (χ_λ).

Theorem 3.12. $Fqsint(\chi_\lambda \vee \chi_\delta \succ Fqsint(\chi_\lambda) \vee Fqsint(\chi_\delta)$.

Theorem 3.13. χ_λ is a fuzzy q-semi closed set if and only if χ_λ = Fqscl (χ_λ).

Theorem 3.14. Let χ_λ and χ_δ be two fuzzy subsets of $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ and $\chi_{\{x\}} \leq \tilde{1}_X$

a) χ_λ is fuzzy q-pre closed if and only if χ_λ =Fqpcl (χ_λ)

b) If $\chi_\lambda \leq \chi_\delta$, then $Fqpcl\ (\chi_\lambda) \prec Fqpcl(\chi_\delta)$

c) $\chi_{\{x\}} \prec Fqpcl(\chi_\delta)$ if and only if $\chi_\lambda \wedge \chi_\delta \neq \tilde{0}_X$ for every fuzzy q-pre-open set $f(\chi_\lambda)$ containing $f(\chi_{\{x\}})$.

Theorem 3.15. Let χ_λ be a fuzzy subset of $(X, \tau_1, \tau_2, \tau_3, \tau_4)$**,** if there exist a fuzzy q-pre-open set χ_δ such that $\chi_\lambda \prec \chi_\delta \prec Fqcl(\chi_\lambda)$, then χ_λ is fuzzy q-pre-open.

Theorem 3.16. In a fuzzy q-topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ the union of any two fuzzy q-semi-open sets is always a fuzzy q-semi-open set.

Proof: Letχ_λ and χ_δ be any two fuzzy q-semi-open sets in *X*. Now

$$\chi_\lambda \vee \chi_\delta \leq Fqcl(Fqint\ \chi_\lambda) \vee Fqcl(Fqint\ \chi_\delta)$$
$$\Rightarrow \chi_\lambda \vee \chi_\delta \leq Fqcl(Fqint\ (\chi_\lambda \vee \chi_\delta))$$

Hence, $(\chi_\lambda \vee \chi_\delta$ fuzzy q-semi-open sets.

Remark 3.17. The intersection of any two fuzzy q-semi-open sets may not be fuzzy q-semi-open sets as show in the following example

Example 3.18. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X

$$\tau_1 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}\},$$

$$\tau_2 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,d\}}\},$$

$$\tau_3 = \{\tilde{1}X, \tilde{0}X, \chi_{\{c,d\}}\},$$

$$\tau_4 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,c,d\}}\}.$$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies. Then Fuzzy q-open sets of

$$FqSO(X) = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \chi_{\{a,c,d\}}\}.$$

Fuzzy q-semi-open set of X is denoted by

$$FqSO(X) = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \chi_{\{a,c,d\}}\}.$$

Here

$\chi_{\{a,d\}} \wedge \chi_{\{c,d\}} = \chi_{\{d\}} \succ FqSO(X)$.

Theorem 3.19. If χ_λ is fuzzy q-open sets then χ_λ is fuzzy q-semi-open set.
Proof: Let χ_λ is a fuzzy q-open set.
Therefore $\chi_\lambda = Fqint(\chi_\lambda)$
.
Now $\chi_\lambda \prec Fqcl(\chi_\lambda) = Fqcl(Fqint(\chi_\lambda))$ hence χ_λ is fuzzy q-semi-open set.

Theorem 3.20. Let χ_λ and χ_δ be two fuzzy subsets of *X* such that $\chi_\delta \prec \chi_\lambda \prec Fqcl(\chi_\delta)$. If χ_δ is a fuzzy q-semi-open set then χ_λ is also fuzzy q-semi-open set.
Proof: Given χ_δ is fuzzy q-semi-open set.
So, we have $\chi_\delta \le Fqcl(Fqint(\chi_\delta)) \le Fqcl(Fqint(\chi_\lambda))$.
Thus $Fqcl(\chi_\delta) \le Fqcl(Fqint(\chi_\lambda))$ hence χ_λ is also fuzzy q-semi-open set.

4. Fuzzy q-semi-continuity and fuzzy q-pre-continuity in fuzzy q-topological space

Definition 4.1. A fuzzy function f from a fuzzy q-topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ into another fuzzy q-topological space $(Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is called fuzzy q-semi-continuous if $f^{-1}(\chi_\lambda)$ is fuzzy q-semi-open set in *X* for each fuzzy q-open set χ_λ in Y.

Example 4.2. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X

$$\tau_1 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}\},$$

$$\tau_2 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,d\}}\},$$

$$\tau_3 = \{\tilde{1}X, \tilde{0}X, \chi_{\{c,d\}}\},$$

$$\tau_4 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,c,d\}}\}.$$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.
Then fuzzy q-open sets of

$$X = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \chi_{\{a,c,d\}}\}.$$

Fuzzy q-semi-open set of X is denoted by

$$FqSO(X) = X = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,,d\}}, \chi_{\{a,c,d\}}\}.$$

Let $Y = \{1,2,3,4\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on Y

$$\tau'_1 = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,4\}}\}, \qquad \tau'_2 = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}\},$$

$$\tau'_3 = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,2\}}\}, \ \ \tau'_4 = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,2,4\}}\}.$$

Fuzzy q-open sets of

$$Y = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}.$$

Fuzzy q-semi-open set of *Y* is

$$FqSO(Y) = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}.$$

Consider the fuzzy function $f: I^x \to I^Y$ is defined as

$$f^{-1}(\chi_{\{4\}}) = \chi_{\{a\}}, \qquad f^{-1}(\chi_{\{1,2\}}) = \chi_{\{c,d\}}, \qquad f^{-1}(\chi_{\{1,4\}}) = \chi_{\{a,d\}},$$

$$f^{-1}(\chi_{\{1,2,4\}}) = \chi_{\{a,c,d\}}, f^{-1}(\tilde{0}_Y) = (\tilde{0}_X)\ ,\ f^{-1}(\tilde{1}_Y) = (\tilde{1}_x).$$

Since the inverse image of each fuzzy q-open set in *Y* under *f* is fuzzy q-semi-open set in *X*. Hence *f* is fuzzy q-semi-continuous function.

Definition 4.2. A fuzzy function f defined from a fuzzy q-topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ into another fuzzy q-topological space $(Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is called fuzzy q-pre-continuous function if $f^{-1}(\chi_\lambda)$ is fuzzy q-pre-open set in *X* for each fuzzy q-open set χ_λ in *Y*.

Example 4.3. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X

$$\tau_1 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}\}, \qquad \tau_2 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,d\}}\},$$

$$\tau_3 = \{\tilde{1}X, \tilde{0}X, \chi_{\{b,d\}}\}, \ \ \tau_4 = \{\tilde{1}X, \tilde{0}X, \chi_{\{a,b,d\}}\}.$$

Fuzzy open-sets in fuzzy q-topological spaces are union of all four fuzzy topologies.
Then fuzzy q-open sets of

$$X = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \ \chi_{\{b,,d\}}, \chi_{\{a,b,d\}}\}.$$

Fuzzy q-pre-open set of *X* is denoted by

$$FqPO(X) = \{\tilde{1}X, \tilde{0}X, \chi_{\{a\}}, \chi_{\{a,d\}}, \ \chi_{\{b,,d\}}, \chi_{\{a,b,d\}}\}.$$

Let $Y = \{1,2,3,4\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on Y

$$\tau_1' = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,4\}}\}, \qquad \tau_2' = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}\},$$

$$\tau_3' = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,2\}}\}, \ \ \tau_4' = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{1,2,4\}}\}.$$

Fuzzy q-open sets of $Y = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}$.

Fuzzy q-pre-open set of *Y* is denoted by

$$FqPO(Y) = \{\tilde{1}Y, \tilde{0}Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}..$$

Consider the fuzzy function $f: I^x \to I^Y$ is defined as

$$f^{-1}(\chi_{\{4\}}) = \chi_{\{a\}}, \qquad f^{-1}(\chi_{\{1,2\}}) = \chi_{\{b,d\}}, \qquad f^{-1}(\chi_{\{1,4\}}) = \chi_{\{a,d\}},$$

$$f^{-1}(\chi_{\{1,2,4\}}) = \chi_{\{a,b,d\}}, f^{-1}(\tilde{0}_Y) = (\tilde{0}_X)\ , \ f^{-1}(\tilde{1}_Y) = (\tilde{1}_x).$$

Since the inverse image of each fuzzy q-open set in *Y* under *f* is fuzzy q-pre-open set in *X*. Hence *f* is fuzzy q-pre-continuous function.

Theorem 4.4. Let $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau_1', \tau_2', \tau_3', \tau_4')$ be a fuzzy q-pre-continuous open function .If χ_λ is a fuzzy q-pre-open set of *X*, then $f(\chi_\lambda)$ is a fuzzy q-pre-open set in *Y*.

Proof: First, let χ_λ be fuzzy q-pre-open set in *X* .There exist a fuzzy q-open set χ_δ in *X* such that $\chi_\lambda \prec \chi_\delta \prec Fqcl(\chi_\lambda)$. Since f is a fuzzy q-open function then $f(\chi_\delta)$ is a fuzzy q-open set in *Y*. Since f is a fuzzy q-continuous function, we have

$$f(\chi_\lambda) \prec f(\chi_\delta) \prec f(Fqcl(\chi_\lambda)) \prec Fqcl(f(\chi_\lambda)).$$

This show that $f(\chi_\lambda)$ is fuzzy q-pre-open in *Y*.

Let χ_λ be a fuzzy q-pre-open in *X* .There exist a fuzzy q-pre-open set χ_δ such that

$$\chi_\delta \prec \chi_\lambda \prec Fqcl(\chi_\delta).$$

Since *f* is a fuzzy q-continuous function, we have by the proof of first part, $f(\chi_\delta)$ is a fuzzy q-pre-open in *X*. Therefore $f(\chi_\lambda)$ is a fuzzy q-pre-open in *Y*.

Theorem 4.5. Let $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ be a fuzzy q-pre continuous open function .If χ_λ is a fuzzy q-pre-open set of *Y*, then $f^{-1}(\chi_\lambda)$ is a fuzzy q-pre-open in *X*.

Proof: First, let χ_λ be a fuzzy q-pre-open set of *Y* .There exist a fuzzy q-open set χ_δ in *Y*. Such that $\chi_\lambda \prec \chi_\delta \prec Fqcl(\chi_\lambda)$. Since *f* is a fuzzy q-open, we have

$$f^{-1}(\chi_\lambda) \prec f^{-1}(\chi_\delta) \prec f^{-1}\big(Fqcl(\chi_\lambda)\big) \prec Fqcl\big(f^{-1}(\chi_\lambda)\big).$$

Since *f* is a fuzzy q-pre continuous function, $f^{-1}(\chi_\delta)$ is a fuzzy q-pre-open set in *X* .By theorem 3.13, $f^{-1}(\chi_\lambda)$ is a fuzzy q-pre-open set in *X*.

Theorem 4.6. The following are equivalent for a function $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$

a) f is a fuzzy q-pre continuous function;

b) The inverse image of each fuzzy q-closed set of *Y* is fuzzy q-pre closed in *X;*

c) $Fqpcl(f^{-1}(\chi_\lambda)) \prec f^{-1}(Fqpcl((\chi_\lambda))$ for every subset χ_λ of *Y*.

d) $f(Fqpcl(\chi_\delta)) \prec Fqcl(f(\chi_\delta))$ for every subset χ_δ of *X*.

Theorem 4.7. If $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ and

$$g: (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \to (Z, \tau''_1, \tau''_2, \tau''_3, \tau''_4)$$

be two fuzzy q-semi continuous function then

$$fog: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Z, \tau''_1, \tau''_2, \tau''_3, \tau''_4)$$

may not be fuzzy q-semi continuous function .

Theorem 4.8. Every fuzzy q-continuous function is a fuzzy q-semi continuous function.

Theorem 4.9. Let $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ be bijective. Then the following conditions are equivalent:

i) f is a fuzzy q-semi-open continuous function.

ii) f is a fuzzy q-semi closed continuous function and

iii) f^{-1} is a fuzzy q-semi continuous function.

Proof: (i)→(ii) Suppose χ_λ is a fuzzy q-closed set in *X* . Then $\tilde{1}_X - \chi_\lambda$ is a Fuzzy q-open set in *X*. Now by (i) $f(\tilde{1}_X - \chi_\lambda)$ is a fuzzy q-semi-open set in *Y* . Now since f^{-1} is fuzzy bijective function so $f\big(\tilde{1}_X - \chi_\lambda\big) = Y - f(\chi_\lambda)$. Hence $f(\chi_\lambda)$ is a fuzzy q-semi closed set in *Y* .Therefore *f* is a fuzzy q-semi closed continuous function.

(ii)→(iii) Let *f* is a fuzzy q-semi closed map and χ_λ be a fuzzy q-closed set of *X* . Since f^{-1} is bijective so $(f^{-1})^{-1}\chi_\lambda$ which is a fuzzy q-semi-closed set in *Y* . Hence f^{-1} is a fuzzy q-semi continuous function.

(iii)→(i) Let χ_λ be a fuzzy q-open set in *X* . Since f^{-1} is a fuzzy q-semi continuous function so $(f^{-1})^{-1}\chi_\lambda = f(\chi_\lambda)$ is a fuzzy q-semi open set in *Y* . Hence *f* is fuzzy q-semi-open continuous function.

Theorem 4.10. Let *X* and *Y* are two fuzzy q-topological spaces. Then $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau_1', \tau_2', \tau_3', \tau_4')$ is fuzzy q-semi-continuous function if one of the followings holds:

i) $f^{-1}(Fqsint\ \chi_\lambda) \leq Fqsint(f^{-1}(\chi_\lambda)$, for every fuzzy q-open set χ_λ in *Y*.
ii) $Fqscl\big(f^{-1}(\chi_\lambda)\big) \leq f^{-1}\big(Fqsint(\chi_\lambda)\big)$, for every fuzzy q-open set χ_λ in *Y* .
Proof: Let χ_λ be any fuzzy *q*-open set in *Y* and if condition (i) is satisfied then

$$f^{-1}(\ Fqsint\ \chi_\lambda) \leq Fqsint(f^{-1}(\chi_\lambda).$$

We get $f^{-1}(\ \chi_\lambda) \leq Fqsint(f^{-1}(\chi_\lambda)$.

Therefore $f^{-1}(\chi_\lambda)$ is a fuzzy q-semi-open set in *X* .Hence *f* is a fuzzy q-semi-continuous function. Similarly we can prove (ii).

Theorem 4.11. A function $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau_1', \tau_2', \tau_3', \tau_4')$ is called fuzzy q-semi open continuous function if and only if

$$f\big(Fqsint(\chi_\lambda)\big) \leq Fqsint\big(f(\chi_\lambda)\big),$$

for every quad open set χ_λ in *X* .
Proof: Suppose that *f* is a quad semi open continuous function.
Since $Fqsint(f(\chi_\lambda) \leq \chi_\lambda$ so $f(Fqsint(f(\chi_\lambda)) \leq f(\chi_\lambda)$.

By hypothesis $Fqsint(f(\chi_\lambda))$ is a fuzzy q-semi-open set and $Fqsint(f(\chi_\lambda))$ is largest fuzzy q-semi-open set contained in $f(\chi_\lambda)$ so $f\big(Fqsint(\chi_\lambda)\big) \leq Fqsint\big(f(\chi_\lambda)\big)$.
Conversely, suppose χ_λ is a fuzzy q-open set in *X* .So $f\big(Fqsint(\chi_\lambda)\big) \leq Fqsint\big(f(\chi_\lambda)\big)$.

Now since $\chi_\lambda = Fsint(\chi_\lambda)$ so $f(\chi_\lambda) \leq Fqsint(f(\chi_\lambda))$
Therefore $f(\ \chi_\lambda)$ is a fuzzy q-semi-open set in *Y* and *f* is a fuzzy q-semi-open continuous function.

Theorem 4.12. A function $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau_1', \tau_2', \tau_3', \tau_4')$ is called fuzzy q-semi closed continuous function if and only if $f\big(Fqscl(\chi_\lambda)\big) \leq Fqscl\big(f(\chi_\lambda)\big)$ for every fuzzy q-closed set χ_λ in *X* .

Proof: Suppose that *f* is a fuzzy q-semi closed continuous function. Since $\chi_\lambda \leq Fqscl(\chi_\lambda)$ so $f(\chi_\lambda) \leq f\big(Fqscl(\chi_\lambda)\big)$. By hypothesis, $f\big(Fqscl(\chi_\lambda)\big)$, is a fuzzy q-semi closed set and $f\big(Fqscl(\chi_\lambda)\big)$ is smallest fuzzy q-semi closed set containing $f(\chi_\lambda)$ so $f\big(Fqscl(\chi_\lambda)\big) \leq Fqscl\big(f(\chi_\lambda)\big)$.
Conversely, suppose χ_λ is a fuzzy q-closed set in *X*. So $f\big(Fqscl(\chi_\lambda)\big) \leq Fqscl\big(f(\chi_\lambda)\big)$.

Since $\chi_\lambda = Fqscl(\chi_\lambda)$ so $Fqscl(f(\chi_\lambda)) \leq f(\chi_\lambda)$.
Therefore $f(\chi_\lambda)$ is a fuzzy q-semi closed set in *Y* and *f* is fuzzy q-semi closed continuous function.

Theorem 4.13. Every fuzzy q-semi continuous function is fuzzy q-continuous function.

5. Conclusion

In this paper the idea of fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuous function, fuzzy q-semi continuous function and fuzzy q-pre continuous function in fuzzy q-topological spaces were introduced and studied.

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