A Note on Fuzzy Bi-Ideals in Ternary Semigroups

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Abstract. The prime fuzzy bi-ideals of semigroups are introduced by Shabir, Jun and Bano. They characterized those semigroups for which each fuzzy bi-ideal is semiprime and also characterized those semigroups for which each fuzzy bi-ideal is strongly prime. In this paper, we prove some properties of fuzzy bi-ideals in ternary semigroups and the relation between fuzzy quasi ideals and fuzzy bi-ideals is considered.

Keywords: Ternary semigroup, fuzzy set, fuzzy ternary semigroup, fuzzy ideal, fuzzy quasi ideal, fuzzy bi-ideal, characteristic function.

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1. Introduction

Lehmer introduced the ternary algebraic system in 1932, and after such structures were studied by Kasner. In 1965, ideal theory in ternary semigroups studied by Sioson [16]. After the introduction of fuzzy sets by Zadeh [9] reconsideration of the concept of classical mathematics began. Fuzzy set has an important impact over the field of mathematical research in both theory and application. It has found manifold applications in mathematics and related areas. Kuroki introduced and studied the notion of fuzzy semigroups. He also studied the concept of fuzzy bi-ideals [6] (1979) and fuzzy quasi-ideals (1982) of semigroups. Since then many papers have been published in the field of fuzzy algebra [1-5,7,8,10-15,17,18]. Many researchers conducted the researches on the generalizations of the notions of fuzzy sets with huge applications in computer, logics and many branches of pure and applied mathematics.

2. Basic definitions and preliminaries

Definition 2.1. A non-empty set $T$ is said to be ternary semigroup if there exists a ternary operation $\cdot : T \times T \times T \rightarrow T$ written as $(a, b, c) \rightarrow a \cdot b \cdot c$ satisfies the following identity $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in T$.
Example 1. Let \( T = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \) then \( T \) is a ternary semigroup under usual multiplication.

**Definition 2.2.** A non-empty subset \( A \) of a ternary semigroup \( T \) is called a ternary subsemigroup of \( T \) if \( AAA \subseteq A \).

**Definition 2.3.** A non-empty subset \( A \) of a ternary semigroup \( T \) is called a quasi ideal in \( T \) if 
\[
(ATT) \cap (TAT) \cap (TTA) \subseteq A \quad \text{and} \quad (ATT) \cap (TTATT) \cap (TTA) \subseteq A.
\]

**Definition 2.4.** A ternary subsemigroup \( A \) of a ternary semigroup \( T \) is said to be a bi-ideal in \( T \) if \( AFATA \subseteq A \).

**Definition 2.5.** Let \( T \) be a non-empty set. A fuzzy subset of a ternary semigroup \( T \) is a function \( \mu : T \to [0,1] \).

**Definition 2.6.** Let \( \mu \) be a fuzzy subset of a non-empty set \( T \) for any \( t \in [0,1] \), the subset \( \mu_t = \{ x \in T : \mu(x) \geq t \} \) of \( T \) is called a level set of \( \mu \).

**Definition 2.7.** For any two fuzzy subsets \( \mu_1 \) and \( \mu_2 \) of a non-empty set \( T \), the union and the intersection of \( \mu_1 \) and \( \mu_2 \) denoted by \( \mu_1 \cup \mu_2 \) and \( \mu_1 \cap \mu_2 \) are fuzzy subsets of \( T \) and defined as 
\[
(\mu_1 \cup \mu_2)(x) = \max\{\mu_1(x), \mu_2(x)\} = \mu_1(x) \vee \mu_2(x)
\]
and 
\[
(\mu_1 \cap \mu_2)(x) = \min\{\mu_1(x), \mu_2(x)\} = \mu_1(x) \wedge \mu_2(x) \quad \text{for all} \ x \in T.
\]
Where \( \vee \) denotes maximum or supremum and \( \wedge \) denotes minimum or infimum.

**Definition 2.8.** Let \( \mu_1, \mu_2 \) and \( \mu_3 \) are any three fuzzy sets of a ternary semigroup \( T \). Then their fuzzy product \( \mu_1 \circ \mu_2 \circ \mu_3 \) is defined by
\[
\sum_{a=xyz} \{ \mu_1(x) \wedge \mu_2(y) \wedge \mu_3(z) \} \quad \text{if} \ a \ \text{is expressible as} \ a=xyz \quad \text{for all} \ x,y,z \in T.
\]
\[
(\mu_1 \circ \mu_2 \circ \mu_3)(a) = \begin{cases} 
0 & \text{otherwise} \\
\end{cases}
\]

**Definition 2.9.** A fuzzy set \( \mu \) of a ternary semigroup \( T \) is called a fuzzy ternary subsemigroup of \( T \) if \( \mu(xyz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \} \) for all \( x, y, z \in T \).

**Definition 2.10.** A fuzzy ternary subsemigroup \( \mu \) of a ternary semigroup \( T \) is called a fuzzy bi-ideal in \( T \) if \( \mu(xmynz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \} \) for all \( x, y, z, m, n \in T \).
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Definition 2.11. A fuzzy set $\mu$ of a ternary semigroup $T$ is called a fuzzy left (right, lateral) ideal in $T$ if $\mu(xyz) \geq \mu(z)$, $(\mu(xyz) \geq \mu(x), \mu(xyz) \geq \mu(y))$ for all $x, y, z \in T$.

Definition 2.12. A fuzzy set $\mu$ of a ternary semigroup $T$ is a fuzzy ideal in $T$ if it is fuzzy left, right and lateral ideal in $T$.

Definition 2.13. Let $A$ be a non-subset of a ternary semigroup $T$. Then the characteristic function of $A$ is defined by $C_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$

We denote the characteristic function $C_T$ of $T$. i.e., $T = C_T$ thus $T(x) = 1$ for all $x \in T$.

Definition 2.14. A fuzzy set $\mu$ of a ternary semigroup $T$ is called a fuzzy quasi ideal of $T$ if $(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$ and $(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$.

i.e., $[(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$ and $[(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$

3. Main results

Theorem 3.1. A non-empty subset $A$ of a ternary semigroup $T$ is a bi-ideal in $T$ if and only if characteristic function $C_A$ of $A$ is a fuzzy bi-ideal in $T$.

Proof: Let $C_A$ is a characteristic function of $A$ in ternary semigroup $T$.

Assume that $A$ is a bi-ideal in $T$. Then we have $ATATA \subseteq A$.

Now consider $C_A \circ T \circ C_A \circ T \circ C_A = C_A \circ C_T \circ C_A \circ C_T \circ C_A$

$= C_{ATATA} \subseteq C_A$

$C_A \circ T \circ C_A \circ T \circ C_A \subseteq C_A$

Therefore $C_A$ is a fuzzy bi ideal in $T$.

Conversely, suppose $C_A$ is fuzzy bi-ideal in $T$.

Then $C_A \circ T \circ C_A \circ T \circ C_A \subseteq C_A$

Let $x \in ATATA$. Then $C_A(x) \geq (C_A \circ T \circ C_A \circ T \circ C_A)(x)$

$= (C_A \circ C_T \circ C_A \circ C_T \circ C_A)(x)$

$= C_{ATATA}(x)$

$\geq 1$ (\because x \in ATATA)

$C_A(x) \geq 1 \Rightarrow x \in A$

Therefore $ATATA \subseteq A$.

Hence $A$ is a bi-ideal in a ternary semigroup $T$.
**Theorem 3.2.** If $\mu$ be a fuzzy bi-ideal in a ternary semigroup $T$, then the level set $\mu_t$ is a bi-ideal in $T$ for every $t \in [0, 1]$.

**Proof:** Let $\mu$ be a fuzzy subset of a ternary semigroup $T$ and let $x \in T$. For $t \in [0, 1]$, let $u \in \mu, T\mu, T\mu_t$, where $\mu_t$ be the level set of $\mu$. Then there exists $x, y, z \in \mu_t$, $m, n \in T$ and such that $u = xmy.nz$.

Consider $[\mu \circ T \circ (\mu \circ T \circ \mu)](u) = \bigvee_{u=abc} \{\mu(a) \land \mu(b) \land \mu(c)\}$

$= \bigvee_{u=abdef} \{\mu(a) \land T(b) \land \{\bigvee_{v=def} \{\mu(d) \land T(e) \land \mu(f)\}\}\}$

$= \bigvee_{u=abdef} \{\mu(a) \land 1 \land \mu(d) \land 1 \land \mu(f)\}$

$= \mu(x) \land \mu(y) \land \mu(z)$

$\geq t \land t \land t = t$

Since $\mu$ is a fuzzy bi-ideal in $T$ $\Rightarrow \mu(u) \geq t$ for all $u \in T$

$\Rightarrow u \in \mu_t$ $\Rightarrow \mu_t, T\mu_t, T\mu \subseteq \mu_t$.

Hence $\mu_t$ is a bi-ideal in $T$.

**Theorem 3.3.** Every fuzzy quasi-ideal in a ternary semigroup $T$ is a fuzzy bi-ideal in $T$.

**Proof:** Suppose $\mu$ is fuzzy quasi-ideal in a ternary semigroup $T$. Then we have

$$(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \subseteq \mu \land (T \circ T \circ \mu) \cap (\mu \circ T \circ T) \subseteq \mu.$$

Consider $\mu(xyz) \geq [(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](xyz)$

$\geq (T \circ T \circ \mu)(xyz) \cap (T \circ \mu \circ T)(xyz) \cap (\mu \circ T \circ T)(xyz)$

$\geq \bigvee_{xyz=abc} \{T(a) \land T(b) \land \mu(c)\} \land \bigvee_{xyz=uvw} \{T(u) \land T(v) \land \mu(w)\}$

$\geq [T(x) \land T(y) \land \mu(z)] \land [T(x) \land \mu(y) \land T(z)] \land [\mu(x) \land T(y) \land T(z)]$

$\geq [1 \land \mu(z)] \land [1 \land \mu(y) \land 1] \land [\mu(x) \land 1 \land 1]$

$\geq \mu(z) \land \mu(y) \land \mu(x)$

$\geq \mu(x) \land \mu(y) \land \mu(z)$

$\mu(xyz) \geq [\mu(x) \land \mu(y) \land \mu(z)]$

Therefore $\mu$ is a fuzzy ternary subsemigroup of $T$.

Again consider

$\mu(xmynz) \geq [(T \circ T \circ \mu) \cap (T \circ T \circ T \circ \mu) \cap (\mu \circ T \circ T)](xyz)$
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≥ [(T ∘ T ∘ μ)(x m y n z) ∩ (T ∘ T ∘ μ ∘ T ∘ T)(x m y n z) ∩ (μ ∘ T ∘ T)(x m y n z)]

≥ [ ∨_{x m y n z = a b c d e} \{T(a) ∩ T(b d c) ∩ μ(e)\} ] ∩

[ ∨_{x m y n z = p q r s t} \{T(p) ∩ T(q) ∩ μ(r) ∩ T(r) ∩ T(t)\} ]

∧ [ ∨_{x m y n z = i j k l o} \{μ(i) ∩ T(j k l) ∩ T(o)\} ]

≥ [T(x) ∩ T(m y n) ∩ μ(z)] ∩ [T(x) ∩ T(m) ∩ μ(y) ∩ T(n) ∩ T(z)]

∧ [μ(x) ∩ T(m y n) ∩ T(z)]

≥ [1 ∧ 1 ∧ μ(z)] ∩ [1 ∧ 1 ∧ μ(y) ∧ 1 ∧ 1] ∩ [μ(x) ∧ 1 ∧ 1]

≥ μ(z) ∧ μ(y) ∧ μ(x)

μ(x m y n z) ≥ μ(x) ∧ μ(y) ∧ μ(z)

Therefore μ is a fuzzy bi-ideal in T.

Theorem 3.4. Let μ be a fuzzy bi-ideal in a ternary semigroup T. Then the fuzzy subset μ* defined by μ* = μ(x) + μ(0) for all x ∈ T is also a fuzzy bi-ideal of T.

Proof: Given that T be a ternary semigroup and μ be a fuzzy bi-ideal in T. Then μ is a fuzzy ternary sub semigroup of T. That is for all x, y, z ∈ T,

μ(x y z) ≥ μ(x) ∧ μ(y) ∧ μ(z)

or

μ(x y z) ≥ min {μ(x), μ(y), μ(z)}.

and μ(a m y n z) ≥ min {μ(a), μ(b), μ(c)} for all a, b, c, m, n ∈ T.

Let μ* be a fuzzy subset of T where μ* = μ(x) + μ(0) for all x ∈ T.

We have μ*(x y z) = μ(x y z) + μ(0)

≥ min {μ(x), μ(y), μ(z)} + μ(0)

= min {μ(x) + μ(0), μ(y) + μ(0), μ(z) + μ(0)}

μ*(x y z) ≥ min {μ*(x), μ*(y), μ*(z)}

Therefore μ* is a fuzzy bi-ideal in T.

Let a, b, c, m, n ∈ T

We have μ*(a m b n c) = μ(a m b n c) + μ(0)

≥ min {μ(a), μ(b), μ(c)} + μ(0)

= min {μ(a) + μ(0), μ(b) + μ(0), μ(c) + μ(0)}

Therefore μ* is a fuzzy ternary sub semigroup of T.
Therefore $\mu^*$ is a fuzzy bi-ideal in $T$.

**Theorem 3.5.** Let $T$ be a left zero ternary semigroup and $\mu$ be a fuzzy left ideal in $T$. Then $\mu(x) = \mu(z)$ for all $x, y, z \in T$.

**Proof:** Let $T$ be a left zero ternary semigroup.  
i.e., $x, y, z \in T \Rightarrow xyz = x$ and $zyx = z$.  
since $\mu$ is fuzzy left ideal in $T$.
Consider  
$$\mu(x) = \mu(xy) \geq \mu(z)$$
and we have  
$$\mu(x) = \mu(yz) \geq \mu(x)$$
$$\mu(z) \geq \mu(x)$$
from (1) and (2), We have $\mu(x) = \mu(z)$ for all $x, y, z \in T$.

**Theorem 3.6.** A non-empty fuzzy subset $\mu$ of a ternary semigroup $T$ is a fuzzy ternary sub semigroup of $T$ if and only if $\mu \circ \mu \circ \mu \subseteq \mu$.

**Proof:** Let $\mu$ be a non-empty fuzzy subset of a ternary semigroup $T$.
Suppose $\mu \circ \mu \circ \mu \subseteq \mu$
we have to prove that $\mu$ is a fuzzy ternary sub semigroup of $T$.
$$\mu(xyz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z) \text{ for all } x, y, z \in T.$$  
Let $x, y, z \in T$, such that $p = xyz$
Consider  
$$\mu(xyz) \geq (\mu \circ \mu \circ \mu)(xyz)$$
$$= \vee_{p=abc} \{ \mu(a) \wedge \mu(b) \wedge \mu(c) \}$$
$$= \vee_{xyz=abc} \{ \mu(a) \wedge \mu(b) \wedge \mu(c) \}$$
$$= \mu(x) \wedge \mu(y) \wedge \mu(z)$$
$$\mu(xyz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}.$$  
Conversely, assume that $\mu$ is a fuzzy ternary sub semigroup of $T$.
\i.e., $\mu(xyz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}$.  
We have to prove that $\mu \circ \mu \circ \mu \subseteq \mu$.
\i.e., $$(\mu \circ \mu \circ \mu)(a) \leq \mu(a) \text{ for all } a \in T$$
Let $a = xyz$ for all $x, y, z \in T$.
Consider  
$$(\mu \circ \mu \circ \mu)(a) = \vee_{a=xyz} \{ \mu(p) \wedge \mu(q) \wedge \mu(r) \}$$
$$= \vee_{xyz=abc} \{ \mu(p) \wedge \mu(q) \wedge \mu(r) \}$$
$$= \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}$$
$$\leq \mu(xyz)$$
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\[ \mu(a) \]

\[ (\mu \circ \mu \circ \mu)(a) \leq \mu(a) \implies \mu \circ \mu \circ \mu \subseteq \mu. \]

If \( a \neq xyz \) then \( (\mu \circ \mu \circ \mu)(a) = 0 \leq \mu(a) \implies (\mu \circ \mu \circ \mu)(a) \leq \mu(a). \)

Therefore \( \mu \circ \mu \circ \mu \subseteq \mu. \)

**Theorem 3.7.** For any nonempty fuzzy subset \( \mu \) of a ternary semigroup \( T \), the following conditions are equivalent.

(i) \( \mu \) is a fuzzy bi-ideal in \( T \).

(ii) \( \mu \circ \mu \circ \mu \subseteq \mu \) and \( \mu \circ C_T \circ \mu \circ \mu \subseteq \mu \) where \( C_T \) is the characteristic function of \( T \).

**Proof:** Let \( T \) be a ternary semigroup and let \( \mu \) be a fuzzy subset of \( T \).

First to prove that (i) \( \implies \) (ii):

Let \( \mu \) is a fuzzy bi-ideal in \( T \) then \( \mu \) is a fuzzy ternary sub semigroup of \( T \). i.e.,

\[ \mu(xyz) \geq \{ \mu(x) \land \mu(y) \land \mu(z) \} \quad \text{for all } x, y, z \in T. \]

and

\[ \mu(a m y n z) \geq \{ \mu(a) \land \mu(b) \land \mu(c) \} \quad \text{for all } a, b, c, m, n, T. \]

From the theorem 3.6, we have \( \mu \circ \mu \circ \mu \subseteq \mu \).

To prove only \( \mu \circ C_T \circ \mu \circ \mu \subseteq \mu \)

i.e., \( (\mu \circ C_T \circ \mu \circ \mu)(a) \leq \mu(a) \) for all \( a \in T \).

Let \( a = x m y n z \) for all \( x, y, z, m, n, p, q, r \in T \).

Consider

\[ \mu \circ C_T \circ \mu \circ \mu(a) = \bigvee_{x m y n z = p a q r} \{ \mu(p) \land \mu(q) \land C_T(v) \land \mu(r) \} \]

\[ = \{ \mu(x) \land C_T(m) \land \mu(y) \land C_T(n) \land \mu(z) \} \]

\[ = \{ \mu(x) \land 1 \land \mu(y) \land 1 \land \mu(z) \} \]

\[ = \{ \mu(x) \land \mu(y) \land \mu(z) \} \]

\[ \leq \mu(x m y n z) \]

\[ \mu(a) \]

\[ \implies (\mu \circ C_T \circ \mu \circ \mu)(a) \leq \mu(a). \]

Therefore \( \mu \circ C_T \circ \mu \circ \mu \subseteq \mu \)

Next to prove that (ii) \( \implies \) (i):

We assume that \( \mu \circ \mu \circ \mu \subseteq \mu \rightarrow (1) \) and \( \mu \circ C_T \circ \mu \circ \mu \subseteq \mu \rightarrow (2). \)

From theorem 3.6 and by equation (1), we have \( \mu \) is a fuzzy ternary sub semigroup of \( T \).

It is enough to prove that \( \mu(a m y n z) \geq \min \{ \mu(a), \mu(b), \mu(c) \} \).

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Consider $\mu(xmynz) \geq \{ \mu \circ C_T \circ \mu \circ C_T \circ \mu \} (xmynz)$.

$$= \bigvee_{xmnz=pqvr} \{ \mu(p) \land C_T(u) \land \mu(q) \land C_T(v) \land \mu(r) \}.$$ 

$$= \{ \mu(x) \land C_T(m) \land \mu(y) \land C_T(n) \land \mu(z) \}.$$ 

$$= \{ \mu(x) \land 1 \land \mu(y) \land 1 \land \mu(z) \}.$$ 

$$= \{ \mu(x) \land \mu(y) \land \mu(z) \}.$$ 

$\mu(xmynz) \geq \{ \mu(x) \land \mu(y) \land \mu(z) \}$ for all $a,b,c,m,n \in T$.

Therefore $\mu$ is a fuzzy bi-ideal in $T$.

**Definition 3.8.** Let $T_1$ and $T_2$ be two ternary semigroups. A mapping $f : (T_1, *) \rightarrow (T_2, *)$ is called a ternary homomorphism if $f(x*y_1 y_2 z) = (f(x) *_2 f(y_2) *_2 f(z))$ for all $x, y, z \in T$.

**Definition 3.9.** Let $f$ be a mapping from a set $X$ to $Y$ and $\mu$ be a fuzzy subset of $Y$, then the pre image of $\mu$ under $f$, denoted by $f^{-1}(\mu)$, is defined as $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

**Theorem 3.10.** Let $f : T_1 \rightarrow T_2$ be a homomorphism of ternary semigroups. If $\mu$ is a fuzzy bi-ideal in $T_2$ then the pre image $f^{-1}(\mu)$ is a fuzzy bi-ideal in $T_1$.

**Proof:** Let $T_1$ and $T_2$ be two ternary semigroups and given that $f : T_1 \rightarrow T_2$ is a homomorphism. Then $f(xyz) = (f(x)f(y)f(z))$ for all $x, y, z \in T_1$.

Let $\mu$ be a fuzzy bi-ideal in $T_2$.

We have to prove that $f^{-1}(\mu)$ is a fuzzy bi-ideal in $T_1$.

Consider $f^{-1}(\mu(xyz)) = \mu(f(xyz))$.

$$= \mu(f(x)f(y)f(z)).$$

$$\geq \min\{ \mu(f(x)), \mu(f(y)), \mu(f(z)) \}.$$ 

$$= \min\{ f^{-1}(\mu(x)), f^{-1}(\mu(y)), f^{-1}(\mu(z)) \}.$$ 

$$f^{-1}(\mu(xyz)) \geq \min\{ f^{-1}(\mu(x)), f^{-1}(\mu(y)), f^{-1}(\mu(z)) \}.$$ 

Therefore, $f^{-1}(\mu)$ is a fuzzy ternary subsemigroup of $T$.

Let $x, y, z, m, n \in T_1$

Again consider $f^{-1}(\mu(xmynz)) = \mu(f(xmynz)) = \mu(f(x)f(m)f(y)f(n)f(z))$. 

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\[ \mu(f(x)) \geq \mu(f(y)) \mu(f(z)). \]
\[ \min\{ f^{-1}(\mu(x)), f^{-1}(\mu(y)), f^{-1}(\mu(z)) \}. \]

Therefore, \( f^{-1}(\mu) \) is a fuzzy bi-ideal in ternary semigroup \( T_1 \).

4. Conclusions

We introduced the notion of fuzzy ideal, fuzzy quasi ideal, fuzzy bi-ideal in an ternary semigroup and studied their properties and relations between them. We characterize the fuzzy bi-ideals in an ternary semigroup with respect to bi ideals. In continuous of this paper we propose to study fuzzy bi-ideals over Ternary semigroups.

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