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On Minimal Topological Totally Closed Graphs

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Abstract. In this paper, we introduced and studied some properties of new functions such as quasi mwg-continuous, totally mwg-continuous functions with m_{wg}- closed graph and totally m_{wg}- closed graph in minimal structures.

Keywords: m_{wg}- closed graph, totally m_{wg}- closed graph, m_{wg}- compact, m_{wg}- connected

AMS Mathematics Subject Classification (2010): 54C10

1. Introduction

In 2000, Popa and Noiri [12] investigated the concept of minimal structure which is more general than a topological space. Moreover, he studied properties of M-continuous function's concept between spaces with minimal structures and obtained some characterizations and aspects of these functions.

On the other hand, they gave the definitions of m- closed graph [8] and strongly m-closed graph [8] together with their properties. In 2012, Min et al. [5] studied m-semi closed graph and strongly m-semi closed graph. Many mathematicians have defined some types of open sets, continuities and closed graphs which are generalizations of m-open sets, M-continuity and m-closed graphs, in spaces with minimal structures. Since the advent of these notions, several research papers with interesting results in different respects came to existence [3, 4, 6, 7, 13, 14]. Recently, Ghosh [2] studied separation axioms and graph functions in nano topological spaces. In 1995, Nour et al., investigated totally semi-continuous Functions [10]. In 2009, Caldas et al., studied the properties of totally b-continuous functions [1] in topological spaces.

In this paper, we introduced and investigated some properties of new functions such as quasi mwg-continuous, totally mwg-continuous functions with m_{wg}- closed graph and totally m_{wg}- closed graph. Also, we defined some new spaces called m_{wg}-Haussdroff space, totally m_{wg}-Compact, totally m_{wg}-Connected and etc., in order to characterize these spaces by using the notion of closed graphs.

Throughout the paper (X, m_X) and (Y, m_Y) are denoted by topological spaces with minimal structure (briefly. m-space). The interior and closure of a subset A of (X, m_X) are denoted by m_X -Int(A) and m_X -Cl(A) respectively.

2. Preliminaries

In this section, we list some definitions which are used in this sequel.

Definition 2.1. [8] Let X be a non empty set and P(X) the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) on X if $\Phi \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be m_X -open and the complement of an m_X -open set is said to be m_X -closed.

Definition 2.2. [8] An m-structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belong to m_X belongs to m_X.

Definition 2.3. [8] Let X be a nonempty set and m_X an m-structure on X. For subset A of X, the m_X-closure of A and the m_X -interior of A are defined in as follows

i. m_X -Cl(A) = $\cap\{F : A \subset F, X - F \in m_X\}$,
ii. m_X -Int(A) = $\cup\{U : U \subset A, U \in m_X\}$.

Definition 2.4. [11] A subset A of a m-space (X, m_X) is said to be

i. minimal generalized closed (mg-closed) sets if m_X - Cl(A) ⊂ U whenever A ⊂ U and U is open in m_X.
ii. minimal weakly generalized closed (mwg-closed) sets if m_X- Cl(m_X - Int(A)) ⊂ U whenever A ⊂ U and U is open in m_X.

The complement of mg-closed set (resp. mwg-closed set) is said to be mg-open set (resp. mwg-open set). The family of all mg-open sets (resp. mwg-open set) is denoted by m_X-GO(X) (resp. m_X -WGO(X)). We set m_X -GO(X, x) = {V ∈ m_X -GO(X) / x ∈ V} for x ∈ m_X . We define similarly, m_X -WGO(X, x) = {V ∈ m_X -WGO(X) / x ∈ V} for x ∈ m_X .

Definition 2.5. [8] A function f: (X, m_X) → (Y, m_Y) is said to be M-closed graph (resp. strongly M-closed graph) if for each (x, y) ∈ (X × Y) - G(f), there exist m_X -open set U containing x and m_Y -open set V containing y such that (U × V) ∩ G(f) = Φ ((U × m_Y - Cl(V)) ∩ G(f) = Φ).

Definition 2.6. [6] A m-space(X, m_X) is said to be

i. m-T_2 if for any distinct points x, y there exists U, V ∈ m_X such that x ∈ U, y ∈ V and U ∩ V = Φ.
ii. m-Urysohn if for any distinct points x, y there exists U, V ∈ m_X such that x ∈ U, y ∈ V and m_X -Cl(U) ∩ m_X -Cl(V) = Φ.
iii. m-Lindelöf **[9]** if every m_X-open cover of X has a countable subcover.

Definition 2.7. A function f: (X, m_X) →(Y, m_Y) is said to be

i. m-continuous [6] if the inverse image of every m - closed set in (Y, m_Y) is m – closed in (X, m_X).

ii. mwg-continuous [11] if $f^{-1}(V)$ is mwg-closed in (X, m_X) for every mwg-closed set V in (Y, m_Y).

Lemma 2.8. [8] Let (X, m_X) be a space with minimal structure, let A be a subset of X and x ∈ X. Then $x \in m_X$ -Cl(A) if and only if U ∩ A ≠ Φ, for every $U \in m_X$ containing the point x.

3. Minimal weakly generalized closed graph (m_{wg}-closed graph)

In this section, we defined and studied some functions with minimal weakly generalized closed graph.

Definition 3.1. A function f: $(X, m_X) \rightarrow (Y, m_Y)$ is said to be minimal weakly generalized closed graph (briefly. m_{wg}- closed graph) if for each (x, y) ∈ (X × Y) - G(f), there exist $U \in m_{wg}$-WGO(X, x) and $V \in m_{wg}$- WGO(Y, y) such that (U × V) ∩ G(f) = Φ.

Lemma 3.2. A function f: $(X, m_X) \rightarrow (Y, m_Y)$ is said to be m_{wg}-closed graph if for each (x, y) ∈ (X × Y) - G(f), there exist $U \in m_X$ -WGO(X, x) and $V \in m_Y$ -WGO(Y, y) such that f(U) ∩ V = Φ.
Proof is obvious from the Definition 3.1.

Theorem 3.3. Every function with m-closed graph has a m_{wg}-closed graph.
Proof follows from the Lemma 3.4 [11] that a m-closed set is mwg-closed set.

Theorem 3.4. Every function with a mg-closed graph has a m_{wg}-closed graph.
Proof follows from the Theorem 3.2 [11] that a mg-closed set is mwg-closed set.

Remark 3.5. Every m-closed set is mg-closed set. But converse need not be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}$ be endowed with the minimal structures $m_X = \{X, \emptyset, \{a\}, \{b\}, \{c\}\}$. Here $\{a\}$, $\{b\}$ and $\{c\}$ are mg-closed sets. But which are not m-closed set.

Theorem 3.7. Every function with m-closed graph has a mg-closed graph.
Proof follows from the Remark 3.5 that a m-closed set is mg-closed set.

From above discussion we have the following implications:

m -closed graph ⟶ m_{wg}-closed graph

↘ mg -closed graph ↗

Figure 1:

Remark 3.8. The converse need not be true for the above implications as shown by the following examples stated below.

Example 3.9. Let X = {a, b, c}and Y = {a, b, c, d} be endowed with the minimal structures m_X = {X, ∅, {a}, {b}, {c}} and m_Y = {Y, ∅, {a, b}, {a, d}, {a, b, d}} respectively. Let f: (X, m_X) → (Y, m_Y) be the mapping defined by f(a) = a, f(b) = b. Then f has m_{wg}-closed graph. But it is not m-closed graph.

Example 3.10. Let X = {a, b, c, d} = Y be endowed with the minimal structures m_X = {X, ∅, {a}, {a, c}} and m_Y = {Y, ∅, {a, b}, {a, d}, {a, b, d}} respectively. Let f: (X, m_X) → (Y, m_Y) be the mapping defined by f(a) = a, f(b) = b. Then f has m_{wg}-closed graph. But it is not mg-closed graph.

Example 3.11. Let X = {a, b, c}and Y = {a, b, c, d} be endowed with the minimal structures m_X = {X, ∅, {a}, {b}, {c}} and m_Y = {Y, ∅, {a, b}, {c, d}} respectively. Let f: (X, m_X) → (Y, m_Y) be the mapping defined by f(a) = a, f(b) = b. Then f has mg-closed graph. But it is not m-closed graph.

Definition 3.12. A m-space (X, m_X) is called

i. m_{wg}-T_1 space if for every pair of distinct points x, y in X there exists a mwg-open set U ∈ X containing x but not y and a mwg-open set V ∈ X containing y but not x.

ii. m_{wg}-Haussdroff space (i.e. m_{wg}-T_2 space) if for every pair of distinct points x, y in X there exists disjoint mwg-open sets U ∈ X and V ∈ X containing x and y respectively.

Theorem 3.13. If f: (X, m_X) → (Y, m_Y) is an injective function with the m_{wg}-closed graph G(f), then X is m_{wg}-T_1.

Proof: Let x and y be two distinct points of X. Since f is injection, f(x) ≠ f(y) *in Y*. (x, f(y)) ∈ (X × Y) − G(f). But G(f) is m_{wg}-closed graph. So, by the Lemma 3.2, there exist mwg-open sets U and V containing x and f(y) respectively, such that f(U) ∩ V = ∅. Hence y ∉ U. Similarly, there exist mwg-open sets M and N containing y and f(x) such that f(M) ∩ N = ∅. Hence x ∉ M. It follows that X is m_{wg}-T_1 space.

Theorem 3.13. If f: (X, m_X) → (Y, m_Y) is a surjective function with the m_{wg}-closed graph G(f), then Y is m_{wg}-T_1.

Proof: Let y and z be two distinct points of Y. Since f is surjective, there exist a point x in X such that f(x) = z. Therefore (x, y) ∉ G(f), by the Lemma 3.2 there exist mwg-open sets U and V containing x and y respectively such that f(U) ∩ V = ∅. It follows that z ∉ V.

Similarly, there exist w ∈ X such that f(w) = y. Hence (w, z) ∉ G(f). Similarly, there exist mwg-open sets M and N containing w and z respectively such that f(M) ∩ N = ∅. Thus y ∉ N, hence the space Y is m_{wg}-T_1.

Theorem 3.14. If a function $f:(X,\ m_X) \to (Y, m_Y)$ is mwg-continuous and Y is m_{wg}-T_2 space, then G(f) is m_{wg}-closed.

Proof: Let (x, y) ∉ G(f) or $(x, y) \in X \times Y - G(f)$, then y ≠ f(x) and Y is m_{wg}-T_2 space. There exist two mwg-open sets U and V such that $f(x) \in U, y \in V$ in Y and $U \cap V = \emptyset$. Since f is mwg-continuous, there exist a mwg-open neighbourhood W of x such that $f(W) \subset U$. Hence $f(W) \cap V = \emptyset$ and this implies that $(W \times V) \cap G(f) = \emptyset$. Hence f has a m_{wg}-closed graph.

Definition 3.15. A function $f:(X,\ m_X) \to (Y, m_Y)$ is called quasi-mwg-continuous if for each x ∈ X and each $V \in m_X$ containing f(x), there exists a $U \in m_X$-GO(X, x) such that $f(U) \subset m_X$-Cl(V).

Remark 3.16. Every mwg-continuous function is quasi-mwg-continuous. But converse need not be true as seen from following example.

Example 3.17. Let X = {a, b, c} and Y = {a, b,c,d} be endowed with the minimal structures $m_X = \{X, \emptyset, \{a, b\}, \{b, c\}\}$ and $m_Y = \{Y, \emptyset, \{a, b\}, \{a, d\}, \{a, b, d\}\}$respectively. Let $f:(X, m_X) \to (Y, m_Y)$ be the mapping defined by f(a) = a, f(b) = b. Then f has quasi-mwg-continuous. But it is not mwg-continuous.

Theorem 3.18. If $f:(X,\ m_X) \to (Y, m_Y)$ is quasi-mwg-continuous and Y is m-T_2, then f has the following property:

(P) For each (x, y) ∉ G(f) there exist $U \in m_X$-WGO(X, x) and $V \in m_Y$ containing y, such that f(U) ∩ m_Y-Int(m_Y-Cl(V)) = Φ.

Proof: Suppose (x, y) ∉ G(f). Then y ≠ f(x). Since Y is m-T_2, there exist V, W ∈ m_Y such that y ∈ V, f(x) ∈ W and V ∩ W = Φ. It is easy to verify that m_Y-Int(m_Y-Cl(V)) ∩ m_Y-Cl(W) = Φ. The quasi-mwg-continuous of f gives a U ∈ m_X-WGO(X, x) such that f(U) ⊂ m_X -Cl(V) and hence f(U) ∩ m_Y-Int(m_Y-Cl(V)) = Φ.

Theorem 3.19. if $f:(X,\ m_X) \to (Y, m_Y)$ is quasi-mwg-continuous and Y is m-T_2, then G(f) m_{wg}-closed.

Proof: If$(x, y) \in X \times Y - G(f)$, then there exist a U ∈ m_X-WGO(X, x) and V ∈ m_Y containing y, such that f(U) ∩ m_Y-Int(m_Y-Cl(V)) = Φ. Hence f(U) ∩ V = Φ so that (U × V) ∩ G(f) = Φ. Thus (x, y) ∈ (U × V) ⊂ $X \times Y - G(f)$ where U × V is mwg-open set in X × Y. Hence G(f) is m_{wg}-closed.

Definition 3.20. A subset K of a nonempty set X with a minimal structure m_X is said to be m_{wg}-compact relative to (X, m_X) if any cover of K by every mwg-open sets has a finite subcover.

Theorem 3.21. Let $f:(X,\ m_X) \to (Y, m_Y)$ be a function. Assume that m_X- is a base for a topology. If the graph G(f) is m_{wg}-closed, then m_X-Cl(f^{-1}(K)) = f^{-1} (K) whenever the set K ⊆ Y is m_{wg}-Compact relative to (Y, m_Y).

Proof: Let K ⊆ Y be m_{wg}-Compact relative to (Y, m_Y) and x ∈ X - f^{-1}(K), for each y ∈ K we have $(x, y) \in X \times Y - G(f)$, hence by the lemma 3.2 there exist an mwg - open sets

U_y containing x and mwg-open set V_y containing y such that f(U) ∩ V = Φ. The family {V_y: y ∈ K} is a cover of K by mwg - open sets. Since K ⊆ Y is m_{wg}-Compact relative to (Y, m_Y), there exists a finite subset of K, say {y_1, y_2, …., y_n}, such that K ⊆ ⋃{ V_{y_k} : $k = 1, 2,, \dots n$}. Then f^{-1}(K) ⊆ ⋃{$f^{-1}(V_{y_k} : k = 1, 2, \dots, n$}. Hence f^{-1}(K) ⊆ ⋃{X\ Uyk : k = 1, 2, …, n} = X \ ∩{U_{yk} : k = 1, 2, …,n}. Assume that m_X- is a base for a topology, there exist U ∈ m_X containing x such that U ⊆ ∩{U_{yk} : k = 1, 2, …,n}. Then U ∩ f^{-1}(K) = Φ, which shows, according to Lemma 2. 8, that x ∈ X\ m_X-Cl(f^{-1}(K)). We proved that X\ f^{-1}(K) ⊆ X\ m_X-Cl(f^{-1}(K)), whence Cl(f^{-1}(K)) = f^{-1}(K).

4. Totally m_{wg}-closed graph

In this section, we defined and studied some functions with totally m_{wg}-closed graph.

Definition 4.1. A subset A of space (X, m_X) is called

i. m-clopen if A is m-closed and m-open sets in X.

ii. mwg-clopen if A is mwg-closed and mwg-open sets in X.

The family of all m-clopen sets (resp. mwg-clopen set) is denoted by m_X-CO(X) (resp. m_X-WGCO(X)). We set m_X -CO(X, x) = {V ∈ m_X -CO(X) / x ∈ V} for x ∈ m_X . we define similarly, m_X -WGCO(X, x) = {V ∈ m_X -WGCO(X) / x ∈ V} for x ∈ m_X .

Definition 4.2. A graph G(f) of a function f : (X, m_X) → (Y, m_Y) is said to be totally m_{wg}-closed if for each (x, y) ∈ (X × Y) - G(f), there exists U ∈ m_X -WGCO(X, x) and V ∈ m_X -O(Y, y) such that (U×V) ∩ G(f) = Φ.

Lemma 4.3. A graph G(f) of a function f : (X, m_X) → (Y, m_Y) is totally m_{wg}-closed in (X × Y) if and only if for each (x, y) ∈ (X × Y) - G(f), there exists U ∈ m_X -WGCO(X, x) and V ∈ m_X -O(Y, y) such that (U×V) ∩ G(f) = Φ.

Proof: It is an immediate consequence of Definition 4.2.

Definition 4.4. A m-space (X, m_X) is called

i. m_{wg}- clopen T_1 (briefly. Mwgco-T_1) space if for every pair of distinct points x, y in X there exists a mwg-clopen set U ⊂ X containing x but not y and a mwg-clopen set V ⊂ X containing y but not x.

ii. m_{wg}-ultra hausdroff (briefly. Mwgco-T_2) space if for every pair of distinct points x, y in X there exists disjoint mwg-clopen sets U ⊂ X and V ⊂ X containing x and y respectively.

Theorem 4.5. Let f: (X, m_X) → (Y, m_Y) has totally m_{wg}-closed graph G(f). If f is injective, then X is m_{wg}- clopen T_1.

Proof: Let x and y be any two distinct points of X. Then, we have (x, f(y)) ∈ (X × Y) - G(f), By Lemma, there exists a mwg-clopen set U of X and m-open set V of Y such that (x, f(y)) ∈ U×V and f(U) ∩ V = Φ. Hence U ∩ f^{-1}(V) = Φ and y ∉ U. This implies that X is m_{wg}- clopen T_1.

Definition 4.6. A function f : (X, m_X) → (Y, m_Y) is called

i. totally m- continuous at a point x ∈ X if $f^{-1}(V)$ is m-clopen set in (X, m_X) for each m-open set V of (Y, m_Y).
ii. totally m_{wg}- continuous at a point x ∈ X if $f^{-1}(V)$ is mwg-clopen set in (X, m_X) for each m-open set V of (Y, m_Y).

Remark 4.7.
i. Every totally m_{wg}- continuous is mwg- continuous functions. But converse need not be true from the following Example 4.8.
ii. Every totally m- continuous is totally m_{wg}- continuous functions. But converse need not be true from the following Example 4.9.

Example 4.8. Let X = {a, b, c} and Y = {p, q, r} be the two topological spaces with m_X ={X, Φ, {a}, {b}, {a, b}} and m_Y = {Y, Φ, {p}}. If f: (X, m_X) → (Y, m_Y) is totally m_{wg}- continuous function defined by f(a) = {p}, f(b) = {q} and f(c) = {r}, then f is not mwg- continuous functions.

Example 4.9. Let X = {a, b, c} and Y = {p, q, r} be the two topological spaces with m_X ={X, Φ, {a}, {b}, {a, b}} and m_Y = {Y, Φ, {q}, {p, q}}. If f: (X, m_X) → (Y, m_Y) is totally m_{wg}- continuous function defined by f(a) = {p}, f(b) = {q} and f(c) = {r}, then f is not totally m- continuous functions.

Theorem 4.10. If f : (X, m_X) → (Y, m_Y) is totally m_{wg}- continuous injection and Y is m_{wg}-T_2, then X is m_{wg}-ultra hausdroff.

Proof: Let x_1, x_2 ∈ X and $x_1 \neq x_2$. Then, since f is injective, $f(x_1) \neq f(x_2)$ in (Y, m_Y). Since Y is m_{wg}-T_2, there exist disjoint mwg-open sets U ⊂ Y and V ⊂ Y containing $f(x_1)$ and $f(x_2)$ respectively, and U ∩ V = Φ. This implies $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Since f is totally m_{wg}- continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are mwg-clopen sets in X. Also $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V)$ = Φ. Thus every two distinct points of X can be separated by disjoint mwg-clopen sets. Therefore X is m_{wg}-ultra hausdroff.

Theorem 4.11. If f: (X, m_X) → (Y, m_Y) is totally m_{wg}- continuous and Y is m-T_2 then G(f) is totally m_{wg}-closed graph in product space X × Y.

Proof: Let (x, y) ∈ X × Y. Then y ≠ f(x) and there exists m-open sets V_1 and V_2 such that f(x) ∈ V_1, y ∈ V_2 and $V_1 \cap V_2$ = Φ. From the hypothesis there exists U ∈ m_X-WGCO(X, x) such that f(U) ⊂ V_1. Therefore, we obtain f(U) ∩ V_2 = Φ.

Definition 4.12. A m-space (X, m_X) is called
i. mwg-normal (resp. m_{wg}-ultra normal) if for each pair of non empty disjoint m-closed sets can be separated by disjoint mwg-open (resp. mwg-clopen) sets.
ii. mwg-regular (resp. m_{wg}-ultra regular) if for each mwg-closed set F of X and each x ∉ F, there exist disjoint mwg-open (resp. mwg-clopen) sets U and V such that F ⊂ U and x ∈ V.

Theorem 4.13. if f : $(X,\ m_X) \rightarrow (Y, m_Y)$ is totally m_{wg}- continuous, m-closed injective and Y is mwg-normal, then X is m_{wg}-ultra normal.

Proof: Let U_1 and U_2 be disjoint m-closed subsets of X. Since f is m-closed and injective, $f(U_1)$ and $f(U_2)$ are disjoint m-closed subsets of Y. Since Y is mwg-normal, $f(U_1)$ and $f(U_2)$ are separated by disjoint mwg-open sets V_1 and V_2 respectively. Therefore we obtain, $U_1 \subset f^{-1}(V_1)$ and $U_2 \subset f^{-1}(V_2)$. Since f is totally m_{wg}- continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are mwg-clopen sets in X. Also, $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \Phi$. Thus each non-empty disjoint m-closed in X can be separated by disjoint mwg-clopen sets in X. Therefore X is m_{wg}-ultra normal.

Definition 4.14. A function f : $(X,\ m_X) \rightarrow (Y, m_Y)$ is called mwg-closed if f(U) is mwg-closed in Y for each m-closed set U in X.

Theorem 4.15. Let f : $(X,\ m_X) \rightarrow (Y, m_Y)$ is totally m_{wg}- continuous, mwg-closed injective. If Y is mwg-regular, then X is m_{wg}-ultra regular.

Proof: Let U be a mwg-closed set not containing x. Since f is mwg-closed, we have f(U) is a mwg-closed set in Y not containing f(x). Since Y is mwg-regular, there exist disjoint mwg-open sets V_1 and V_2 such that $f(x) \in V_1$ and $f(U) \in V_2$, which implies $x \in f^{-1}(V_1)$ and $U \subset f^{-1}(V_2)$, where $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are mwg-clopen sets, because f is totally m_{wg}-continuous function. Moreover, since f is injective, $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = f^{-1}(\Phi) = \Phi$. Thus for each pair of point and a mwg-closed set not containing the point, they can be separated by disjoint mwg-clopen sets. Therefore X is is m_{wg}-ultra regular.

Definition 4.16. A function f: $(X,\ m_X) \rightarrow (Y, m_Y)$ is said to be totally m_{wg}- open if the image of every mwg-clopen subset of X is mwg-clopen.

Theorem 4.17. Let f: $(X,\ m_X) \rightarrow (Y, m_Y)$ has a totally m_{wg}-closed graph G(f). If f is surjective totally m_{wg}- open function, then Y is m_{wg}-ultra hausdroff.

Proof: Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) \backslash G(f)$. By the definition, there exists a mwg-clopen set U of X and $V \in O(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \Phi$. Then, we have $f(U) \cap V = \Phi$. Since f is totally m_{wg}- open, then f(U) is mwg-clopen such that $f(x) = y_1 \in f(U)$. This implies that Y is m_{wg}-ultra hausdroff.

Definition 4.18. A space $(X,\ m_X)$ is said to be

i. m_{wg}- space if every mwg-open set of X is m-open in X.

ii. m_{wg}- connected if it cannot be written as the union of two nonempty disjoint mwg-open sets.

Theorem 4.19. If the function f : $(X,\ m_X) \rightarrow (Y, m_Y)$ is totally m- continuous and X is m_{wg}- space, then f is totally m_{wg}- continuous.

Proof of the theorem is obvious.

Theorem 4.20. If f: (X, m_X) → (Y, m_Y) is a totally m_{wg}- continuous function from a m_{wg}- connected space X onto any space Y, then Y is an indiscrete space.

Proof: Suppose that Y is not indiscrete. Let A be a proper non-empty m-open subset of Y. Then f^{-1}(A) is a proper non-empty mwg-clopen subset of (X, m_X), which is a contradiction to the fact that X is m_{wg}- connected.

Theorem 4.21. Let X be m_{wg}- connected, if f: (X, m_X) → (Y, m_Y) is a totally m_{wg}-continuous function with totally m_{wg}-closed graph, then f is constant.

Proof: Suppose that f is not constant. Then there exist two points x and y of X such that f(x) ≠ f(y). Then we have (x, f(y)) ∉ G(f). Since G(f) is totally m_{wg}-closed graph, there exist a mwg-clopen set U of X and V ∈ O(Y) such that f(U) ∩ V = Φ. Hence U ∩ f^{-1}(V) = Φ. This is contradiction with the m_{wg}- connectedness of X.

Theorem 4.22. If f: (X, m_X) → (Y, m_Y) is a totally m_{wg}- continuous surjective function and X is m-connected, then Y is m_{wg}- connected space.

Proof: Suppose Y is not m_{wg}- connected space. Let U and V from disconnection of Y. Then U and V are mwg-open sets in Y and Y = U ∪ V where U ∩ V = Φ. Also X = f^{-1}(Y) = f^{-1}(U ∪ V) = f^{-1}(U) ∪ f^{-1}(V), where f^{-1}(U) and f^{-1}(V) are non empty mwg-clopen sets in X, because f is totally m_{wg}- continuous. Further f^{-1}(U) ∩ f^{-1}(V) = f^{-1}(U ∩ V) = = Φ. This implies X is not connected, which is a contradiction. Hence Y is m_{wg}- connected space.

Definition 4.23. A space (X, m_X) is said to be

i. Totally m_{wg}-Compact if every mwg-clopen cover of X has a finite subcover.
ii. Countably m_{wg}- Compact if every mwg-clopen countably cover of X has a finite subcover.
iii. Totally m_{wg}- Lindelof if every mwg-clopen cover of X has a countable subcover.

Definition 4.24. A subset A of a space X is said to be totally m_{wg}-Compact relative to X if every cover of A mwg-clopen sets of X has a finite subcover.

Theorem: 4.25. If a function f (X, m_X) → (Y, m_Y) is totally m_{wg}- continuous and A is totally m_{wg}-Compact relative to X, then f(A) is m-compact in Y.

Proof: Let {B_α : α ∈ I} be any cover of f(A) by m-open sets of the subspace f(A). For each α ∈ I, there exists a m-open set A_α of Y such that $B_\alpha = K_\alpha$ ∩ f(A). For each x ∈ A, there exists α_x ∈ I such that f(x) ∈ $A_{\alpha x}$ and there exists U_x ∈ m_X-WGCO(X) containing x such that f(U_x) ⊂ $A_{\alpha x}$. Since the family {U_x : x ∈ K} is cover of A by mwg-clopen sets of K, there exists a finite subset A_0 of A such that A ⊂ { U_x : x ∈ A_0}. Therefore, we obtain f(A) ⊂ ∪{f(U_x): x ∈ A_0} which is subset of ∪{$A_{\alpha x}$: x ∈ A_0}. Thus, f(A) = ∪{$A_{\alpha x}$: x ∈ A_0} and f(A) is m-compact.

Theorem 4.26. Let f: (X, m_X) → (Y, m_Y) be a totally m_{wg}- continuous surjective function, then the following statements hold:

i. If X is totally m_{wg}- Lindelof, then Y is m- Lindelof
ii. If X is countably m_{wg}- Compact, then Y is m-countably compact.

Proof: Let $\{B_\alpha : \alpha \in I\}$ be an m-open cover of Y. Since f is totally m_{wg}- continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a mwg-clopen cover of X. Since X is totally m_{wg}- Lindelof, there exists a countable subset I_0 of I such that $X = \cup \{f^{-1}(V_{\alpha x}) : \alpha \in I_0\}$ and Y is m-Lindelof. (ii) similar to (i).

Definition 4.27. A m_{wg}- frontier of a subset A of X is m_{wg}- fr(A) = m_{wg}- Cl(A) $\cap$ m_{wg}- Cl(X \A).

Theorem 4.28. The set of all points x ∈ X in which a function f: $(X, m_X) \rightarrow (Y, m_Y)$ is not totally m_{wg}- continuous is the union of m_{wg}- frontier of the inverse image of m-open sets containing f(x).

Proof: Suppose that f is not totally m_{wg}- continuous at x ∈ X. Then there exists a m-open set V of Y containing f(x) such that f(U) is not contained in V for each U ∈ m_X-WGO(X) containing x and hence x ∈ m_{wg}- Cl(X \ f^{-1}(V)). On the other hand, x ∈ f^{-1}(V) ⊂ m_{wg}- Cl(f^{-1}(V)) and hence x ∈ m_{wg}- fr(f^{-1}(V)).

Conversely, suppose that f is totally m_{wg}- continuous at x ∈ X and let V be a m-open set of Y containing f(x). Then there exists U ∈ MWGO(X) containing x such that U ⊂ f^{-1}(V). Hence, x ∈ m_{wg}-Int (f^{-1}(V)). Therefore, x ∈ m_{wg}- fr(f^{-1}(V)) for each m-open set V of Y containing f(x).

5. Conclusion

In this paper, we introduced the new class of graph functions called as minimal weakly generalized closed graph (m_{wg}-closed graph) and totally m_{wg}-closed graph in minimal structure space. Many of the their properties with some new continuous functions such as quasi mwg-continuous and totally mwg-continuous functions are studied and their characterisations with separation axioms, compact spaces, connected spaces and Lindelof spaces as introduced in m-spaces using minimal weakly generalized closed sets are analyzed.

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