On Decompositions of \((r*g^*)^*\) Closed Set in Topological Spaces

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Abstract. The aim of this paper is to obtain decompositions of \((r*g^*)^*\) closed set. The concept of \((r*g^*)^*\) locally closed sets and \((r*g^*)^*\) locally continuous functions are introduced and some of their properties are investigated. Furthermore the notions of \(P^*\) sets, \(P^{**}\) sets, \(Q^{**}\) sets, \(W^*\) sets and \(A^*\) sets are introduced and are used to obtain the decompositions of \((r*g^*)^*\) closed sets.

Keywords: \((r*g^*)^*\) closed set, \((r*g^*)^*\) closure, \((r*g^*)^*\) continuous functions, \((r*g^*)^*\) irresolute functions, \((r*g^*)^*\) open sets.

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1. Introduction
Levin [10] introduced the concept of generalized closed set in topological spaces. The concept of locally closed sets in a topological space was introduced by Bourbaki [4]. Ganster and Reilly [5] further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. Balachandran et al. [3] introduced the concept of generalized locally closed sets and GLC-continuous functions and investigated some of their properties. Arockiarani, Balachandran and Ganster [2] introduced regular generalized locally closed sets and RGLC- continuous functions. The Authors [12] have already introduced \((r*g^*)^*\) closed sets and investigated some of their properties. The aim of this paper is to introduce \((r*g^*)^*\) locally closed set and \((r*g^*)^*\) locally continuous function and investigate some of their properties. Furthermore the notions of \(P^*\) sets, \(P^{**}\) sets, \(Q^{**}\) sets, \(W^*\) sets and \(A^*\) sets are used to obtain the decompositions of \((r*g^*)^*\) closed sets.

2. Preliminaries

Definition 2.1. A subset \(A\) of a Topological space \(X\) is called

1) A generalized closed set \((g-closed)\) [10] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
A regular generalized closed set (rg-closed) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular open.

A \((r^g*)^*\) closed set if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((r^g*)^*\)-open. The complement of \((r^g*)^*\)-closed set is \((r^g*)^*\)-open.

A locally closed set if \( A = S \cap F \) where \( S \) is open and \( F \) is closed.

A generalized locally closed set if \( A = S \cap F \) where \( S \) is \( g \)-open and \( F \) is \( g \)-closed.

A \( glc^*\)-set if \( A = S \cap F \) where \( S \) is \( g \)-open and \( F \) is closed.

A \( glc^{**}\)-set if \( A = S \cap F \) where \( S \) is open and \( F \) is \( g \)-closed.

A regular generalized locally closed set is \( S = G \cap F \) where \( G \) is \( rg \)-open and \( F \) is \( rg \)-closed in \((X, \mathcal{T})\).

A \( rglc^*\) if there exists a \( rg \)-open set \( G \) and a closed set \( F \) of \((X, \mathcal{T})\) such that \( S = G \cap F \).

A \( rglc^{**}\) if there exists an open set \( G \) and a \( rg \)-closed set \( F \) such that \( B = G \cap F \).

**Definition 2.2.** A subset \( S \) of a topological space is called a

1. \( t \)-set if \( \text{int}(S) = \text{int}(\text{cl}(S)) \).
2. \( t^* \)-set if \( \text{cl}(S) = \text{cl}(\text{int}(S)) \).
3. \( \alpha^* \)-set if \( \text{int}(S) = \text{int}(\text{cl}(\text{int}(S))) \).
4. \( C \)-set if \( S = G \cap F \) where \( G \) is open and \( F \) is a \( t \)-set.
5. \( Cr \)-set if \( S = L \cap M \) where \( L \) is \( rg \)-open and \( M \) is a \( t \)-set.
6. \( Cr^* \)-set if \( S = L \cap M \) where \( L \) is \( rg \)-open and \( M \) is a \( \alpha^* \)-set.
7. A set if \( S = G \cap F \) where \( G \) is open and \( F \) is a regular closed set.

**Definition 2.3.** Let \( X \) be a topological space. Let \( A \) be a subset of \( X \). A \((r^g*)^*\) closure if \( A \) is defined as the intersection of all \((r^g*)^*\)-closed sets containing \( A \).

**Definition 2.3.** A function \( f : (X, \mathcal{T}) \to (Y, \sigma) \) is called

(i) \( g \)-continuous if \( f^{-1}(V) \) is \( g \)-closed in \((X, \mathcal{T})\) for every closed set \( V \) of \((Y, \sigma)\).

(ii) \((r^g*)^*\)-continuous if the inverse image of every closed set in \((Y, \sigma)\) is \((r^g*)^*\)-closed in \((X, \mathcal{T})\).

(iii) \((r^g*)^*\)-irresolute map if \( f^{-1}(V) \) is a \((r^g*)^*\)-closed set in \((X, \mathcal{T})\) for every \((r^g*)^*\)-closed set \( V \) of \((Y, \sigma)\).

(iv) LC-continuous if \( f^{-1}(V) \) is a locally closed set in \((X, \mathcal{T})\) for every open set \( V \) of \((Y, \sigma)\).

(v) G LC-continuous if \( f^{-1}(V) \) is a gl-closed set in \((X, \mathcal{T})\) for every open \( V \) of \((Y, \sigma)\).

(vi) Rgl continuous if \( f^{-1}(V) \) is a rgl closed set in \((X, \mathcal{T})\) for every open \( V \) of \((Y, \sigma)\).

**3. \((r^g*)^*\)-locally closed sets**

**Definition 3.1.** A Subset \( S \) of \((X, \mathcal{T})\) is called \((r^g*)^*\)-Locally closed if \( S = A \cap B \) where \( A \) is \((r^g*)^*\)-open and \( B \) is \((r^g*)^*\)-closed.

**Example 3.2.** Let \( X = \{ a, b, c \} \). Let \( \mathcal{T} = (\varnothing, X, \{a\}, \{b\}, \{a,b\}) \).
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Closed sets are \(\{\varnothing, X, \{c\}, \{b,c\}, \{a,c\}\}\)
\((r^g)^*\) closed sets are \(\{\varnothing, X, \{c\}, \{b,c\}, \{a,c\}\}\)
\((r^g)^*\) open sets are \(\{\varnothing, X, \{a\}, \{a,b\}\}\)
Now \(\{a\} = \{a,b\} \cap \{a,c\}\) where \(\{a,b\}\) is \((r^g)^*\) open and \(\{a,c\}\) \((r^g)^*\) closed and hence \(\{a\}\) is a \((r^g)^*\) locally closed set.
Here \(\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\) are \((r^g)^*\) Locally closed sets.

**Definition 3.3.** A Subset \(S\) of \((X, \mathcal{I})\) is called \((r^g)^*\) Locally * closed if \(S = A \cap B\) where \(A\) is \((r^g)^*\) open and \(B\) is closed.

**Example 3.4.** In Example 3.2 \(\{b\} = \{a,b\} \cap \{b,c\}\) is \((r^g)^*\) Locally * closed.

**Definition 3.5.** A subset \(S\) of \((X, \mathcal{I})\) is called \((r^g)^*\) locally ** closed if \(S = A \cap B\) where \(A\) is open and \(B\) is \((r^g)^*\) closed.

**Example 3.6.** In Example 3.2 \(\{a\} = \{a,b\} \cap \{a,c\}\) is \((r^g)^*\) locally **closed.

**Remark 3.7.** Every closed set is \((r^g)^*\) locally closed set.

**Theorem 3.8.**
(i) Every Locally closed sets is \((r^g)^*\) locally closed.
(ii) Every \(g^*\) locally closed set is \((r^g)^*\) locally closed
(iii) Every \((r^g)^*\) locally closed set is gpr locally closed
(iv) Every \((r^g)^*\) locally closed set is rwg locally closed
(v) Every \((r^g)^*\) locally closed set is rg locally closed

**Proof:**
(i) Let \(S = A \cap B\) where \(A\) is open and \(B\) is closed in \(X\). But every open set is \((r^g)^*\) open and every closed set is \((r^g)^*\) closed and hence \(S\) is \((r^g)^*\) locally closed set.
(ii) Proof follows from the fact that every \(g^*\) closed set is \((r^g)^*\) closed set [12].
(iii) Proof follows from the fact that every \((r^g)^*\) closed set is gpr closed set [12].
(iv) Proof follows from the fact that every \((r^g)^*\) closed set is rwg closed set [12].
(v) Proof follows from the fact that every \((r^g)^*\) closed set is rg closed set [12].
The converse of the above statements need not true as seen from the following example.

**Example 3.9.**
(i) Let \(X = \{a, b, c\}\). Let \(\mathcal{I} = \{\varnothing, \{c\}, \{b,c\}\}\)
Closed sets are \(\{\varnothing, X, \{a\}, \{a,b\}\}\)
\((r^g)^*\) closed sets are \(\{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}\}\)
\((r^g)^*\) open sets are \(\{\varnothing, X, \{b,c\}, \{c\}, \{b\}\}\)
Here \(\{a,c\}\) is \((r^g)^*\) locally closed set but not locally closed set.
Let $X = \{a, b, c\}$, $\mathcal{I} = \{\emptyset, X, \{a\}\}$. Here $\{b\}$ is not a $g^*$ locally closed set but it is a $(r^*g^*)^*$ closed set.

Let $X = \{a, b, c, d\}$, $\mathcal{I} = \{\emptyset, X, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Here $\{b, c, d\}$ is not a $g^*$ locally closed set but it is a $(r^*g^*)^*$ closed set.

Here $\{c, d\}$ is $\alpha$ locally closed set but not $(r^*g^*)^*$ closed set.

In the above example $\{a, c, d\}$ is $wg$ locally closed set but not $(r^*g^*)^*$ closed set.

In the above example $\{c, d\}$ is $rg$ closed set but not $(r^*g^*)^*$ closed set.

Remark 3.10.

$(r^*g^*)^*$ locally closed sets are independent of semilocally closed sets, $\alpha$ locally closed sets, $wg$ locally closed sets and the following examples support our statement.

Example 3.11.
Let $X = \{a, b, c, d\}$, $\mathcal{I} = \{\emptyset, X, \{a\}, \{c, d\}\}$. Here $\{a\}$ is $(r^*g^*)^*$ locally closed but not semi locally closed set.

Let $X = \{a, b, c, d\}$, $\mathcal{I} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Here $\{c, d\}$ is semi locally closed set but not $(r^*g^*)^*$ locally closed set.

Example 3.12.
Let $X = \{a, b, c, d\}$, $\mathcal{I} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Here $\{a, c\}$ is not $\alpha$ locally closed set but $\{a, c\}$ is $(r^*g^*)^*$ locally closed set.

Here $\{c, d\}$ is $\alpha$ locally closed set but $\{c, d\}$ is not $(r^*g^*)^*$ locally closed.

Example 3.13. From the above example, $\{c, d\}$ is $wg$ locally closed set but $\{c, d\}$ is not $(r^*g^*)^*$ locally closed set.

Let $X = \{a, b, c, d\}$, $\mathcal{I} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Here $\{a\}$ is $(r^*g^*)^*$ locally closed set but not $wg$ locally closed set.

Theorem 3.14. Every locally closed set is $(r^*g^*)^*$ locally closed.

The converse need not be true as seen from the following example.

Example 3.15. In example 3.8 $\{a, c\}$ is $(r^*g^*)^*$ locally closed but not locally closed.

Theorem 3.16. Every locally closed set is $(r^*g^*)^*$ locally closed.

The converse need not be true as seen from the following example.

In example 3.9 $\{a, c\}$ is $(r^*g^*)^*$ locally closed but not locally closed.

Theorem 3.17. If $A$ is $(r^*g^*)^*$ locally closed in $X$ and $B$ is $(r^*g^*)^*$ open then $A \cap B$ is $(r^*g^*)^*$ locally closed in $X$.

Proof: Since $A$ is $(r^*g^*)^*$ locally closed $A = P \cap Q$ where $P$ is $(r^*g^*)^*$ closed and $Q$ is $(r^*g^*)^*$ open. Now $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$.

Since $(Q \cap B)$ is $(r^*g^*)^*$ open and $P$ is $(r^*g^*)^*$ closed $A \cap B$ is $(r^*g^*)^*$ locally closed.
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**Theorem 3.18.** A subset $S$ of $(X, \mathcal{I})$ is (r*g*)* locally closed $(X - S)$ is the union of a (r*g*)* open and a (r*g*)* closed set.

**Proof:** If $S$ is (r*g*)* locally closed then $S = P \cap Q$ where $P$ is (r*g*)* closed and $Q$ is (r*g*)* open. Now $X - S = X - (P \cap Q) = (P^c \cup Q^c)$. Now $P^c$ is (r*g*)* open and $Q^c$ is (r*g*)* closed. Hence the result.

**Result 3.19.** The complement of a (r*g*)* locally closed set need not be locally closed.

**Example 3.20.** Let $X = \{a,b,c\}$, $\mathcal{I} = \{\varnothing, X, \{c\}, \{b,c\}\}$. Closed sets are $\{\varnothing, X, \{a,b\}, \{a\}\}$. Here $\{b\}$ is (r*g*)* locally closed. But its complement $\{a,c\}$ is not locally closed.

**Theorem 3.21.** Let $A$ and $B$ are subsets of $(X, \mathcal{I})$. If $A$ is (r*g*)* locally closed and $B$ is open then $A \cap B$ is (r*g*)* locally closed.

**Proof:** Let $A$ be (r*g*)* locally closed in $(X, \mathcal{I})$. Then there exists an open set $P$ and (r*g*)* closed set $Q$ such that $A = P \cap Q$. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$ which is (r*g*)* locally closed.

**Theorem 3.22.** If $A$ is (r*g*)* locally closed subset of $(X, \mathcal{I})$ and $B$ is (r*g*)* closed then $A \cap B$ is (r*g*)* locally closed.

**Proof:** Let $A$ be (r*g*)* locally closed. Then $A = P \cap Q$ where $P$ is (r*g*)* open and $Q$ is closed. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$. Hence $A \cap B$ is (r*g*)* locally closed.

**Theorem 3.23.** If $A$ is (r*g*)* locally closed subset of $(X, \mathcal{I})$ and $B$ is (r*g*)* open then $A \cap B$ is (r*g*)* locally closed.

**Proof:** Let $A$ be (r*g*)* locally closed. Then $A = P \cap Q$ where $P$ is (r*g*)* open and $Q$ is (r*g*)* closed. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$. Hence $A \cap B$ is (r*g*)* locally closed.

4. (r*g*)* locally continuous functions

**Definition 4.1.** A function $f : (X, \mathcal{I}) \rightarrow (Y, \sigma)$ is called (r*g*)* locally continuous if $f^{-1}(V)$ is (r*g*)* locally closed in $(X, \mathcal{I})$ for every open set $V$ in $(Y, \sigma)$.

**Example 4.2.** Let $X = \{a,b,c\}$ and $\mathcal{I} = \{\varnothing, X, \{c\}, \{b,c\}\}$. Closed sets are $\{\varnothing, X, \{a\}, \{a,b\}\}$. (r*g*)* Closed sets are $\{X, \varnothing, \{a\}, \{a,b\}\}$. (r*g*)* open sets are $\{X, \varnothing, \{b,c\}, \{c\}\}$. (r*g*)* locally closed set are $\{X, \varnothing, \{a\}, \{a,b\}, \{a,c\}, \{b,c\}, \{c\}\}$. Let $Y = \{a,b,c\}$ $\sigma = \{Y, \varnothing, \{b\}\}$ Define $f : (X, \mathcal{I}) \rightarrow (Y, \sigma)$ defined by $f(a) = a$, $f(b) = c$ $f(c) = b$. Now $\{b\} \in \sigma$ and $f^{-1}(\{b\}) = \{c\}$ which is (r*g*)* locally closed in $(X, \mathcal{I})$. Hence $f$ is (r*g*)* locally continuous function.
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**Definition 4.3.** A function \( f : (X, \mathcal{S}) \to (Y, \sigma) \) is said to be a \((r^*g^*)^*\) locally irresolute function if \( f^{-1}(V) \) is a \((r^*g^*)^*\) locally closed set in \((X, \mathcal{S})\) for every \((r^*g^*)^*\) locally closed set \( V \) of \((Y, \sigma)\).

**Example 4.4.** Let \( X = \{a,b,c\} \) and \( \mathcal{S} = \{\emptyset, X, \{a\}\} \). Closed sets = \{\emptyset, X, \{b\}, \{a,b\}, \{a\}\}.

\((r^*g^*)^*\) closed sets are \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a\}\}.

\((r^*g^*)^*\) locally closed sets are \{\emptyset, X, \{a\}, \{a,b\}, \{a\}\}.

\(Y = \{a,b,c\}, \sigma = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}\). Closed set of \( Y = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}\). \((r^*g^*)^*\) closed set of \( Y \) are \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}.

\((r^*g^*)^*\) open sets of \( Y \) are \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}.

Here let \( f : (X, \mathcal{S}) \to (Y, \sigma) \) be defined by \( f(c) = a, f(b) = b, f(a) = c \).

\( f^{-1}(\{a\}) = \{b\}, f^{-1}(\{b\}) = \{a\}, f^{-1}(\{c\}) = \{a\}. \) \( f^{-1}(\{a,b\}) = \{a\}, f^{-1}(\{a,c\}) = \{b\} \).

The converse need not be true as seen from the following example.

**Theorem 4.5.** Every locally continuous function is \((r^*g^*)^*\) locally continuous.

**Proof:** Let \( f : (X, \mathcal{S}) \to (Y, \sigma) \) be a locally continuous map. Let \( F \) be an open set in \((Y, \sigma)\). Then \( f^{-1}(F) \) is locally closed in \((X, \mathcal{S})\). Since every locally closed set is \((r^*g^*)^*\) locally closed, \( f^{-1}(F) \) is \((r^*g^*)^*\) locally closed set. Therefore \( f \) is \((r^*g^*)^*\) locally continuous.

Similarly we can prove the following results.

**Theorem 4.7.**

(i) Every \( g^* \) locally continuous function is \((r^*g^*)^*\) locally continuous.

(ii) Every \((r^*g^*)^*\) locally continuous function is \( gpr \) locally continuous function.

(iii) Every \((r^*g^*)^*\) locally continuous function set is \( rwg \) locally continuous function.

(iv) Every \((r^*g^*)^*\) locally continuous function set is \( rg \) locally continuous function.
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**Definition 4.8.** A function \(f: (X, \Im) \to (Y, \sigma)\) is called \((r\ast g\ast)^*\)locally *continuous, if \(f^{-1}(V)\) is \((r\ast g\ast)^*\)locally closed in \((X, \Im)\) for every \(V \in \sigma\).

**Example 4.9.** In example 4.6 the function \(f\) is \((r\ast g\ast)^*\)locally*continuous function.

**Definition 4.10.** A map \(f: (X, \Im) \to (Y, \sigma)\) is said to be a \((r\ast g\ast)^*\) locally* irresolute map if \(f^{-1}(V)\) is \((r\ast g\ast)^*\)locally*closed set in \((X, \Im)\) for every \((r\ast g\ast)^*\) locally*closed set \(V\) of \((Y, \sigma)\).

**Example 4.11.** In example 4.4 \(f\) is \((r\ast g\ast)^*\)locally*irresolute.

**Definition 4.12.** A function \(f: (X, \Im) \to (Y, \sigma)\) is  \((r\ast g\ast)^*\)locally **continuous,  if \(f^{-1}(V)\) is \((r\ast g\ast)^*\)locally** closed in \((X, \Im)\) for every \(V\) open in \((Y, \sigma)\).

In example 4.6 the function \(f\) is \((r\ast g\ast)^*\)locally**closed continuous

**Definition 4.13.** A function \(f: (X, \Im) \to (Y, \sigma)\) is said to be a \((r\ast g\ast)^*\) locally** irresolute function if \(f^{-1}(V)\) is \((r\ast g\ast)^*\)locally**closed set in \((X, \Im)\) for every \((r\ast g\ast)^*\)locally**closed set \(V\) of \((Y, \sigma)\).

**Example 4.14.** The function \(f\) defined in example 4.4 is a \((r\ast g\ast)^*\) locally** irresolute.

**Theorem 4.15.** Let \(f: (X, \Im) \to (Y, \sigma)\) be a function. If \(f\) is locally continuous, then it is \((r\ast g\ast)^*\)locally*continuous and \((r\ast g\ast)^*\)locally**continuous.

**Proof:** Let \(f: (X, \Im) \to (Y, \sigma)\) be a locally continuous function. Let \(F \in \sigma\). Then \(f^{-1}(F)\) is locally closed in \((X, \Im)\). Since every locally closed set is \((r\ast g\ast)^*\)locally*closed, \(f^{-1}(F)\) is \((r\ast g\ast)^*\)locally*closed set. Therefore \(f\) is \((r\ast g\ast)^*\)locally*continuous. Also since every locally closed set is \((r\ast g\ast)^*\)locally**closed, \(f^{-1}(F)\) is \((r\ast g\ast)^*\)locally**closed set. Hence \(f\) is \((r\ast g\ast)^*\)locally**continuous.

The converse need not be true as seen from the following example.

**Example 4.16.** Let \(X = \{a,b,c\}\) and \(\Im = \{X, \emptyset, \{a\}, \{b,c\}\}\).

Closed sets are \(\{X, \emptyset, \{a\}, \{b,c\}\}\).

Locally closed sets of \(X\) are \(\{X, \emptyset, \{a\}, \{b,c\}\}\).

\((r\ast g\ast)^*\)Closed sets of \(X\) are \(\{X, \emptyset, \{a\}, \{b,c\}\}\).

\((r\ast g\ast)^*\)open sets of \(X\) are \(\{X, \emptyset, \{a\}, \{b\}\}\).

\((r\ast g\ast)^*\)locally closed set of \(X\) are \(\{X, \emptyset, \{a\}, \{b\}\}\).

\((r\ast g\ast)^*\)locally** closed set of \(X\) are \(\{X, \emptyset, \{a\}, \{b\}\}\).

Let \(Y = \{a,b,c\}\) and \(\sigma = \{\emptyset, \{a\}\}\) Closed set = \(\{\emptyset, \{b,c\}\}\).

Define a mapping \(f: (X, \Im) \to (Y, \sigma)\) by \(f(a) = a, f(b) = c, f(c) = b\).

Here \(f^{-1}\{a\} = \{a\}\) is \((r\ast g\ast)^*\)locally*closed and \((r\ast g\ast)^*\)locally**closed but not a locally closed set. Hence \(f\) is \((r\ast g\ast)^*\)locally*closed continuous and \((r\ast g\ast)^*\)locally**continuous but not locally continuous.
**Theorem 4.17.** Let $f : (X, \Im) \rightarrow (Y, \sigma)$ be a function. If $f$ is $(r^\ast g^\ast)$ locally continuous, then it is $(r^\ast g^\ast)$ locally continuous.

**Proof:** Let $f$ be $(r^\ast g^\ast)$ locally continuous. Let $V \in \sigma$. Then $f^{-1}(V)$ is $(r^\ast g^\ast)$-locally closed. \:\: $f^{-1}(V) = F \cap G$ where $F$ is $(r^\ast g^\ast)$ open and $G$ is closed. But every closed set is $(r^\ast g^\ast)$ closed. \:\: $G$ is $(r^\ast g^\ast)$ closed and. \:\: $f^{-1}(V)$ is $(r^\ast g^\ast)$-locally closed. Hence $f$ is $(r^\ast g^\ast)$-locally continuous.

The converse need not be true as seen from the following example.

**Example 4.18.** In example 4.16 let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{b, c\}\}$. Closed set $= \{\emptyset, Y, \{b, c\}\}$. Define $f$ by $f(a) = b$, $f(c) = c$, $f(b) = a$. \:\: $f^{-1}(b, c) = \{a, c\}$ is $(r^\ast g^\ast)$-locally closed and hence $f$ is $(r^\ast g^\ast)$-locally continuous but $f^{-1}(b, c) = \{a, c\}$ is not $(r^\ast g^\ast)$-locally closed. Therefore $f$ is not $(r^\ast g^\ast)$-locally continuous.

Similarly, we can prove the following theorem.

**Theorem 4.19.** Let $f : (X, \Im) \rightarrow (Y, \sigma)$ be a map. If $f$ is $(r^\ast g^\ast)$ locally continuous, then it is $(r^\ast g^\ast)$ locally continuous.

The converse need not be true as seen from the following example.

In example 4.16 let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{a, c\}\}$. Define $f$ by $f(b) = b$, $f(c) = a$, $f(a) = c$. Now $f^{-1}(b) = \{b\}$ is $(r^\ast g^\ast)$-locally closed and hence $f$ is $(r^\ast g^\ast)$-locally continuous. But $f^{-1}(b) = \{b\}$ is not $(r^\ast g^\ast)$ locally closed in $X$. Therefore $f$ is not $(r^\ast g^\ast)$-locally continuous.

**Theorem 4.20.** Let $f : (X, \Im) \rightarrow (Y, \sigma)$ be a map. If $f$ is $(r^\ast g^\ast)$ locally irresolute, then it is $(r^\ast g^\ast)$ locally continuous.

**Proof:** Let $V \in \sigma$. Then $V = V \cap Y$. Hence $V$ is $(r^\ast g^\ast)$-locally closed in $Y$. Since $f$ is $(r^\ast g^\ast)$ locally irresolute, $f^{-1}(V)$ is $(r^\ast g^\ast)$ locally closed. Now $f^{-1}(V) = F \cap G$, where $F$ is $(r^\ast g^\ast)$ open and $G$ is closed. But every closed set is $(r^\ast g^\ast)$ closed. \:\: $f^{-1}(V)$ is $(r^\ast g^\ast)$ locally closed. Hence $f$ is $(r^\ast g^\ast)$ locally continuous.

The converse need not be true as seen from the following example.

In example 4.16, let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$. Closed set $= \{\emptyset, Y, \{a, b\}\}$. Let $f$ be defined by $f(a) = a$, $f(c) = b$, $f(b) = c$. Now $f^{-1}(a, b) = \{a, c\}$ is $(r^\ast g^\ast)$-locally closed but not $(r^\ast g^\ast)$-locally closed. Hence $f$ is $(r^\ast g^\ast)$-locally continuous but not $(r^\ast g^\ast)$-locally irresolute.

**Remark 4.21.** Composition of two $(r^\ast g^\ast)$ locally continuous functions need not be $(r^\ast g^\ast)$ locally continuous.

Let $X = Y = \{a, b, c, d\}$.

Let $f : (X, \Im) \rightarrow (Y, \sigma)$ where $\Im = \{\emptyset, X, \{a, c\}, \{a, d\}, \{a\}, \{a, c, d\}\}$. $\sigma = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Let $f$ be defined by $f(a) = a$, $f(d) = c$, $f(c) = b$, $f(b) = d$ $f^{-1}(a, b) = \{a, c\}$, $f^{-1}(c, d) = \{d, b\}$ $f^{-1}(a, b)$ is $(r^\ast g^\ast)$ locally closed. $f^{-1}(\{c, d\})$ is $(r^\ast g^\ast)$ locally closed. Hence
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\(f : X \rightarrow Y\) is \((r^*g^*)^*\) locally continuous

let \(g : (Y, \sigma) \rightarrow (Z, \eta)\) where \(\eta = \{\emptyset, X, \{a, b\}\}\) be defined by \(g(c) = b, g(a) = c, g(d) = d, g(b) = a\).

\(g^{-1}(\{a, b\}) = \{b, c\}\) is \((r^*g^*)^*\) locally closed and hence \(g\) is \((r^*g^*)^*\) locally continuous but \((g \circ f)^{-1}(\{a, b\}) = f^{-1}(g^{-1}(\{a, b\}) = f^{-1}(\{b, c\}) = \{c, d\}\) is not \((r^*g^*)^*\) locally closed. Hence \(g \circ f\) is not \((r^*g^*)^*\) locally continuous.

The following theorem gives the condition under which the composition of two functions is \((r^*g^*)^*\)locally continuous.

**Theorem 4.22.** Let \(f : (X, \mathcal{I}) \rightarrow (Y, \sigma)\) and \(g : (Y, \sigma) \rightarrow (Z, \eta)\) be two function. Then

1) \(g \circ f\) is \((r^*g^*)^*\)locally continuous if \(g\) is \((r^*g^*)^*\)locally continuous and \(f\) is \((r^*g^*)^*\)locally irresolute.

2) \(g \circ f\) is \((r^*g^*)^*\)locally irresolute if both \(f\) and \(g\) are \((r^*g^*)^*\)locally irresolute.

3) \(g \circ f\) is \((r^*g^*)^*\)locally continuous if \(g\) is \((r^*g^*)^*\)locally continuous and \(f\) is \((r^*g^*)^*\)locally irresolute.

5. Another decomposition of \((r^*g^*)^*\) closed sets

The following definitions are introduced to obtain decompositions of \((r^*g^*)^*\)closed set.

**Definitions 5.1.** A subset \(A\) of a topological space \(X\) is called a

1) \(P^*\)set if \(A = L \cap M\) where \(L\) is \((r^*g^*)^*\)open and \(M\) is a \(t\) set.

2) \(P^{**}\) set if \(A = L \cap M\) where \(L\) is \((r^*g^*)^*\)open and \(M\) is a \(t^*\) set.

3) \(Q^{**}\) set if \(A = L \cap M\) where \(L\) is \((r^*g^*)^*\)open and \(M\) is a \(C^*\) set.

4) \(W^*\) set if \(A = L \cap M\) where \(L\) is \((r^*g^*)^*\)open and \(M\) is a \(\alpha^*\) set.

5) \(A^*\) set if \(A = L \cap M\) where \(L\) is \((r^*g^*)^*\)open and \(M\) is a regular closed set.

**Propositions 5.2.**

1. Every \(C^*\) set is a \(P^*\) set.
2. Every \(P^*\) set is \(C^*\) set.
3. Every \(W^*\) set is \(C^*\) set.
4. Every \(A^*\) set is \(P^{**}\) set.
5. Every \(t^*\) set is \(P^*\) set.
6. Every \(C^*\) set is \(Q^{**}\) set.
7. Every \(\alpha^*\) set is \(W^*\) set.
8. Every \((r^*g^*)^*\)open set is \(P^*\) set.
9. Every \((r^*g^*)^*\)open set is \(W^*\) set.

**Remark 5.3.** The converses need not be true as seen from the following examples.

**Example 1.** Let \(X = \{a, b, c\}\) \(\mathcal{Z} = \{\emptyset, X, \{a\}, \{b, c\}\}\).

Here \(\{b\}\) is \(P^*\) but not \(C^*\).

**Example 2.** Let \(X = \{a, b, c\}\) \(\mathcal{Z} = \{\emptyset, X, \{b\}, \{a, b\}\}\).

Here \(\{b, c\}\) is a \(C^*\) set but not a \(P^*\) set.

**Example 3.** In example 2 \(\{b, c\}\) is \(C^*\) but not \(W^*\).
Example 4. In example 2 \{b,c\} is P** but not A*.
Example 5. In example 2 \{a\} is A* but not A.
Example 6. In example 2 \{a,b\} is P* but not t
Example 7. In example 1 \{a,b\} is Q** but not C.
Example 8. In example 2 \{b\} is w* but not \(\alpha\)
Example 9. In example 2 \{c\} is P* but not (r*g*)*open.
Example 10. In example 2 \{a,c\} is W* but not (r*g*)*open.

REFERENCES