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Analysis of Priority Queuing Models: L - R Method

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Abstract. Priority queuing models have a wide range of application in computer network system. In this paper the performance measures of fuzzy priority queuing model are computed using L- R method. L -R method is convenient and flexible compared to other methods. Numerical illustration is given to check the validity of the proposed method.

Keywords: Fuzzy queue, priority discipline performance measure, L-R method and triangular fuzzy numbers.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

Queuing models play an indispensable role for capacity planning in practical situations. In today's networking environment, queuing models are basically relevant to service organizations and suggest ways and means to improve the efficiency of the service. A queue provides service at different alternatives on the basis of first-in-first out, last come first served, random selection and priority selection.

An efficient priority scheme is of great importance in the design and construction of telecommunication networks. We discuss and analyze the priority discipline fuzzy queuing model in two cases: no priority discipline, preemption priority by L- R Method. In a preemptive priority queue, an element with higher priority is served before an element with lower priority is already present in the service when the higher element arrives to the system. The order of the entries in the queue may change according to the priority scheme.

In fuzzy logic literature, fuzzy queues are extensively studied by researchers like Li and Lee [1], Buckley [16], Negi and Lee [2], Kao et al. [8], Chen [8] have analyzed fuzzy queues using Zadeh's extension principle, Ritha and Robert [15] analyzed fuzzy queues using DSW Algorithm, Ritha and Menon [11]. In this paper, L-R fuzzy number, arithmetic of L-R fuzzy numbers and triangular fuzzy number concepts are utilized for analysis of priority queuing models.

2. Preliminaries

L-R fuzzy number: A fuzzy number $\widetilde{M}$ is said L-R fuzzy number if only if there exists three real numbers m, a>0, b>a and two positive, continuous and decreasing functions L and R, from R to [0,1], such that

L(0) = R(0) = 1

L(1) = 0, L(x) > 0

R(1) = 0, R(x) > 0,

$$\eta_{\widetilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{a}\right) & if\ x \epsilon\ [m-a, m] \\ R\left(\frac{x-m}{b}\right) & if\ x \epsilon\ [m, m+b] \\ 0 & otherwise \end{cases}$$

The L-R representation of the fuzzy number $\widetilde{M}$ is $\widetilde{M}$=⟨m,ab⟩$_{LR}$ where m is called the mean/ mode/ modal value of $\widetilde{M}$, a and b are the left spread and right spread of $\widetilde{M}$.

Arithmetic of L-R fuzzy numbers:

Suppose there are two L-R fuzzy numbers of the same type $\widetilde{M}$ = ⟨ m, a, b ⟩$_{LR}$ and $\widetilde{N}$ = ⟨n,c,d⟩$_{LR}$ then

1. $\widetilde{M} + \widetilde{N}$ = <m+n , a+c, b+d>$_{LR}$
2. $\widetilde{M} - \widetilde{N}$ = < m-n , a+d , b+c>$_{LR}$
3. $\widetilde{M} . \widetilde{N}$ = <mn, mc+na-ac , md+nb+bd>$_{LR}$
4. $\frac{\widetilde{M}}{\widetilde{N}} = \frac{\langle m,a,b \rangle_{LR}}{\langle n,c,d \rangle_{LR}} = \left\langle \frac{m}{n}, \frac{md}{n(n+d)} + \frac{a}{n} - \frac{ad}{n(n+d)}, \frac{mc}{n(n-c)} + \frac{b}{n} + \frac{bc}{n(n-c)} \right\rangle_{LR}$

Triangular fuzzy numbers:

A fuzzy number $\tilde{A}$ is said to be triangular fuzzy number if and only if there exists three real numbers a<b<c such that:

$$\eta_{\widetilde{M}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) & if\ a \leq x \leq b \\ \left(\frac{c-x}{c-b}\right) & if\ b < x \leq c \\ 0 & otherwise \end{cases}$$

Denoted by $\tilde{A}$ = (a/b/c) or $\tilde{A}$ = (a, b, c)

Remark: A triangular fuzzy number $\tilde{A}$ = (a/ b/c) is always an L-R fuzzy number.

In L-R representation,

$\tilde{A}$ = (a/ b/c) = < b, b-a, c-b >$_{LR}$ for L(x) = R (x) = max (0, 1-x)

The following notations are used:

$\tilde{\lambda}$: Fuzzy rate of arrival

$\tilde{\mu}$: Fuzzy rate of service

$\tilde{C}$: Average total cost of inactivity when there is no priority discipline

$\widetilde{C}'$: Average total cost of inactivity when there is preemption priority

Fuzzy priority queues:

Fuzzy priority queues are based on the concept of fuzzy set theory. Consider a priority queuing system with single server, infinite calling population in which arrival rate is $\tilde{\lambda}$ and service rate is $\tilde{\mu}$

We establish the priority discipline fuzzy queuing model using L-R technique for two cases:

(i) No priority discipline
(ii) Preemption priority discipline

(a) No priority queuing model:

Average total cost of inactivity when there is no priority discipline

$\tilde{C} = (\tilde{C}_1 \tilde{\lambda}_1 + \tilde{C}_2 \tilde{\lambda}_2)\widetilde{W}$ with $\widetilde{W} = \frac{1}{\tilde{\mu} - \tilde{\lambda}}$

(b) Preemption priority queuing model:

Average total cost of inactivity when there is preemption priority

$\tilde{C}' = \tilde{C}_1 \tilde{\lambda}_1 \widetilde{W}_1 + \tilde{C}_2 \tilde{\lambda}_2 \widetilde{W}_2$ $W_{q,i} = \frac{\lambda}{\mu^2} \frac{1}{(1-\sigma_i)(1-\sigma_{i+1})}$

where $\sigma_1 = \frac{\lambda 1}{\mu}$, where $\sigma_2 = \frac{\lambda 2}{\mu}$ and $\sigma_3 = 0$

L-R method description:

Let the arrival rate and service rate be triangular fuzzy numbers such that

$\tilde{\lambda} = (\lambda_1/\lambda_2/\lambda_3)$ and $\tilde{\mu} = (\mu_1/\mu_2/\mu_3)$

$\widetilde{C}_1 = (C_{11}/C_{12}/C_{13} = < C_{12}, C_{12} - C_{11}, C_{13} - C_{12} >_{LR}$

$\tilde{C}_2 = (C_{21}/C_{22}/C_{23} = < C_{22}, C_{22} - C_{21}, C_{23} - C_{22} >_{LR}$

$\widetilde{\lambda}_1 = (\lambda_{11}/\lambda_{12}/\lambda_{13}) = <\lambda_{12}, \lambda_{12} - \lambda_{11}, \lambda_{13} - \lambda_{12} >_{LR}$

$\tilde{\lambda}_2 = (\lambda_{21}/\lambda_{22}/\lambda_{23}) = <\lambda_{22}, \lambda_{22} - \lambda_{21}, \lambda_{23} - \lambda_{22} >_{LR}$

$\widetilde{\lambda} = (\lambda_1/\lambda_2/\lambda_3) = <\lambda_2, \lambda_2 - \lambda_1, \lambda_3 - \lambda_2 >_{LR}$

$\tilde{\mu} = (\mu_1/\mu_2/\mu_3) = <\mu_2, \mu_2 - \mu_1, \mu_3 - \mu_2 >_{LR}$

The cost of inactivity when there is no priority discipline is given as:

$$\tilde{c} = \frac{\widetilde{C_1}\ \widetilde{\lambda_1} + \widetilde{C_2}\ \widetilde{\lambda_2}}{\tilde{\mu} - \tilde{\lambda}}$$

$$\frac{<C_{12}, C_{12}-C_{11}, C_{13}-C_{12}>_{LR} <\lambda_{12}, \lambda_{12}-\lambda_{11}, \lambda_{13}-\lambda_{12}>_{LR} + <C_{22}, C_{22}-C_{21}, C_{23}-C_{22}>_{LR} <\lambda_{22}, \lambda_{22}-\lambda_{21}, \lambda_{32}-\lambda_{22}>_{LR}}{<\mu_2, \mu_2-\mu_1, \mu_3-\mu_2>_{LR} - <\lambda_2, \lambda_2-\lambda_1, \lambda_3-\lambda_2>_{LR}}$$

$$\frac{C_{12}\lambda_{12}+C_{22}\lambda_{22}}{\mu_2-\lambda_2}, \left\{\frac{[C_{12}\lambda_{12}+C_{22}\lambda_{22}][\mu_3-\mu_2+\lambda_2-\lambda_1]}{(\mu_2-\lambda_2)(\mu_3-\lambda_1)} + \frac{C_{12}\lambda_{12}-C_{11}\lambda_{11}+C_{22}\lambda_{22}-C_{21}\lambda_{21}}{\mu_2-\lambda_2} - \frac{C_{12}\lambda_{12}-C_{11}\lambda_{11}+C_{22}\lambda_{22}-C_{21}\lambda_{21}}{(\mu_2-\lambda_2)(\mu_3-\lambda_1)}\right\},$$

$$\left\{\frac{[C_{12}\lambda_{12}+C_{22}\lambda_{22}][\mu_2-\mu_1+\lambda_3-\lambda_2]}{(\mu_2-\lambda_2)(\mu_1-\lambda_3)} + \frac{C_{13}\lambda_{13}-C_{12}\lambda_{12}+C_{23}\lambda_{23}-C_{22}\lambda_{22}}{\mu_2-\lambda_2} + \frac{[C_{13}\lambda_{13}-C_{12}\lambda_{12}+C_{23}\lambda_{23}-C_{22}\lambda_{22}][\mu_2-\mu_1+\lambda_3-\lambda_2]}{(\mu_2-\lambda_2)(\mu_1-\lambda_3)}\right\}$$

The lower and upper bound of $\widetilde{N}$ are $N_1 - N_2$ and $N_1 + N_3$. Noting m = N_1, N = $N_1 - N_2$,

V = $N_1 + N_3$.

The cost of inactivity for Preemptive priority queuing model is given as:

$$\tilde{C}' = \tilde{C}_1 \tilde{\lambda}_1 \widetilde{W_1} + \widetilde{C}_2 \tilde{\lambda}_2 \widetilde{W}_2$$

where $\widetilde{W_1}$ = $(W_{11}/W_{12}/W_{13}$ = $<W_{12}, W_{12}-W_{11}, W_{13}-W_{12}>_{LR}$ and
$\widetilde{W}_2 = (W_{21}/W_{22}/W_{23}) = <W_{22}, W_{22}-W_{21}, W_{23}-W_{22}>_{LR}$

$\tilde{C} = <C_{12}\lambda_{12}, C_{12}\lambda_{12} - C_{11}\lambda_{11}, C_{13}\lambda_{13} - C_{12}\lambda_{12}>_{LR} < W_{12}, W_{12}-W_{11}, W_{13}-W_{12}>_{LR}$ +
$< C_{22}\lambda_{22}, C_{22}\lambda_{22} - C_{21}\lambda_{21}, C_{23}\lambda_{23} - C_{22}\lambda_{22}>_{LR} < W_{22}, W_{22}-W_{21}, W_{23}-W_{22}>_{LR}$

$\tilde{C} = < C_{12}\lambda_{12}W_{12} + C_{22}\lambda_{22}W_{22}, C_{12}\lambda_{12}W_{12} - C_{11}\lambda_{11}W_{11} + C_{22}\lambda_{22}W_{22} - C_{21}\lambda_{21}W_{21},$
$C_{13}\lambda_{13}W_{13} - C_{12}\lambda_{12}W_{12} + C_{23}\lambda_{23}W_{23} - C_{22}\lambda_{22}W_{22}>_{LR}$

Let us illustrate the above model with an example:

No priority discipline fuzzy queuing model are given with their fuzzy arrival rates and service rates with their costs

$\tilde{\lambda}$ = (44 /45 /46) = $< 45, 1, 1>_{LR}$ and $\tilde{\mu}$ = (47/48/49) = $< 48,1,1>_{LR}$
$\widetilde{\lambda_1}$ = (34/35/36) = $<35,1, 1>_{LR}$ and $\widetilde{\lambda_2}$ = (32/33/34) = $<33,1,1>_{LR}$
$\widetilde{C_1}$= (41/42/43) = $<42,1,1>_{LR}$ and $\widetilde{C_2}$ = (37/38/39) = $<38, 1, 1>_{LR}$

The cost of inactivity is given as

$\widetilde{C}$ = < 908, 392, 1966 $>_{LR}$

On simplification,

Modal value = 908, Left spread= 392, Right spread = 1966

From the above calculation, we consider the values for $\tilde{\lambda}, \tilde{\mu}, \widetilde{\lambda_1}, \widetilde{\lambda_2}, \widetilde{C_1}, \widetilde{C_2}$ with

$\widetilde{W_1}$= (44/45/46) =<45,1,1$>_{LR}$ and $\widetilde{W_2}$ = (47/48/49) = <48, 1, 1$>_{LR}$

The cost of inactivity for non-preemptive priority fuzzy queue model is obtained as

$\widetilde{C'}$ = <126342,9358, 9840 $>_{LR}$

6. Conclusion

In this paper, we have analyzed fuzzy priority queues by L-R Method, based on L-R fuzzy numbers, L-R fuzzy arithmetic using the concept of triangular fuzzy numbers. Fuzzy concept gives more appropriate result when compared with crisp results. L-R Method is short, flexible, convenient and approximate method for best explanatory results. L-R Method helps in further research work for evaluation of fuzzy queuing operating system.

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