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A Study on Intuitionistic L-Fuzzy Metric Spaces

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Abstract. Intuitionistic fuzzy set can be utilized as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. Atanassov introduced and studied the concept of Intuitionistic fuzzy sets.The intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the classical and fuzzy models. In this paper, a new concept of Intuitionistic L-fuzzy metric space is introduced and some theorems about Inuitionistic L-fuzzy metric space are discussed.

Keywords: Intuitionistic fuzzy set, intuitionistic l-fuzzy metric space, operations on intuitionisitc L-metric spaces.

AMS Mathematics Subject Classification (2010): 05C72, 54E50, 03F55

1. Introduction

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets are introduced by Zadeh [7] as an extension of the classical notion of sets. The concept of an intuitionistic fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In general, the theory of intuitionistic fuzzy sets is the generalization of fuzzy sets.

In the year 1989 Atanassov[1] introduced Interval valued intuitionistic fuzzy sets and many researchers have shown interest in the IFS theory and successfully applied in many other fields.

This paper is organized as follows. The definition of fuzzy metric space and intuitionistic fuzzy metric space and are introduced in section 2. In section 3, we extend the fuzzy metric space to L-fuzzy metric space, In that, the range of the function will be a lattice. Also the definitions of intuitionistic L- Fuzzy metric space are introduced. In section 4, we dicuss about the theorems on intuitionistic L- Fuzzy metric space in weakly compaitable.

2. Preliminaries

Definition 2.1. Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(u, \mu_A(u)) | u \in X\}$.

Example 2.1. **T**he membership function of the fuzzy set of real numbers ”close to 1”, is can be defined as A(t) = exp(−β(t − 1)2)where β is a positive real number.

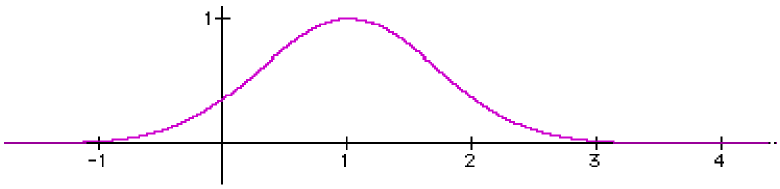

Figure 1: A membership function for ”x is close to 1”.

Definition 2.2. An *L*-fuzzy set ϕ on U is a mapping ϕ:U→ L, where *L* is a ‘transitive partially ordered set’. In this work, we assume that $(L,\le)$ is a preordered set. Notice that it is natural to assume that the relation ≤ is not antisymmetric; if $x, y \in L$ are synonyms, that is, words or expressions that are used with the same meaning, then $x \le y$ and $x \ge y$, but still *x* and *y* are distinct words.

Example 2.2. Suppose that *U* consists of a group of people. The *L*-fuzzy set, whose membership function ϕ , describes how well the persons in *U* can ski. For instance, there exist people who can ski very well, some ski badly, and some are moderate skiers.

Definition 2.3. Let a set E be fixed. An IFS A in E is an object of the following . $A = \{ (x,\mu_A(x) , \upsilon_A(x)), x\in E \}$, Where the functions $\mu_A (x) : E \rightarrow [0, 1]$ and $\upsilon_A (x) : E \rightarrow [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x\in E$, respectively, and for every $x\in E$: $0 \le \mu_A(x)+ \upsilon_A (x) \le 1$ When $\upsilon_A (x) = 1 - \mu_A (x)$ *for all* $x\in E$ is ordinary fuzzy set. In addition, for each IFS A in E, if $\pi_A(x)= 1- \mu_A (x) - \upsilon_A (x)$ Then $\pi_A(x)$ is called the degree of indeterminacy of X to A or called the degree of hesitancy of x to A. It is obvious that $0 \le \pi_A (x) \le 1$, for each $x\in E$.

Example 2.3. An IF-set is a pair of mappings $\mu:X \rightarrow [0,1]$, $\upsilon: X \rightarrow [0,1]$ such that $\mu (x) + \upsilon (x) \le 1$ for any $x \in X$. In our case X is the set of all pupils in the considered class. If A(x) is the number of acceptation of the pupil x (hence $A(x) \in \{0,1,\ldots n\}$ where n is the number of pupils in the class), then we put $\mu(x) = A(x)/ n$

Similarly $\mu(x) = N(x) /n$,where N(x) is the numbers of non-acceptation of the pupil x. Since $A(x) + N(x) \le n$. We obtain $\mu(x) + \upsilon(x) = A(x)/n+N(x)/n \le 1$, hence the pair (μ,υ) is an example of an IF-set.

Definition 2.4. A binary operation *: [0,1] x [0,1]→[0,1] is called at- norm if ([0,1],*) is anabelian topological monoid with unit 1 such that $a*b \le c*d$ whenever $a\le c$ and $b \le d$ for a, b, c, d ϵ [0,1].

Example 2.4. t- norm are a*b = ab and a*b = min{a, b}.

Definition 2.5. A binary operation * : [0,1] x [0,1] → [0,1] is a ***continuous t-norm*** if it satisfies the following conditions

a) * is a associative and commutative
b) a*1 = a for all a ϵ [0,1]
c) * is continuous
d) a*b ≤ c*d whenever a ≤ c and b ≤ d, for each a, b, c, d ϵ [0,1].

Example 2.5. a*b = ab and a*b =(a,b)

Definition 2.6. The 3-tuple(X,M,*) is said to be a ***fuzzy metric space*** if X be a non empty set and * be a continuous t-norm. A fuzzy set X^2 x (0,∞) is called a fuzzy metric on X if x, y, z ∈X and s, t >0 , the following condition holds

a) M (x, y, t) =0
b) M (x, y, t) = 1 iff x=y
c) M (x, y, t) = M(y, x, t);
d) M (x, y, t + s) ≥ M(x, y, t) * M(x, y, s)
e) M (x, y, .) : (0, +∞) → [0, 1] is ***left continuous***

The function M(x,y,t) denote the degree of nearness between x and y with respect to t respectively.

Definition 2.7. A 5-tuple(X, M, N, *, ◊) is said to be an ***intuitionistic fuzzy metric space*** if *X* is an arbitrary set, * is a continuous *t*-norm, ◊ is a continuous*t*- conorm and, M, Nare fuzzy sets on X^2× [0, ∞) satisfying the conditions:

1) M(x, y, t) + N(x, y, t) ≤ 1 for all, *x*,y *ϵX* and ; t > 0
2) M(x, y, 0) =*0* for all, x,y *ϵX*,
3) M(x, y, t) = 1 for all, x,y *ϵX*, and t > 0 if and only if x = y;
4) M(x, y, t) = M(y, x, t) for all, x,y *ϵX* and *t*> 0;
5) M(x, y, t)*M(y, z, s) ≤ M(x, z, t+s), for all, x, y, z ϵ X and s,t >0;
6) M(x, y, .):[0,∞)→[0,∞]is left continuous, for all, x,y ϵ X;
7) $\lim_{n\to\infty} \mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t}) = 1$ for all, x,y ϵ X and t > 0;
8) N(x, y, 0) = 1for all, x,y ϵ X ;
9) N(x, y, t) = 0, for all x,y ϵ X and t > 0 if and only if x=y;
10) N(x, y, t) = N(y, x, t) for all, x,y ϵ X, and *t*> 0;
11) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t+s) for all x, y, z ϵX and s,t >0;
12) N(x, y, .):[0,∞)→[0,1]is right continuous, for all x,y ϵ X;
13) $\lim_{t\to\infty} \mathrm{N}(\mathrm{x},\mathrm{y},\mathrm{t}) = 0$ for all x,y ϵ X.

The functions M(x, y, t)and N(x, y, t) denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. *t* respectively.

Definition 2.8. Let X be a non-empty set and f:X → X be a mappings . A point x ϵ X is called a ***fixed point*** of f if x remains invariant under f i.e. fx = x. In graphical terms, a fixed point means the point (x, fx) is on the line y = x, or in other words, the graph of f has a point in common with line y = x.

Example 2.6. A mappings f : R→R defined by f(x)= 3x, for all x ϵ R, x = 0 is the unique fixed point.

3. Intuitionistic L-fuzzy metric space

Definition 3.1. A 5-tuple(X, M, N, *, ◊) is said to be an ***intuitionistic L-fuzzy metric space*** if *X* is an arbitrary set, * is a continuous *t*-norm, ◊ is a continuous *t*- conorm and, M, N are L-fuzzy sets on $X^2 \times [0, \infty)$ satisfying the conditions:

1. M(x, y, t) + N(x, y, t) ≤ 1 for all, *x*,y ϵ*X* and ; t > 0
2. M(x, y, 0) = *0* for all, x,y ϵ*X*,
3. M(x, y, t) = 1 for all, x,y ϵ*X*, and t > 0 if and only if x = y;
4. M(x, y, t) = M(y, x, t) for all, x,y ϵ*X* and *t*> 0;
5. M(x, y, t)*M(y, z, s) ≤ M(x, z, t+s), for all, x, y, z ϵ X and s,t >0;
6. M(x, y, .):[0,∞)→[0,∞]is left continuous, for all, x,y ϵ X;
7. $\lim_{n\to\infty} \mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t}) = 1$ for all, x,y ϵ X and t > 0;
8. N(x, y, 0) = 1for all, x,y ϵ X ;
9. N(x, y, t) = 0, for all x,y ϵ X and t > 0 if and only if x=y;
10. N(x, y, t) = N(y, x, t) for all, x,y ϵ X, and *t*> 0;
11. N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t+s) for all x, y, z ϵX and s,t >0;
12. N(x, y, .):[0,∞)→[0,1]is right continuous, for all x,y ϵ X;
13. $\lim_{t\to\infty} \mathrm{N}(\mathrm{x},\mathrm{y},\mathrm{t}) = 0$ for all x,y ϵ X.

The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. *t* respectively.

Definition 3.2. Let (X, M, N, *, ◊) be an intuitionistic L-fuzzy metric space. Then 1) A sequence $\{x_n\}$ in X is said to be ***Cauchy sequence*** if, for all and t>0 and *p* > 0,
$\lim_{n\to\infty} \mathrm{M}(\mathrm{x}_{n+p},\mathrm{x}_n,\mathrm{t}) = 1$ and$\lim_{n\to\infty} \mathrm{N}(\mathrm{x}_{n+p},\mathrm{x}_n,\mathrm{t}) = 1$
2) A sequence $\{x_n\}$ in *X* is said to be ***convergent*** to a point x ϵ X, if for all t > 0.
$\lim_{n\to\infty} \mathrm{M}(\mathrm{x}_n,\mathrm{x},\mathrm{t}) = 1$ and $\lim_{n\to\infty} \mathrm{N}(\mathrm{x}_n,\mathrm{x},\mathrm{t}) = 0$

Definition 3.3. An intuitionistic L-fuzzy metric space (X, M, N, *, ◊) is said to be ***complete*** if and only if every Cauchy sequence in *X* is convergent.

Example 3.1. Let $X = \left\{\frac{1}{n}\middle| n \in N\right\} \cup \{0\}$ and let * be the continuous t-norm and ◊ be the continuous *t*- conorm defined by a*b = ab, and a◊b = min {1,a+b} respectively, for all a,b ϵ[0,1]. For each t ϵ (0,∞) and x,y ϵ X, define *M* and *N* by

$$M(x,y,t) = \left\{\begin{matrix} \frac{t}{t+|x-y|}, t > 0 \\ 0, t = 0 \end{matrix}\right\} \text{ and } N(x,y,t) = \left\{\begin{matrix} \frac{|x-y|}{t+|x-y|}, t > 0 \\ 1, t = 0 \end{matrix}\right\}$$

Clearly (X, M, N, *, ◊) is complete intuitionistic L- fuzzy metric space.

Definition 3.4. A pair of self mappings (A,S)of a intuitionistic L-fuzzy metric space (X, M, N, *, ◊)is said to be ***commuting*** if and M(ASx, SAx, t) =1 and N(ASx,SAx, t) = 0 for all x ϵX.

Definition 3.5. A pair of self mappings (A,S) of a intuitionistic L-fuzzy metric Space (X, M, N, *, ◊) is said to be ***weakly commuting*** if M(ASx, SAx, t) ≥ M(ASx, Sx, t) and N(ASx, SAx, t) ≤ N(Ax, Sx, t) for all x ϵ X and t > 0.

Definition 3.6. A pair of self mappings (A,S),of a intuitionistic L-fuzzy metric space (X, M, N, *, ◊) is said to be ***point wise R-weakly commuting***, if given x ϵ X, there exist such that for all t>0. M(ASx, SAx, t) ≥ M(Ax, Sx, 1/R), and N(ASx, SAx, t) ≥ N(Ax, Sx, 1/R) clearly, every pair of weakly commuting mappings is point wise *R*-weakly commuting with R=1.

Definition 3.7. Two mappings A and S of a Intuitionistic L-fuzzy metric space (X, M, N, *, ◊) are called *reciprocally continuous* if $ASu_n \to Az$, $SAu_n \to Sz$, whenever $\{x_n\}$ is a sequence such that $Au_n \to z$, $Su_n \to z$, for some z in *X*. If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true.

Theorem 3.8. Let (X, M, N, *, ◊) be a complete intuitionistic L-fuzzy metric space with continuous *t*-norm * and continuous *t*-conorm ◊ defined by t*t ≥ t and (1-t) ◊ (1-t) ≤ (1-t) for all tϵ[0,1].Further, let (A,S)and (B,T) be point wise *R*-weakly commuting pairs of self mappings of X satisfying:
A(X) ≤ T(X), B(X) ≤ S(X),….. (1)
there exists a constant k ϵ (0,1) such that ……(2)
M(Ax, By, kt) ≥ M(Ty, By, t)*M(Sx, Ax, t)*M(Sx, By, αt)*M(Ty, Ax,(2-α)t)*M(Ty, Sx, t)
N(Ax, By, kt) ≤ N(Ty, By, t) ◊ N(Sx, Ax, t)◊ N(Sx, By, αt)◊N(Ty, Ax,(2-α)t)◊N(Ty, Sx, t)
for all x,y ϵ X, t>0 and α(0,2) . Then the continuity of one of the mappings in compatible pair (A,S) or (B,T) on (X, M, N, *, ◊) implies their reciprocal continuity.
Proof: First, assume that *A* and *S* are compatible and *S* is continuous. We show that *A* and *S* are reciprocally continuous. Let $\{x_n\}$ be a sequence such that $Au_n \to z$ and $Su_n \to z$ for some z ϵX as n→∞.Since *S* is continuous, we have $SAu_n \to Sz$ and $SSu_n \to Sz$ as n→∞ and $SSu_n \to Sz$ as n→∞ and since (A,S) is compatible, we have
$\lim_{n\to\infty} M(ASU_n, SAU_n, t) = 1, \lim_{n\to\infty} N(ASU_n, SAU_n, t) = 0$
$\Rightarrow \lim_{n\to\infty} M(ASU_n, Sz, t) = 1, \lim_{n\to\infty} M(ASU_n, Sz, t) = 0$
That is $ASu_n \to Sz$ as n→∞. By (1), for each n, there exists for all n ϵ X such that $ASu_n = Tv_n$. Thus, we have, $SSu_n \to Sz$, $SAu_n \to Sz$, and $Tv_n \to Sz$ as n→∞ whenever $ASu_n = Tv_n$.Now we claim that $Bv_n \to Sz$ as n→∞.
Suppose not, then taking α=1in (2), we have
M(*ASun* , *Bvn* , *kt*)≥*M* (*Tvn* , *Bvn* ,*t*) * *M* (*SSun* , *ASun* ,*t*) * *M* (*SSun* , *Bvn* , α*t*) * *M* (*Tvn* , *ASun* , (2-α)*t*) **M* (*Tvn* , *SSun* ,*t*) *N* (*ASun* , *Bvn* , *kt*) ≤ *N* (*Tvn* , *Bvn* , *t*)◊N(SSun,ASun,t)◊N(SSun,Bvn, αt) ◊N(Tvn, ASun, (2-α)t) ◊ *N* (*Tvn* , *SSun* , *t*)
Taking n→∞, we get
$M(Sz, Bv_n, kt) \geq M(Sz, Bv_n, t) * M(Sz, Sz, t) * M(Sz, Bv_n, t) * M(Sz, Sz, t) * M(Sz, Sz, t)$
$N(SZ, Bv_n, kt) \leq N(Sz, Bv_n, t) \lozenge N(Sz, Sz, t) \lozenge N(Sz, Bvn, t) \lozenge N(Sz, Sz, t) \lozenge N(Sz, Sz, t)$
That is,
$M(Sz, Bv_n, kt) \geq M(Sz, Bv_n, t)$,
$N(Sz, Bv_n, kt) \leq N(Sz, Bv_n, t)$.
we have $Bv_n \to Sz$ as n→∞
Now, we claim that .*Az*=*S*z. Again take α=1in (2), we have
$M(Az, Bv_n, kt) \geq M(Tv_n, Bv_n, t) * M(Sz, Az, t) * M(Sz, Bv_n, t) * M(Tv_n, Az, t) * M(Tv_n, Sz, t)$

$N(Az, Bv_n, kt) \leq N(Tv_n, Bv_n, t) \lozenge N(Sz, Az, t) \lozenge N(Sz, Bv_n, t) \lozenge N(Tv_n, Az, t) * N(Tv_n, Sz, t)$

As $n \to \infty$

$M(Az, Sz, kt) \geq M(Sz, Sz, t) * M(Sz, Az, t) * M(Sz, Sz, t) * M(Sz, Az, t) * M(Sz, Sz, t)$

$N(Az, Sz, kt) \leq N(Sz, Sz, t) \lozenge N(Sz, Az, t) \lozenge N(Sz, Sz, t) \lozenge N(Sz, Az, t) \lozenge N(Sz, Sz, t)$

i.e. $M(Az, Sz, kt) \geq M(Sz, Az, t)$

$N(Az, Sz, kt) \leq M(Sz, Az, t)$ Therefore, we have. $Az = Sz$

Hence, $SAu_n \to Sz$, $ASu_n \to Sz = Az$ as $n \to \infty$

This proves that *A* and *S* are reciprocally continuous on *X*. Similarly, it can be proved that *B* and *T* are reciprocally continuous if the pair (B,T)is assumed to be compatible and *T* is continuous.

4. Intuitionistic L-fuzzy metric space in weakly compatible

Theorem 4.1. Let (X, M, N, *, ◊) be a complete intuitionistic L-fuzzy metric space with continuous *t*-norm * and continuous *t*-conorm ◊ defined by $t*t \geq t$ and $(1-t)\lozenge(1-t) \leq (1-t)$ for all $t \in [0,1]$. Further, let (A, S) and (B, T) be point wise *R*-weakly commuting pairs of self mappings of *X* satisfying (1), (2). If one of the mappings in compatible pair (*A*,*S)* or (*B*,*T)* is continuous, then *A*, *B*, *S* and *T* have a unique common fixed point.

Proof: Let $x_0 \in X$. By (1), we define the sequences$\{x_n\}$ and$\{y_n\}$ in *X* such that for all, *n* = 0,1,2...

$Y_{2n}=Ax_{2n}=Tx_{2n+1}, y_{2n+1}= Bx_{2n}=1Sx_{2n+1}$, We show that$\{y_n\}$ is a Cauchy sequence in *X*. By (2) take $\alpha=1-\beta$, $\beta \in (0,1)$,we have

$M(y_{2n+1}, y_{2n+2}, kt) = M(Bx_{2n+1}, Ax_{2n+2}, kt) = M(Ax_{2n+2}, Bx_{2n+1}, kt) \geq M(Tx_{2n+1}, Bx_{2n+1}, t) * M(Sx_{2n+2}, Ax_{2n+2}, t)$

$M(Sx_{2n+2}, Bx_{2n+1}, (1-\beta)t) * M(Tx_{2n+1}, Ax_{2n+2}, (1-\beta)t) * M(Tx_{2n+1}, Sx_{2n+2}, t)$

$= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+1}, y_{2n+2}, (1-\beta)t) *$

$M(y_{2n}, y_{2n+2}, (1-\beta)t) *1* M(y_{2n}, y_{2n+1}, t)$

$= M(y_{2n+1}, y_{2n+2}, \beta t) * M(y_{2n}, y_{2n+1}, t)$

$\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+1}, y_{2n+2}, (1-\beta)t)$

Now, taking $\beta \to 1$, we have

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+1}, y_{2n+2}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) \geq M(y_{2n}, y_{2n+1}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$

Similarly, we can show that

$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$

Also,

$N(y_{2n+1}, y_{2n+2}, kt) = N(Bx_{2n+1}, Ax_{2n+2}, kt) = N(Ax_{2n+2}, Bx_{2n+1}, kt) \leq N(Tx_{2n+1}, Bx_{2n+1}, t) \lozenge$

$N(Sx_{2n+2}, Ax_{2n+2}, t) \lozenge N(Sx_{2n+2}, Bx_{2n+1}, (1-\beta)t) \lozenge N(tx_{2n+1}, Ax_{2n+2}, (1-\beta)t) \lozenge N(Tx_{2n+1}, Sx_{2n+2}, t)$

$= N(y_{2n}, y_{2n+1}, kt) \lozenge N(y_{2n+1}, y_{2n+2}, t) \lozenge N(y_{2n+1}, y_{2n+2}, (1-\beta)t) \lozenge N(y_{2n}, y_{2n+2}, (1+\beta)t) \lozenge N(y_{2n}, y_{2n+1}, t)$

$= N(y_{2n}, y_{2n+1}, kt) \lozenge N(y_{2n+1}, y_{2n+2}, t) \lozenge 0 \lozenge N(y_{2n+1}, y_{2n+2}, t) \lozenge N(y_{2n}, y_{2n+2}, \beta t) \lozenge N(y_{2n}, y_{2n+1}, t)$

$\leq N(y_{2n}, y_{2n+1}, t) \lozenge N(y_{2n+1}, y_{2n+2}, t) \lozenge N(y_{2n+1}, y_{2n+2}, \beta t)$

Taking $\beta \to 1$, we get

$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \lozenge N(y_{2n+1}, y_{2n+2}, t) \lozenge N(y_{2n+1}, y_{2n+2}, \beta t)$

$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \lozenge N(y_{2n+1}, y_{2n+2}, t) \leq N(y_{2n}, y_{2n+1}, t)$
$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$
Similarly, it can be shown that
$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$
Therefore, for any n and t, we
$N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$
Hence $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, so $\{y_n\}$ converges to z in X. Its subsequences $\{Ax_{2n}\},\{Tx_{2n+1}\},\{Bx_{2n+1}\}$ and $\{Sx_{2n+2}\}$ also converge to z.
Now, suppose that (A, S) is a compatible pair and S is continuous. Then A and S are reciprocally continuous, then $SAx_n \to Sz$, $ASx_n \to Az$ as $n \to \infty$.
As, (A, S) is a compatible pair. This implies

$$\lim_{n-\infty} M(ASx_n, Sz, t) = 1, N(Az, Sz, t) = 0$$

This gives M(Az, Sz, t) = 1, N(Az, Sz, t) = 0 as $n \to \infty$ Hence, Sz = Az.
Since $A(X) \subseteq T(X)$, therefore there exists a point $p \in X$ such that Sz = Az =Tp.
Now, again by taking $\alpha = 1$ in (2), we have
$M(Az,Bp,kt) \geq M(Tp, Bp,t) * M(Sz,Az,t) * M(Sz,Bp,t) * M(Tp, Az,t) * M(Tp, Sz,t)$
$M(Az,Bp,kt) \geq M(Az, Bp,t) * M(Az,Az,t) * M(Az,Bp,t) * M(Az, Az,t) * M(Az,Az,t)$
And
$N(Az,Bp,kt) \leq N(Tp, Bp,t) \lozenge N(Sz,Az,t) \lozenge N(Sz,Bp,t) \lozenge N(Tp, Az,t) \lozenge N(Tp, Sz,t)$
$N(Az,Bp,kt) \leq N(Az, Bp,t) \lozenge N(Az,Az,t) \lozenge N(Az,Bp,t) \lozenge N(Az, Az,t) \lozenge N(Az, Az,t)$
Thus, Az = Bp = Sz = Tp
Since, A and S are point wise R-weakly commuting mappings, therefore there exists R > 0 Such that,
M(ASz, SAz, t) ≥ M(Az, Sz, t/R)= 1
And N(ASz, SAz, t) ≤N(Az, Sz, t/R)= 0.
Hence, ASz = SAz and ASz = SAz = AAz = SSz.
Similarly B and T are point wise R- weakly commuting mappings, we have
BBp = BTp = TBp =TTp.
$M(AAz, Bp, kt) \geq M(Tp, Bp,t) * M(SAz, AAz,t) * M(SAz, Bp,t) * M(Tp, AAz,t) * M(Tp, SAz,t)$
$M(AAz, Az, kt) \geq M(Tp,Tp,t) * M(AAz, AAz,t) * M(AAz, Az,t) * M(Az, AAz,t) * M(Az, AAz,t)$
And
$N(AAz, Bp,kt) \leq N(Tp, Bp,t) \lozenge N(SAz, AAz,t) \lozenge N(SAz, Bp,t) \lozenge N(Tp, AAz,t) \lozenge N(Tp, SAz,t)$
$N(AAz, Az, kt) \leq N(Tp,Tp,t) \lozenge N(AAz, AAz,t) \lozenge N(AAz, Az,t) \lozenge N(Az, AAz,t) \lozenge N(Az, AAz,t)$
M(AAz, Az, kt) ≥ M(AAz, Az, t),
N(AAz, Az, t) ≤ N(AAz, Az, t).
By theorem, we have SAz = AAz = Az. Hence Az is common fixed point of A and S. Similarly by (2) Bp = Az is a common fixed point of B and T. Hence Az is a common fixed point of A, B, S and T.

Uniqueness:

Suppose that Ap (≠Az) is another common fixed point of A, B, S and T.
Then by (2), take $\alpha = 1$
M(AAz, BAp, kt) ≥ M(TAp, BAp, t)*M(SAz, AAz, t)*M(SAz, BAp, t)* M(TAp, AAz, t)*M(TAp, SAz, t)
M(Az, Ap, kt) ≥ M(Ap, Ap, t)*M(Az, Az, t)*M(Az, Ap, t)*M(Ap, Az, t)* M(Ap,Az, t)

And

N(AAz, BAp, kt) ≤ N(TAp, BAp, t)◊N(SAz, AAz, t)◊N(SAz, BAp, t) ◊N(TAp, AAz, t)◊N(TAp, SAz, t)

N(Az, Ap, kt) ≤ N(Ap, Ap, t)◊N(Az, Az, t)◊N(Az, Ap, t)◊N(Ap, Az, t)◊N(Ap, Az, t)

This gives M(Az, Ap, kt) ≥ M(Az, Ap, t) and N(Az, Ap, kt) ≤ N(Az, Ap, t) By theorem, Ap = Az Thus uniqueness follows.

Taking S = T = Ix in above theorem, we get following result.

Corollary 4.1.1. Let (X, M, N, *, ◊) be a complete intuitionistic L-fuzzy metric space with continuous t- norm * and continuous t- conorm ◊ defined by t*t ≥ t and (1-t) ◊ (1-t) ≤ (1-t) for all t ϵ [0,1].

Further, let A and B are reciprocally continuous mapping on X satisfying M(Ax, By, kt) ≥ M(y, By, t)*M(x, Ax, t)*M(x, By, αt)*M(y, Ax, (2-α)t)*M(y, x, t) N(Ax, By, kt) ≤ N(y, By, t) ◊ N(x, Ax, t) ◊ N(x, By, αt) ◊ N(y, Ax, (2-α)t) ◊ N(y, x, t) For all u, v ϵ X, t > 0 and α ϵ (0,2) then pair A and B has a unique common fixed point We give now example to illustrate the above theorem.

Example 4.1. Let X = [0,∞] and let M and N be defined by M(u, v, t) $=\frac{1}{t+|u-v|}$ and N(u,v,t) = $\frac{|u-v|}{t+|u-v|}$. Then (X, M, N, *, ◊) is complete Intuitionistic L-fuzzy metric space. Let A, B, S, and T be self maps on x defined as, Ax = Bx = 3x/4 and Sx = Tx = 2x for all x ϵ X. Clearly,

1) Either of pair (A,S) or (B,T) be continuous self mapping on X.
2) A(x) ⊆ T(x), B(x)⊆ S(x);
3) {A, S} and {B, T} are R- weakly commuting pairs as both pairs commute at coincidence points.
4) {A, S} and {B, T} satisfies inequality (2), for all x, y ϵ X, where k ϵ (0,1)

Hence all conditions of theorem are satisfied and x = 0 is a unique common fixed point of A, B, S, T.

5. Conclusion

Last three decades were very productive for fuzzy mathematics and the recent literature has observed the fuzzy application in almost every direction of mathematics such as arithmetic, topology, graph theory, probability theory, logic etc. In this work a general analysis has been done to reveal the links between the fixed point properties and the fuzzy metric spaces. The analysis can be used to form some new fixed point theorems by using different types.

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