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Fixed Point Theorem and Semi-Compatibility in Menger Probabilistic Metric Space

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Abstract. The present paper deals with a fixed point theorem for six self maps using the concept of semi-compatible self maps in a Menger PM-space. Our result generalizes the result of Singh and Sharma [12].

Keywords: Common fixed points, compatible maps, semi-compatible maps, t-norm.

AMS Mathematics Subject Classification (2010): 47H10, 54H25

1. Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [7]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [9] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [10] extended the notion of contraction mapping to the setting of the Menger space. They obtained a generalization of the classical Banach contraction principle on complete Menger spaces.

The notion of compatible mapping in a Menger space has been introduced by Mishra [8]. Singh and Sharma [12] have proved a common fixed point theorem for four compatible maps in Menger space by taking a new inequality. Using the concept of compatible mappings of type (A) and weak compatible mappings, Jain et al. [2, 3, 4] proved some interesting fixed point theorems in Menger space. Cho, Sharma and Sahu [1] introduced the concept of semi-compatibility in a d-complete topological space. In Menger space, Singh et al. [11] defined the concept of semi-compatibility of pair of self-maps. Using the concept of occasionally weakly compatible mappings, Jha et. al. [5] proved fixed point theorems in semi-metric space. Afterwards, Jha et al. [6] proved a common fixed point theorem for reciprocal continuous compatible mappings in metric space. In the sequel, Srinivas et al. [13] gave Djoudi's common fixed point theorem on compatible mappings of type (P).

In this paper, we generalize the result of Singh and Sharma [12] by introducing the notion of semi-compatible self maps.

2. Preliminaries

Definition 2.1. [8] A mapping $\mathcal{F} : R \to R^+$ is called a *distribution* if it is non-decreasing left continuous with inf $\{\mathcal{F}(t) \mid t \in R\} = 0$ and sup $\{\mathcal{F}(t) \mid t \in R\} = 1$.

We shall denote by L the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & t \le 0, \\ 1, & t > 0 \end{cases}$$

Definition 2.2. [8] A mapping $t : [0, 1] \times [0, 1] \to [0, 1]$ is called a *t-norm* if it satisfies the following conditions :

(t-1) $t(a, 1) = a, \quad t(0, 0) = 0$;

(t-2) $t(a, b) = t(b, a)$;

(t-3) $t(c, d) \ge t(a, b)$; for $c \ge a$, $d \ge b$,

(t-4) $t(t(a, b), c) = t(a, t(b, c))$.

Definition 2.3. [8] A *probabilistic metric space (PM-space)* is an ordered pair $(X, \mathcal{F})$ consisting of a non empty set X and a function $\mathcal{F} : X \times X \to L$, where L is the collection of all distribution functions and the value of $\mathcal{F}$ at $(u, v) \in X \times X$ is represented by $F_{u, v}$. The function $F_{u,v}$ assumed to satisfy the following conditions:

(PM-1) $F_{u,v}(x) = 1$, for all $x > 0$, if and only if $u = v$;

(PM-2) $F_{u,v}(0) = 0$;

(PM-3) $F_{u,v} = F_{v,u}$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(x + y) = 1$,

for all $u, v, w \in X$ and $x, y > 0$.

A *Menger space* is a triplet $(X, \mathcal{F}, t)$ where $(X, \mathcal{F})$ is a PM-space and t is a t-norm such that the inequality

(PM-5) $F_{u,w}(x + y) \ge t\{F_{u, v}(x), F_{v, w}(y)\}$, for all $u, v, w \in X$, $x, y \ge 0$.

Definition 2.4. [8] A sequence $\{x_n\}$ in a Menger space $(X, \mathcal{F}, t)$ is said to be *convergent* and *converges to a point* x in X if and only if for each $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x}(\varepsilon) > 1 - \lambda \qquad \text{for all } n \ge M(\varepsilon, \lambda).$$

Further, the sequence $\{x_n\}$ is said to be *Cauchy sequence* if for $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x_m}(\varepsilon) > 1 - \lambda \qquad \text{for all } m, n \ge M(\varepsilon, \lambda).$$

A Menger PM-space $(X, \mathcal{F}, t)$ is said to be *complete* if every Cauchy sequence in X converges to a point in X.

Definition 2.5. [8] Self maps S and T of a Menger space $(X, \mathcal{F}, t)$ are said to be *compatible* if $F_{STx_n, TSx_n}(x) \to 1$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to u$ for some u in X, as $n \to \infty$.

Definition 2.6. [11] Self maps S and T of a Menger space $(X, \mathcal{F}, t)$ are said to be *semi-compatible* if $F_{STx_n, Tu}(x) \to 1$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to u$ for some u in X, as $n \to \infty$.

It follows that if the pair (S, T) is semi-compatible and $Sy = Ty$ then $STy = TSy$.

Proposition 2.1. [11] If (S, T) is a semi-compatible pair of self maps in a Menger PM-space $(X, \mathcal{F}, t)$ and T is continuous then (S, T) is compatible.

Proposition 2.2. [11] If (X, d) is a metric space, then the metric d induces a mapping $F : X \times X \to L$, defined by

$$F_{p,q}(x) = H(x - d(p, q)),\ p, q \in X \text{ and } x \in R.$$

Further, if $t : [0, 1] \times [0, 1] \to [0, 1]$ is defined by $t(a, b) = \min\{a, b\}$, then $(X, \mathcal{F}, t)$ is a Menger space. It is complete if (X, d) is complete. The space $(X, \mathcal{F}, t)$ is called an induced Menger space.

Remark 2.1. [11] The concept of semi-compatibility of pair of self maps is more general than that of compatibility.

Proposition 2.3. [8] If S and T are compatible self maps of a Menger space $(X, \mathcal{F}, t)$ where t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$ and $Sx_n, Tx_n \to u$ for some u in X. Then $TSx_n \to Su$ provided S is continuous.

Proposition 2.4. [4] Let S and T be compatible self maps of Menger space $(X, \mathcal{F}, t)$ and $Su = Tu$ for some u in X then $STu = TSu = SSu = TTu$.

Lemma 2.1. [4] Let $\{p_n\}$ be a sequence in a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm and $t(x, x) \geq x$. Suppose, for all $x \in [0, 1]$, there exists $k \in (0, 1)$ such that for all $x > 0$ and $n \in N$,

$$F_{p_n, p_{n+1}}(kx) \geq F_{p_{n-1}, p_n}(x)$$

or

$$F_{p_n, p_{n+1}}(x) \geq F_{p_{n-1}, p_n}(k^{-1}x).$$

Then $\{p_n\}$ is a Cauchy sequence in X.

3. Main results

Theorem 3.1. Let A, B, S, T, L and M be self mappings of a complete Menger space $(X, \mathcal{F}, t)$ with $t(a, a) \geq a$, for some $a \in [0, 1]$, satisfying :

(3.1.1) $\quad L(X) \subseteq ST(X),\ M(X) \subseteq AB(X)$;

(3.1.2) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

(3.1.3) either AB or L is continuous;

(3.1.4) (L, AB) is compatible and (M, ST) is semi-compatible;

(3.1.5) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{Lp, Mq}(x) + F_{ABp, Lp}(x)][F_{Lp, Mq}(x) + F_{STq, Mq}(x)] \geq 4[F_{ABp, Lp}(x/\alpha)][F_{Mq, STq}(x)].$$

Then A, B, S, T, L and M have a unique common fixed point in X.

Proof: Let $x_0 \in X$. From condition (3.1.5) $\exists$ $x_1, x_2 \in X$ such that

$$Lx_0 = STx_1 = y_0 \quad \text{and} \quad Mx_1 = ABx_2 = y_1.$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Lx_{2n} = STx_{2n+1} = y_{2n} \quad \text{and} \quad Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$$

for $n = 0, 1, 2, \ldots$.

Step 1. Putting $p = x_{2n}$, $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{Lx_{2n},Mx_{2n+1}}(x)+F_{ABx_{2n},Lx_{2n}}(x)][F_{Lx_{2n}, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/a)][F_{Mx_{2n+1}, STx_{2n+1}}(x)]$$

$$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)][F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n}, y_{2n+1}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/a)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$2 F_{y_{2n}, y_{2n+1}}(x) [F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/a)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x) [F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1},y_{2n}}(x/a)][F_{y_{2n}, y_{2n+1}}(x)]$$

or, $$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1}, y_{2n}}(x/a)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/a). \tag{3.1.6}$$

Similarly,

$$F_{y_{2n-1}, y_{2n}}(x/a) \geq F_{y_{2n-2}, y_{2n-1}}(x/a^2). \tag{3.1.7}$$

From (3.1.6) and (3.1.7), it follows that

$$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/a) \geq F_{y_{2n-2}, y_{2n-1}}(x/a^2).$$

By repeated application of above inequality, we get

$$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/a) \geq F_{y_{2n-2}, y_{2n-1}}(x/a^2) \geq \ldots \geq F_{y_0, y_1}(x/a^n).$$

Therefore, by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X, which is complete.

Hence $\{y_n\} \to z \in X$.

Also its subsequences converges as follows :

$\{Mx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$, (3.1.8)

$\{Lx_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$. (3.1.9)

Case I. AB is continuous.

As AB is continuous,

$$(AB)^2x_{2n} \to ABz \quad \text{and} \quad (AB)Lx_{2n} \to ABz.$$

As (L, AB) is compatible, so by proposition (2.3), we have

$$L(AB)x_{2n} \to ABz.$$

Step 2. Putting $p = ABx_{2n}$ and $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{LABx_{2n},Mx_{2n+1}}(x)+F_{ABABx_{2n},LABx_{2n}}(x)][F_{LABx_{2n},Mx_{2n+1}}(x) + F_{STx_{2n+1},Mx_{2n+1}}(x)] \geq 4[F_{ABABx_{2n},LABx_{2n}}(x/a)][F_{Mx_{2n+1},STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{ABz,z}(x) + F_{ABz,ABz}(x)][F_{ABz,z}(x) + F_{z,z}(x)] \geq 4[F_{ABz,ABz}(x/a)][F_{z,z}(x)],$$

i.e. $F_{ABz,z}(x) \geq 1$, yields $ABz = z$. (3.1.10)

Step 3. Putting $p = z$ and $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{Lz,Mx_{2n+1}}(x) + F_{ABz,Lz}(x)][F_{Lz,Mx_{2n+1}}(x) + F_{STx_{2n+1},Mx_{2n+1}}(x)] \geq 4[F_{ABz,Lz}(x/a)][F_{Mx_{2n+1},STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{Lz,z}(x) + F_{z,Lz}(x)][F_{Lz,z}(x) + F_{z,z}(x)] \geq 4[F_{z,Lz}(x/a)][F_{z,z}(x)],$$

i.e. $F_{Lz,z}(x) \geq 1$, yields $Lz = z$.

Therefore, $ABz = Lz = z$.

Step 4. Putting $p = Bz$ and $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{LBz,Mx_{2n+1}}(x)+F_{ABBz,Bz}(x)][F_{LBz,Mx_{2n+1}}(x) + F_{STx_{2n+1},Mx_{2n+1}}(x)] \geq 4[F_{ABBz,LBz}(x/a)][F_{Mx_{2n+1},STx_{2n+1}}(x)].$$

As $BL = LB$, $AB = BA$, so we have

$L(Bz) = B(Lz) = Bz$ and $AB(Bz) = B(ABz) = Bz$.

Letting $n \to \infty$, we get

$$[F_{Bz,z}(x) + F_{Bz,Bz}(x)][F_{Bz,z}(x) + F_{z,z}(x)] \geq 4[F_{Bz,Bz}(x/a)][F_{z,z}(x)],$$

i.e. $F_{Bz,z}(x) \geq 1$, yields $Bz = z$ and $ABz = z$ implies $Az = z$.

Therefore, $Az = Bz = Lz = z$. (3.1.11)

Step 5. As $L(X) \subseteq ST(X)$, there exists $v \in X$ such that

$$z = Lz = STv.$$

Putting $p = x_{2n}$ and $q = v$ for $x > 0$ in (3.1.5), we get

$$[F_{Lx_{2n}, Mv}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mv}(x) + F_{STv, Mv}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/a)][F_{Mv, STv}(x)].$$

Letting $n \to \infty$ and using equation (3.1.9), we get

$$[F_{z, Mv}(x) + F_{z, z}(x)][F_{z, Mv}(x) + F_{z, Mv}(x)] \geq 4[F_{z, z}(x/a)][F_{Mv, z}(x)],$$

i.e. $F_{z, Mv}(x) \geq 1$, yields $Mv = z$.

Hence, $STv = z = Mv$.

As (M, ST) semi-compatible, we have

$$STMv = MSTv.$$

Thus, $STz = Mz$.

Step 6. Putting $p = x_{2n}$, $q = z$ for $x > 0$ in (3.1.5), we get

$$[F_{Lx_{2n}, Mz}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mz}(x) + F_{STz, Mz}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/a)][F_{Mz, STz}(x)].$$

Letting $n \to \infty$ and using equation (3.1.8) and Step 5, we get

$$[F_{z, Mz}(x) + F_{z, z}(x)][F_{z, Mz}(x) + F_{Mz, Mz}(x)] \geq 4[F_{z, z}(x/a)][F_{Mz, Mz}(x)],$$

i.e. $F_{z, Mz}(x) \geq 1$, yields $z = Mz$.

Step 7. Putting $p = x_{2n}$ and $q = Tz$ for $x > 0$ in (3.1.5), we get

$$[F_{Lx_{2n}, MTz}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, MTz}(x) + F_{STTz, MTz}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/a)][F_{MTz, STTz}(x)].$$

As $MT = TM$ and $ST = TS$, we have $MTz = TMz = Tz$ and $ST(Tz) = T(STz) = Tz$.

Letting $n \to \infty$, we get

$$[F_{z,Tz}(x) + F_{z,z}(x)][F_{z,Tz}(x) + F_{Tz,Tz}(x)] \geq 4[F_{z,z}(x/a)][F_{Tz,Tz}(x)],$$

i.e. $F_{z, Tz}(x) \geq 1$, yields $Tz = z$.

Now $STz = Tz = z$ implies $Sz = z$.

Hence $Sz = Tz = Mz = z$. (3.1.12)

Combining (3.1.11) and (3.1.12), we get

$$Az = Bz = Lz = Mz = Tz = Sz = z.$$

Hence, the six self maps have a common fixed point in this case.

Case II. L is continuous

As L is continuous, $L^2x_{2n} \to Lz$ and $L(AB)x_{2n} \to Lz$.

As (L, AB) is compatible, so by proposition (2.3),

$$(AB)Lx_{2n} \rightarrow Lz.$$

Step 8. Putting $p = Lx_{2n}$ and $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{LLx_{2n}, Mx_{2n+1}}(x) + F_{ABLx_{2n}, LLx_{2n}}(x)][F_{LLx_{2n}, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)] \geq 4[F_{ABLx_{2n}, LLx_{2n}}(x/a)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \rightarrow \infty$, we get

$$[F_{Lz, z}(x) + F_{Lz, Lz}(x)][F_{Lz, z}(x) + F_{z, z}(x)] \geq 4[F_{Lz, Lz}(x/a)][F_{z, z}(x)],$$

i.e. $F_{Lz,z}(x) \geq 1$, yields $Lz = z$.

Now, using steps 5-7, we get $Mz = STz = Sz = Tz = z$.

Step 9. As $M(X) \subseteq AB(X)$, there exists $w \in X$ such that

$$z = Mz = ABw.$$

Putting $p = w$ and $q = x_{2n+1}$ for $x > 0$ in (3.1.5), we get

$$[F_{Lw, Mx_{2n+1}}(x) + F_{ABw,Lw}(x)][F_{Lw,Mx_{2n+1}}(x) + F_{STx_{2n+1},Mx_{2n+1}}(x)] \geq 4[F_{ABw, Lw}(x/a)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \rightarrow \infty$, we get

$$[F_{Lw, z}(x) + F_{z, Lw}(x)][F_{Lw, z}(x) + F_{z, z}(x)] \geq 4[F_{z, Lw}(x/a)][F_{z, z}(x)],$$

i.e. $F_{Lw, z}(x) \geq 1$, yields $Lw = z = ABw$.

Since (L,AB) is compatible and so by proposition (2.4), we have

$$LABw = ABLw.$$

Hence,

$$Lz = ABz.$$

Also, $Bz = z$ follows from step 4.

Thus, $Az = Bz = Lz = z$ and we obtain that z is the common fixed point of the six maps in this case also.

Step 10. (Uniqueness) Let u be another common fixed point of A, B, S, T, L and M; then $Au = Bu = Su = Tu = Lu = Mu = u$.

Putting $p = z$ and $q = u$ for $x > 0$ in (3.1.5), we get

$$[F_{Lz, Mu}(x) + F_{ABz, Lz}(x)][F_{Lz, Mu}(x) + F_{STu, Mu}(x)] \geq 4[F_{ABz, Lz}(x/a)][F_{Mu, STu}(x)].$$

Letting $n \rightarrow \infty$, we get

$$[F_{z, u}(x) + F_{z, z}(x)][F_{z, u}(x) + F_{u, u}(x)] \geq 4[F_{z, z}(x/a)][F_{u, u}(x)],$$

i.e. $F_{z,u}(x) \geq 1$, yields $z = u$.

Therefore, z is a unique common fixed point of A, B, S, T, L and M.

This completes the proof.

Remark 3.1. If we take B = T = I, the identity map on X in theorem 3.1, then the condition (3.1.2) is satisfied trivially and we get

Corollary 3.1. Let A, S, L and M be self mappings of a complete Menger space (X, $\mathcal{F}$, t) satisfying :

(3.1.13) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(3.1.14) Either A or L is continuous;

(3.1.15) (L, A) is compatible and (M, S) is semi-compatible;

(3.1.16) for all p, q $\in$ X, x > 0 and $0 < \alpha < 1$,

$$[F_{Lp,\, Mq}(x) + F_{Ap,\, Lp}(x)][F_{Lp,\, Mq}(x) + F_{Sq,\, Mq}(x)]$$
$$\geq 4[F_{Ap,\, Lp}(x/\alpha)][F_{Mq,\, Sq}(x)].$$

Then A, S, L and M have a unique common fixed point in X.

Next we utilize our Theorem 3.1 to prove another common fixed point theorem in a complete metric space.

Theorem 3.2. Let A, B, S, T, L and M be self mappings of a complete metric sapce (X, d) satisfying (3.1.1), (3.1.2), (3.1.3), (3.1.4) and

(3.1.17) $[d(Lp,Mq)]^{1/2}\{[d(ABp,Lp)]^{1/2} + [d(STq,Mq)]^{1/2}\}$
$$\leq \alpha\{d(ABp,Lp)+ d(Mq, STq)\},$$

for all p, q $\in$ X where $0 < \alpha < 1$.

Then A, B, S, T, L and M have a unique common fixed point in X.

Proof. The proof follows from theorem 3.1 and by considering the induced Menger space (X, $\mathcal{F}$, t), where t(a,b) = min {a, b} and $F_{p,\, q}(x) = H(x - d(p,q))$, H being the distribution function as given in the definition 2.1.

4. Conclusion

In view of remark 3.1, corollary 3.1 is a generalization of the result of Singh and Sharma [12] in the sense that the condition of compatibility of the pairs of self maps has been reduced to compatible and semi-compatible self maps and only one of the compatible maps is needed to be continuous.

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