Mean Time to Recruitment for a Multigrade Manpower System with Two Sources of Depletion when Wastages form an order Statistics and the Breakdown Threshold Distribution Follows SCBZ Property

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Abstract. In this paper a multi graded organization in which depletion of manpower occur due to policy and transfer decisions is considered when the breakdown threshold distribution follows SCBZ property. Mean time to recruitment is obtained by using an univariate CUM policy of recruitment (ie) "The organization survives iff atleast $r$ (1 $\leq r \leq n$) out of $n$ grades survives in the sense that threshold crossing has not take place in these grades" when wastages form an order statistics. The influence of the nodal parameter on the system characteristics is studied and relevant conclusions are presented

Keywords: Loss of man hours, Policy decision, Transfer decision, Order statistics, SCBZ property.

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1. Introduction

Exit of personal which is in other words known as wastage, is an important aspects in the study of manpower planning. Many models have been discussed using different types of wastages and also different types of distribution for the loss of man powers, the thresholds and inter decision times. Such models are seen in [1,2]. In [3,4,5, 6] the authors have obtained the mean time to recruitment in a two grade manpower system based on order statistics by assuming different distribution for thresholds. In [8] for a two grade manpower system with two types of decisions when the wastages form a geometric process is obtained. The problem of time to recruitment is studied by several authors for the organizations consisting of single grade/two grade/ three grades. More specifically for a two grade system, in all the earlier work, the threshold for the organization is minimum or maximum or sum of the thresholds for the loss of manpower in each grades, no attempt has been made so far to design a comprehensive recruitment policy for a system with two or three grades. In [10,11,12] a new design for a comprehensive univariate CUM recruitment policy of manpower system is used with n grades in order to bring results proved independently for maximum, minimum model as a special case. In all
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previous work, the problem of time to recruitment is studied for only an organization consisting of at most three grades. In [11,12] author has worked on this comprehensive univariate policy when wastages form ordinary renewal process and inter decision time form geometric and order statistics. Here an attempt is made for loss of manpower due to policy and transfer decision with high and low attrition rate for inter policy decision.

2. Model description and assumptions

An organization having $n$-grades in which decisions are taken at random epochs $(0, \infty)$ is considered. At every policy decision epoch a random number of person quit the organization and at every transfer decision epoch a random number of persons are transferred. It is assumed that the loss of manpower is linear and cumulative. The loss of manpower process, process of inter policy and inter transfer decision times are statistically independent. The loss of manpower follows order statistics, inter policy decision times are hyper exponential and inter transfer decision times are exponential. The thresholds for the $n$-grades are independent and identically distributed random variables following SCBZ property with same parameter. Univariate CUM policy of recruitment “The organization survives iff at least $r$ ($1 \leq r \leq n$) out of $n$ grades survives in the sense that threshold crossing has not take place in these grades”.

$x_i$: Continuous random variable denoting the amount of depletion of manpower caused due to the $i^{th}$ policy decision in organization.$t_i$: Time of occurrence of the $i^{th}$ decision.$x_{m1}$: Cumulative loss of manpower due to the first $m1$ policy decisions in the Organization.$y_j$: Continuous random variable denoting the amount of depletion of manpowers caused due to the $j^{th}$ transfer decision in organization.$\bar{y}_{n1}$: Cumulative loss of manpower due to the first $n1$ transfer decisions in the Organization.

$W_U(.)$: The distribution function of inter policy decision times with hyper exponential i.i.d random variable.

$W_V(.)$: The distribution function of inter transfer decision times with exponential i.i.d random variable.

$W_{U}^{m1}(.)$: $m1$ fold convolution of $W_U$ with itself.

$W_{V}^{n1}(.)$: $n1$ fold convolution of $W_V$ with itself.

$\bar{x}_{m1} + \bar{y}_{n1}$: The cumulative loss of manpower due to $m1$ policy decisions and $n1$ transfer decision.

$\bar{W}_{\bar{x}_{m1} + \bar{y}_{n1}}$: Distribution function of cumulative loss of manpower due to $m1$ policy decisions and $n1$ transfer decision.

$T$: Time to recruitment.

$E(T)$: Mean time to recruitment.

$N_p(T)$: Number of policy decisions at time T.

$N_{\text{Trans}}(T)$: Number of transfer decisions at time T.

3. Main results

From renewal theory, the survival function of $T$ is

$$P(T > t) = P(\bar{x}_{N_p(T)} + \bar{y}_{N_{\text{Trans}}(T)} < Z)$$

(1)

Conditioning upon $N_p(T)$ and $N_{\text{Trans}}(T)$ and using law of total probability

$$P(T > t) = \sum_{m1=0}^{\infty} P(N_p(T) = m1) \sum_{n1}^{\infty} P(N_{\text{Trans}}(T) = n1) P(\bar{x}_{m1} + \bar{y}_{n1} \leq Z)$$

(2)
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As \( \{N_p(T)\} \) and \( \{N_{Trans}(T)\} \) are two independent ordinary renewal process by hypothesis invoking the result, \( (N_p(T) = m_1) = W_u^{m_1}(t) - W_u^{m_1+1}(t) \) and \( P(N_{Trans}(T) = n_1) = W_v^{n_1}(t) - W_v^{n_1+1}(t) \)

\[
P(T > t) = \sum_{m_1=0}^{\infty} [W_u^{m_1}(t) - W_u^{m_1+1}(t)] \sum_{n_1=0}^{\infty} [W_v^{n_1}(t) - W_v^{n_1+1}(t)] P(\bar{x}_{m_1} + \bar{y}_{n_1} \leq Z) (3)
\]

where \( W_u^{0}(t) = W_v^{0}(t) = 1 \)

Let \( z_j, j = 1, 2, \ldots, n \) follows SCBZ property with same parameter \( \theta \), we have by law of total probability,

\[
H(z) = 1 - pe^{-(\theta + \theta_1)z} - qe^{-(\theta_2)z} (4)
\]

where \( p = \frac{\theta_1 - \theta_2}{\theta + \theta_1 - \theta_2} \) and \( q = \frac{\theta}{\theta + \theta_1 - \theta_2} \) with \( p+q=1 \)

Since \( Z \) is independent of \( \bar{x}_{m_1} \) and \( \bar{y}_{n_1} \), by hypothesis conditioning upon \( z \) and using law of total probability

\[
P(\bar{x}_{m_1} + \bar{y}_{n_1} \leq Z) = \int_0^\infty P(\bar{x}_{m_1} + \bar{y}_{n_1} < z) h(z)dz
\]

where

\[
h(z) = \sum_{i=r}^{n} nC_i \left\{ p^i n(\theta + \theta_1)e_{-n(\theta + \theta_1)z} + \cdots q^i n\theta_2 e^{-n\theta_2 z} \right\} - (n - i)C_i \left\{ p^{i+1}(i + 1)(\theta + \theta_1)e_{-(i+1)(\theta + \theta_1)z} + \cdots q^{i+1}(i + 1)(\theta_2)e_{-(i+1)(\theta_2)z} \right\}
\]

\[
\left\{ \begin{array}{c}
-(-1)^n-i\left\{ p^n n(\theta + \theta_1)e_{-n(\theta + \theta_1)z} + \cdots q^n\theta_2 e^{-n\theta_2 z} \right\}
\end{array} \right\} dz
\]

Substituting equation (8) in (3)

\[
P(T > t) = \sum_{m_1=0}^{\infty} [W_u^{m_1}(t) - W_u^{m_1+1}(t)] \sum_{n_1=0}^{\infty} [W_v^{n_1}(t) - W_v^{n_1+1}(t)]
\]

\[
\int_0^\infty P(\bar{x}_{m_1} + \bar{y}_{n_1} < z)
\]

\[
\sum_{i=r}^{n} nC_i \left\{ p^i (\theta + \theta_1)e_{-(\theta + \theta_1)z} + \cdots q^i \theta_2 e^{-i\theta_2 z} \right\} - (n - i)C_i \left\{ p^{i+1}(i + 1)(\theta + \theta_1)e_{-(i+1)(\theta + \theta_1)z} + \cdots q^{i+1}(i + 1)(\theta_2)e_{-(i+1)(\theta_2)z} \right\}
\]

\[
\left\{ \begin{array}{c}
-(-1)^n-i\left\{ p^n n(\theta + \theta_1)e_{-n(\theta + \theta_1)z} + \cdots q^n\theta_2 e^{-n\theta_2 z} \right\}
\end{array} \right\} dz
\]
\[ P(T > t) = \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) - W_{U}^{m_1+1}(t) \sum_{n_1}^{\infty} W_{V}^{n_1}(t) - W_{V}^{n_1+1}(t) \]
\[ \times \sum_{i=r}^{n} nC_i \left\{ (n-i)C_i \{ p^iD_{l(\theta + \theta_1)}(t) + \ldots + q^iD_{l(\theta_2)}(t) \} - \right. \]
\[ \left. \left( -1 \right)^{n-i} \{ p^nD_{l(\theta + \theta_1)}(t) + \ldots + q^nD_{l(\theta_2)}(t) \} \right\} \]

where \( \bar{\omega}_{x_{m_1+y_{n_1}}(\theta)} = [\bar{\omega}_{\theta_1}(\theta)]^{m_1}[\bar{\omega}_{\theta_1}(\theta)]^{n_1} \) (11)

Using (11) in (10)

\[ P(T > t) = \sum_{i=r}^{n} nC_i \left\{ (n-i)C_i \{ p^iD_{l(\theta + \theta_1)}(t) + \ldots + q^iD_{l(\theta_2)}(t) \} - \right. \]
\[ \left. \left( -1 \right)^{n-i} \{ p^nD_{l(\theta + \theta_1)}(t) + \ldots + q^nD_{l(\theta_2)}(t) \} \right\} \]

where \( D_\theta(t) = \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) - W_{U}^{m_1+1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1}^{\infty} W_{V}^{n_1}(t) - W_{V}^{n_1+1}(t) \bar{\omega}_{\theta_1}(\theta) \)

Expanding and simplifying the equation (13)

\[ D_\theta(t) = 1 - \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]

\[ G_\theta(t) = 1 - D_\theta(t) \]
\[ = \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]
\[ - \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]
\[ - \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]
\[ \frac{d}{dt}(G_\theta(t)) \]
\[ = \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]
\[ - \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]
\[ - \left[ 1 - \bar{\omega}_{\theta_1}(\theta) \right] \sum_{m_1=0}^{\infty} W_{U}^{m_1}(t) \bar{\omega}_{\theta_1}(\theta) \sum_{n_1=0}^{\infty} W_{V}^{n_1}(t) \bar{\omega}_{\theta_1}(\theta) \]

Since \( \bar{\omega}_v(t) \) is exponential with parameter \( \mu_2, \bar{\omega}_v^{n_1}(t) \) is a gamma distribution with parameter \( \mu_2, n_1 \). Therefore,
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\[ 1 - w_{y_i}(\theta) \sum_{n_1=0}^{\infty} W_{y_1}^{n_1}(t) \tilde{w}_{y_i}(\theta)^{n_1-1} = \mu_2 [1 - \tilde{w}_{y_i}(\theta)] e^{-\mu_2 t [1 - \tilde{w}_{y_i}(\theta)]} \]

Therefore

\[ [1 - \tilde{w}_{y_i}(\theta)] \sum_{n_1=0}^{\infty} W_{y_1}^{n_1}(t) (\tilde{w}_{y_i}(\theta))^{n_1-1} = \mu_2 [1 - \tilde{w}_{y_i}(\theta)] e^{-\mu_2 t [1 - \tilde{w}_{y_i}(\theta)]} \] (17)

Substituting equation (16) and (17) in equation (15)

\[ g_\theta(t) = \mu_2 [1 - \tilde{w}_{y_i}(\theta)] e^{-\mu_2 t [1 - \tilde{w}_{y_i}(\theta)]} - \left\{ (1 - \tilde{w}_{x_i}(\theta)) \sum_{m_1=0}^{\infty} W_{y_1}^{m_1}(t) (\tilde{w}_{x_i}(\theta))^{m_1-1} \right\} \times \left\{ \mu_2 [1 - \tilde{w}_{y_i}(\theta)] e^{-\mu_2 t [1 - \tilde{w}_{y_i}(\theta)]} \right\} \]

\[ + \left\{ [1 - \tilde{w}_{x_i}(\theta)] \sum_{m_1=0}^{\infty} W_{y_1}^{m_1}(t) (\tilde{w}_{x_i}(\theta))^{m_1-1} \right\} [e^{-\mu_2 t [1 - \tilde{w}_{y_i}(\theta)]}] \] (18)

Taking Laplace Stelies transform on both side

\[ \tilde{g}_\theta(s) = \mu_2 [1 - \tilde{w}_{y_i}(\theta)] \frac{s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]}{s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]} \]

\[ - \left\{ \mu_2 [1 - \tilde{w}_{y_i}(\theta)] \sum_{m_1=0}^{\infty} \frac{[\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])]^{m_1}}{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]} (\tilde{w}_{x_i}(\theta))^{m_1-1} \right\} \]

\[ + \left\{ [1 - \tilde{w}_{x_i}(\theta)] \sum_{m_1=0}^{\infty} \frac{[\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])]^{m_1}}{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]} (\tilde{w}_{x_i}(\theta))^{m_1-1} \right\} \]

\[ \tilde{g}_\theta(s) = \mu_2 [1 - \tilde{w}_{y_i}(\theta)] [1 - \tilde{w}_{x_i}(\theta)] A(s) + [1 - \tilde{w}_{x_i}(\theta)] B(s) \] (19)

where

\[ A(s) = \left\{ \frac{[\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])]}{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]} \left[ 1 - \frac{[\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])]}{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]} (\tilde{w}_{x_i}(\theta)) \right] \right\} \]

\[ B(s) = \frac{1}{1 - [\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])] (\tilde{w}_{x_i}(\theta))} \]

From equation (20)

\[ \frac{d}{ds} (A(s)) = \left\{ \left[ \frac{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \left[ \frac{[s + \mu_2 [1 - \tilde{w}_{x_i}(\theta)]]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \right\} \]

\[ + \left\{ \left[ \frac{[s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \left[ \frac{[s + \mu_2 [1 - \tilde{w}_{x_i}(\theta)]]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \right\} \]

\[ \frac{d}{ds} (A(s)) \bigg|_{s=0} = \left\{ \left[ \frac{[\tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)])]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \left[ \frac{[\mu_2 [1 - \tilde{w}_{x_i}(\theta)]]}{[1 - \tilde{w}_{y_1}(s + \mu_2 [1 - \tilde{w}_{y_i}(\theta)]) (\tilde{w}_{x_i}(\theta))]} \right] \right\} \]

Again from equation (20)
Since \( x \)

We know that

\[
\int_0^1 \frac{1}{(1 - \bar{w}_x(\theta))(\bar{w}_x(\theta))} \, d\theta = \sum_{j=1}^\infty \frac{1}{j} \left( \frac{1}{1 - \bar{w}_x(\theta)} \right)^j \frac{1}{\bar{w}_x(\theta)} \tag{22}
\]

Similarly \( y \)

Therefore the probability density function of \( X \) and \( M \) are given by (Sheldon Ross 2005)

\[
w_x(t) = p\mu_h e^{-\mu_h t} + (1 - p)\mu_i e^{-\mu_i t}
\]

\[
\bar{w}_x(s) = \frac{p\mu_h}{s + \mu_h} + \frac{(1-p)\mu_i}{s + \mu_i}
\]

\[
\bar{w}_x'(s) = \frac{1}{(s + \mu_h)^2} + \frac{(1-p)\mu_i}{(s + \mu_i)^2}
\]

We know that \( E(T) = -\left[ \int_0^\infty \bar{w}_x(s) \right] \)

From (12)

\[
E(T) = -\sum_{i=1}^n n_i \left\{ p^i \left[ \int_0^\infty \bar{g}_i(\theta) \, d\theta \right] \right\} + \cdots + q^n \left[ \int_0^\infty \bar{g}_n(\theta) \, d\theta \right]
\]

Since \( x \) form an order statistics,

Let the probability function \( x(1) \) and \( x(m1) \) are given by (Sheldon Ross 2005)

\[
w_x(j)(x) = j^m \left( \frac{m!}{m!} \right) [w_x(x)]^{j-1} [1 - w_x(x)]^{m1-j}, j = 1, 2, 3 \ldots m1
\]

Therefore the probability density function of \( x(1) \) and \( x(m1) \) are given by

\[
w_x(1)(x) = m1 w_x(x)[1 - w_x(x)]^{m1-1}
\]

\[
w_x(m1)(x) = m1 w_x(x)[w_x(x)]^{m1-1}
\]

Similarly \( y \) form an order statistics

Let the probability function \( y(1) \) and \( y(n1) \) are given by (SheldonRoss 2005)

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\[ w_{y(j)}(y) = j \binom{n_1}{j} w_y(y) \left[ 1 - w_y(y) \right]^{n_1-1}, j = 1, 2, 3 \ldots n_1 \]  

(30)

Therefore the probability density function of \( y(1) \) and \( y(n_1) \) are given by

\[ w_{y(1)}(y) = n_1 w_y(y) \left[ 1 - w_y(y) \right]^{n_1-1} \]  

(31)

\[ w_{y(n_1)}(y) = n_1 w_y(y) \left[ w_y(y) \right]^{n_1-1} \]  

(32)

We shall now obtain the mean time to recruitment according as

Case (i)

Suppose \( w_{x_1}(x) = w_{x(1)}(x) \) and \( w_{y_1}(y) = w_{y(1)}(y) \)

Since \( w_x(x) = ce^{-cx} \)

\[ w_{x(1)}(x) = m_1 ce^{-cx} (e^{-cx})^{m_1-1} = m_1 ce^{-m_1cx} \]

\[ \bar{w}_{x_1}(\theta) = m_1 c \int_0^\infty e^{-m_1cx} e^{-\theta x} dx = m_1 c \int_0^\infty e^{-(m_1c+\theta)x} dx = m_1 c \left[ \frac{e^{-(m_1c+\theta)x}}{(m_1c+\theta)} \right]_0^\infty = \frac{m_1 c}{m_1c+\theta} \]  

(33)

Similarly \( \bar{w}_{y_1}(\theta) = \frac{n_1 c}{n_1 c+\theta} \)  

(34)

Substituting (33) and (34) in (25)

\[ -\left[ \frac{d}{ds} \left[ \bar{\theta}(s) \right] \right]_{s=0} = \frac{1 - \frac{p\mu_h}{\mu_2} \left[ 1 - \frac{n_1 c}{n_1 c+\theta} \right]}{\mu_2 \left[ 1 - \frac{n_1 c}{n_1 c+\theta} \right] + \mu_1} \left[ 1 - \frac{p\mu_h}{\mu_2} \left[ 1 - \frac{n_1 c}{n_1 c+\theta} \right] + \frac{(1-p)\mu_l}{\mu_2} \left[ 1 - \frac{n_1 c}{n_1 c+\theta} \right] + \mu_l \left[ 1 - \frac{n_1 c}{n_1 c+\theta} \right] + \mu_l \right] \]  

(35)

Using (35) in (26). We get the mean time to recruitment.

Case (ii)

Suppose \( w_{x_1}(x) = w_{x(1)}(x) \) and \( w_{y_1}(y) = w_{y(n_1)}(y) \)

Since \( w_y(y) = ce^{-cy} \)

Using equation (32)

\[ w_{y(n_1)}(y) = n_1 ce^{-cy}(1 - e^{-cy})^{n_1-1} \]

\[ \bar{w}_{y(n_1)}(\theta) = n_1 c \int_0^\infty e^{-cy}(1 - e^{-cy})^{n_1-1}e^{-\theta y} dy = n_1 \int_0^1 (1 - z)^{n_1-1} z^{\frac{\theta}{c}} dz = n_1 \beta \left( \frac{\theta}{c} + 1, n_1 \right) \]

(36)

\[ \bar{w}_{y(n_1)}(\theta) = \frac{n_1 c^{n_1}}{\delta(c, \theta)} \]  

(37)

where \( \delta(c, \theta) = (c + \theta)(2c + \theta)(3c + \theta) \ldots (n_1c + \theta) \)

Substituting (33) and (36) in (25)

\[ \bar{w}_{y(n_1)}(\theta) = \frac{n_1 c^{n_1}}{\delta(c, \theta)} \]  

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\[ - \left[ \frac{d}{ds} [\tilde{g}(s)] \right] \bigg|_{s=0} = \frac{1 - \frac{p\mu_h}{\mu_2} \frac{1 - \frac{n_{1c}}{n_{1c}+\theta}}{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + \frac{(1-p)\mu_i}{\mu_2} \frac{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + m_{1c}}{\delta(c,\theta)} \]

where \( \delta(c, \theta) \) is given by equation (37).

Using (38) in (26), we get the mean time to recruitment.

**Case (iii)**

**Suppose \( w_x(x) = w_{x(m1)}(x) \) and \( w_y(y) = w_{y(1)}(y) \)**

Using (36) for \( x \) we get

\[ \bar{w}_{X(m1)}(\theta) = \frac{m_{1c}m_{1c}}{\delta(c,\theta)} \]

where \( \delta(c, \theta) \) is given by (37).

Substituting (34) and (39) in (25),

\[ - \left[ \frac{d}{ds} [\tilde{g}(s)] \right] \bigg|_{s=0} = \frac{1 - \frac{p\mu_h}{\mu_2} \frac{1 - \frac{n_{1c}}{n_{1c}+\theta}}{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + \frac{(1-p)\mu_i}{\mu_2} \frac{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + m_{1c}}{\delta(c,\theta)} \]

where \( \delta(c, \theta) \) is given by equation (37).

Using (40) in (26), we get the mean time to recruitment.

**Case (iv)**

**Suppose \( w_x(x) = w_{x(m1)}(x) \) and \( w_y(y) = w_{y(1)}(y) \)**

Substituting (39) and (36) in (25),

\[ - \left[ \frac{d}{ds} [\tilde{g}(s)] \right] \bigg|_{s=0} = \frac{1 - \frac{p\mu_h}{\mu_2} \frac{1 - \frac{n_{1c}}{n_{1c}+\theta}}{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + \frac{(1-p)\mu_i}{\mu_2} \frac{1 - \frac{n_{1c}m_{1c}}{\delta(c,\theta)}} + m_{1c}}{\delta(c,\theta)} \]

where \( \delta(c, \theta) \) is given by equation (37).

Using (41) in (26), we get the mean time to recruitment.

3. Numerical illustration

The behavior of the performance measure due to the change in parameter is analyzed numerically for different values of \( n \) and \( r \).

**Case (i)**

**Sub Case (i) \( n=3, r=1 \)**

The mean time to recruitment is given by,

\[ E(T) = \left\{ \left\{ p^3 E_{11} + 3p^2 q^4 E_{12} + 3p^4 q^2 E_{13} + q^3 E_{14} \right\} - 3\left\{ p^2 E_{15} + 2pq E_{16} + q^2 E_{17} \right\} + 3\left\{ p E_{18} + q E_{19} \right\} \right\} \]

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Sub Case (ii)  \( n=3, r=2 \)

The mean time to recruitment is given by

\[
E(T) = \left\{ \left\{ -2\left[ p^3 E_{11} + 3p^2 q^1 E_{12} + 3p^3 q^2 E_{13} + q^3 E_{14} \right] \right\} + 3\left\{ \left[ p^2 E_{15} + 2pq E_{16} + q^2 E_{17} \right] \right\} \}
\]

Sub Case (iii)  \( n=3, r=3 \)

The mean time to recruitment is given by

\[
E(T) = \left\{ \left\{ p^3 E_{11} + 3p^2 q^1 E_{12} + 3p^3 q^2 E_{13} + q^3 E_{14} \right\} \right\}
\]

where

\[
E_{11} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{12} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{13} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{14} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{15} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{16} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{17} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]

\[
E_{18} = \frac{1}{\mu_1} - \frac{\mu_1}{\mu_c + (\delta + \theta_1)} + \frac{\mu_c}{\mu_{1c} + (\delta + \theta_1)} - \frac{(1-p)\mu_1}{\mu_{1c} + (\delta + \theta_1)}
\]
The mean time to recruitment is given by

\[ E_{19} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

**Case (ii)**

**Sub Case (i) n=3, r=1**

The mean time to recruitment is given by

\[ E(T) = \left\{ \left\{(p^3 E_{21} + 3p^2 q^1 E_{22} + 3p^1 q^2 E_{23} + q^3 E_{24}) \right\} - 3\left\{(p^2 E_{25} + 2pq E_{26} + q^2 E_{27}) \right\} 

\[ + 3\left\{(p^1 E_{28} + q^3 E_{29}) \right\} \right\} \]

**Sub Case (ii) n=3, r=2**

The mean time to recruitment is given by

\[ E(T) = \left\{ \left\{-2(p^3 E_{21} + 3p^2 q^1 E_{22} + 3p^1 q^2 E_{23} + q^3 E_{24}) \right\} + 3\left\{(p^2 E_{25} + 2pq E_{26} + q^2 E_{27}) \right\} \right\} \]

**Sub Case (iii) n=3, r=3**

The mean time to recruitment is given by

\[ E(T) = \left\{ \left\{(p^3 E_{21} + 3p^2 q^1 E_{22} + 3p^1 q^2 E_{23} + q^3 E_{24}) \right\} \right\} \]

where

\[ E_{21} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

\[ E_{22} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

\[ E_{23} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

\[ E_{24} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

\[ E_{25} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]

\[ E_{26} = \mu_2 \left\{ \frac{1 - n_{1c} \frac{1}{n_{1c} + (\theta_2)}}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \left\{ 1 - \frac{p \mu_h}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \right\} \]

\[ + \frac{1 - (1-p) \mu_1}{\mu_2 \frac{1}{n_{1c} + (\theta_2)}} \]
Mean Time to Recruitment for a Multigrade Manpower System with Two Sources …

\[ E_{27} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n + \frac{1}{c(\gamma + \theta, \beta_2)}}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\gamma_i \mu + \mu_h} \frac{1}{\frac{1}{c(\gamma_i + \theta, \beta_2)}} \right) \]

\[ E_{28} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n + \frac{1}{c(\gamma + \theta, \beta_2)}}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\gamma_i \mu + \mu_h} \frac{1}{\frac{1}{c(\gamma_i + \theta, \beta_2)}} \right) \]

\[ E_{29} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n + \frac{1}{c(\gamma + \theta, \beta_2)}}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\gamma_i \mu + \mu_h} \frac{1}{\frac{1}{c(\gamma_i + \theta, \beta_2)}} \right) \]

Case (iii)

Sub Case (i) \( n=3, r=1 \)

The mean time to recruitment is given by,

\[ E(T) = \left\{ (p^3 E_{31} + 3 p^2 q E_{32} + 3 p^2 q^2 E_{33} + q^3 E_{34}) \right\} - 3 \left\{ (p^2 E_{35} + 2 pq E_{36} + q^2 E_{37}) \right\} + 3 \left\{ (p E_{38} + q E_{39}) \right\} \]

Sub Case (ii) \( n=3, r=2 \)

The mean time to recruitment is given by

\[ E(T) = \left\{ -2 (p^3 E_{31} + 3 p^2 q E_{32} + 3 p^1 q^2 E_{33} + q^3 E_{34}) \right\} + 3 \left\{ (p^2 E_{35} + 2 pq E_{36} + q^2 E_{37}) \right\} \]

Sub Case (iii) \( n=3, r=3 \)

The mean time to recruitment is given by

\[ E(T) = \left\{ (p^3 E_{31} + 3 p^2 q E_{32} + 3 p^1 q^2 E_{33} + q^3 E_{34}) \right\} \]

where

\[ E_{31} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n \mu + \mu_e}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\mu_i} \frac{1}{1 - \frac{1}{c(\gamma + \theta, \beta, \mu)}} \right) \]

\[ E_{32} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n \mu + \mu_e}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\mu_i} \frac{1}{1 - \frac{1}{c(\gamma + \theta, \beta, \mu)}} \right) \]

\[ E_{33} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n \mu + \mu_e}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\mu_i} \frac{1}{1 - \frac{1}{c(\gamma + \theta, \beta, \mu)}} \right) \]

\[ E_{34} = \frac{1}{\mu_2} \left( 1 - \frac{\mu_h}{1 - \frac{1}{n \mu + \mu_e}} \right) \left( 1 - \sum_{i=1}^{3} \frac{1}{\mu_i} \frac{1}{1 - \frac{1}{c(\gamma + \theta, \beta, \mu)}} \right) \]
The mean time to recruitment is given by,

\[ E(T) = \left\{ \left\{p^3E_{41} + 3p^2q^1E_{42} + 3p^1q^2E_{43} + q^3E_{44}\right\} - 3\left\{p^2E_{45} + 2pqE_{46} + q^2E_{47}\right\} \right\} 
+ 3\left\{p^1E_{48} + q^4E_{49}\right\} \]

Sub Case (ii) \( n=3, r=2 \)

The mean time to recruitment is given by

\[ E(T) = \left\{ \left\{3p^3E_{41} + 3p^2q^1E_{42} + 3p^1q^2E_{43} + q^3E_{44}\right\} - 2\left\{p^2E_{45} + 2pqE_{46} + q^2E_{47}\right\} \right\} \]

Sub Case (iii) \( n=3, r=3 \)

The mean time to recruitment is given by

\[ E(T) = \{p^3E_{41} + 3p^2q^1E_{42} + 3p^1q^2E_{43} + q^3E_{44}\} \]

where

\[ E_{41} = \left\{ \left\{p^3E_{41} + 3p^2q^1E_{42} + 3p^1q^2E_{43} + q^3E_{44}\right\} - 3\left\{p^2E_{45} + 2pqE_{46} + q^2E_{47}\right\} \right\} 
+ 3\left\{p^1E_{48} + q^4E_{49}\right\} \]

\[ E_{42} = \left\{ \left\{3p^3E_{41} + 3p^2q^1E_{42} + 3p^1q^2E_{43} + q^3E_{44}\right\} - 2\left\{p^2E_{45} + 2pqE_{46} + q^2E_{47}\right\} \right\} \]
Mean Time to Recruitment for a Multigrade Manpower System with Two Sources …

\[
E_{43} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{44} = \frac{1 - \frac{p \mu h}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{p \mu h}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{45} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{46} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{47} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{48} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

\[
E_{49} = \frac{1 - \frac{\mu h_1}{\mu_2 - (1-p)\mu}}{\mu_2 - (1-p)\mu + \frac{\mu h_1}{\mu_2 - (1-p)\mu} + \frac{m_1 c_{n_1}}{\delta(c(\theta + \theta_1 + 2\theta_2))}}
\]

4. Comparison table

The influence of parameters on the performance measures namely the mean time for recruitment is studied numerically. In the following tables these performance measures are calculated by varying the parameter \(\lambda_1, \lambda_2, \mu_h, \mu_1, \mu_2, \theta_1\) and \(\theta_2\) one at a time and taking the parameters \(p = 0.3, 1 - p = q = 0.7, \theta = 0.2\)
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Mean Time to Recruitment for a Multigrade Manpower System with Two Sources …

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Findings:

- As ‘c’ the loss of manpower increases, (i.e) the mean loss of manpower decreases, and hence time taken for threshold crossing (i.e) the mean time to recruitment increases.
- As \( \mu_h', \mu_l' \) and \( \mu_z' \) the parameter for policy and transfer decision increases (i.e) mean policy and transfer decision time decreases, and hence the time taken for threshold crossing(i.e) the mean time to recruitment decreases.
- As \( \theta_1' \) and \( \theta_2' \) the the mean threshold level decreases and hence the time taken for threshold crossing i.e., mean time to recruitment decreases.

5. Conclusions

In the context of providing scope for future work, it is worthwhile to mention that the present work can be studied by considering different types of loss in manpower also.

REFERENCES

9. S.Dhivy, V.Vasudevan and A.Srinivasn, Stochastic models for the time to recruitment in a two grademanpower system using same geometric process for theinter decision times, proceedings of mathematical andcomputational models, PSG college of technology (ICMCM), Narosa publishing House, pp.276-283, Dec -2011
Mean Time to Recruitment for a Multigrade Manpower System with Two Sources …

