The Sum Connectivity Revan Index of Silicate and Hexagonal Networks

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Abstract. In Chemical Graph Theory, the connectivity indices are applied to measure the chemical characteristics of chemical compounds. In this paper, we introduce the sum connectivity Revan index of a molecular graph. Also we compute the sum connectivity Revan index for certain networks of chemical importance like silicate networks and hexagonal networks. The first and second Revan indices of silicate networks and hexagonal networks are determined.

Keywords: Sum connectivity Revan index, silicate networks, hexagonal networks.

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1. Introduction

We consider only finite, simple and connected graph \( G \) with vertex set \( V(G) \) and edge set \( E(G) \). The degree \( d_G(v) \) of a vertex \( v \) is the number of vertices adjacent to \( v \). Let \( \Delta(G) \) (\( \delta(G) \)) denote the maximum (minimum) degree among the vertices of \( G \). We refer [1] for undefined notations and terminologies.

Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. Numerous topological indices have been considered in theoretical chemistry, and have found some applications in QSPR/QSAR study, see [2].

The Revan vertex degree of a vertex \( v \) of a graph \( G \) is defined as

\[
r_G(v) = \Delta(G) + \delta(G) - d_G(v).\]

The Revan edge connecting the Revan vertices \( u \) and \( v \) will be denoted by \( uv \). In [3], Kulli introduced the first and second Revan indices of a graph \( G \). These indices are defined as

\[
R_1(G) = \sum_{u \in E(G)} \left[ r_G(u) + r_G(v) \right], \quad R_2(G) = \sum_{u \in E(G)} r_G(u) r_G(v).
\]

In [4], the product connectivity Revan index of a graph \( G \) is defined as
We propose the sum connectivity Revan index of a graph as follows:

The sum connectivity Revan index of a molecular graph \( G \) is defined as

\[
PR(G) = \sum_{uv \in E(G)} \frac{1}{r_G(u) r_G(v)}.
\]

Recently several topological indices were studied, for example, in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,16,17,18].

In this paper, the sum connectivity Revan index of certain important chemical structures like silicate networks and hexagonal networks are computed. Also the first and second Revan indices of these networks are determined. For networks see [19] and references cited therein.

2. Results for silicate networks

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolized by \( SL_n \) where \( n \) is the number of hexagons between the center and boundary of \( SL_n \). A 2-dimensional silicate network is shown in Figure 1.

![Figure 1: A 2-dimensional silicate network](image)

In the following theorem, we compute the exact formulas of \( R_1(SL_n) \), \( R_2(SL_n) \) for silicate networks.

**Theorem 1.** Let \( SL_n \) be the silicate networks. Then

1. \( R_1(SL_n) = 270n^2 + 54n \).
2. \( R_2(SL_n) = 486n^2 + 216n \).

**Proof:** Let \( G \) be the graph of silicate network \( SL_n \) with \( 15n^2 + 3n \) vertices and \( 36n^2 \) edges. From Figure 1, it is easy to see that the vertices of \( SL_n \) are either of degree 3 or 6. In \( SL_n \), by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

- \( E_{33} = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 3 \} \), \( |E_{33}| = 6n \).
- \( E_{36} = \{ uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6 \} \), \( |E_{36}| = 18n^2 + 6n \).
- \( E_{66} = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 6 \} \), \( |E_{66}| = 18n^2 - 12n \).

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as follows:
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We have $\Delta(G) + \delta(G) = 9$, $\rho_G(u) = 9 - d_G(u)$.

$RE_{66} = \{|uv\in E(G) \mid r_G(u) = r_G(v) = 6\}, |RE_{66}| = 6n$.

$RE_{63} = \{|uv\in E(G) \mid r_G(u) = 6, r_G(v) = 3\}, |RE_{63}| = 18n^2 + 6n$.

$RE_{33} = \{|uv\in E(G) \mid r_G(u) = r_G(v) = 3\}, |RE_{33}| = 18n^2 - 12n$.

(1) To compute $R_1(SL_n)$, we see that

$$R_1(G) = \sum_{uv \in E(G)} \left[ r_G(u) + r_G(v) \right]$$

$$= \sum_{R_{E_{66}}} \left[ r_G(u) + r_G(v) \right] + \sum_{R_{E_{63}}} \left[ r_G(u) + r_G(v) \right] + \sum_{R_{E_{33}}} \left[ r_G(u) + r_G(v) \right]$$

$$= 6n \times 12 + (18n^2 + 6n)9 + (18n^2 - 12n)6$$

$$= 270n^2 + 54n.$$ 

(2) To compute $R_2(SL_n)$, we see that

$$R_2(G) = \sum_{uv \in E(G)} \left[ r_G(u) \cdot r_G(v) \right]$$

$$= \sum_{R_{E_{66}}} \left[ r_G(u) \cdot r_G(v) \right] + \sum_{R_{E_{63}}} \left[ r_G(u) \cdot r_G(v) \right] + \sum_{R_{E_{33}}} \left[ r_G(u) \cdot r_G(v) \right]$$

$$= 6n \times 36 + (18n^2 + 6n)18 + (18n^2 - 12n)$$

$$= 486n^2 + 216n.$$ 

In the following theorem, we compute the sum connectivity Revan index of $SL_n$.

**Theorem 2.** Let $SL_n$ be the silicate networks. Then

$$SR(SL_n) = \left(6 + 3\sqrt{6}\right)n^2 + \left(\sqrt{3} + 2 - 2\sqrt{6}\right)n.$$ 

**Proof:** Let $G = SL_n$. From equation (1), and by cardinalities of the revan edge partition of silicate network $SL_n$, we have

$$SR(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{r_G(u) + r_G(v)}}$$

$$= \sum_{R_{E_{66}}} \frac{1}{\sqrt{r_G(u) + r_G(v)}} + \sum_{R_{E_{63}}} \frac{1}{\sqrt{r_G(u) + r_G(v)}} + \sum_{R_{E_{33}}} \frac{1}{\sqrt{r_G(u) + r_G(v)}}$$

$$= \left(\frac{1}{\sqrt{6 + 6}}\right)6n + \left(\frac{1}{\sqrt{6 + 3}}\right)(18n^2 + 6n) + \left(\frac{1}{\sqrt{3 + 3}}\right)(18n^2 - 12n)$$

$$= (6 + 3\sqrt{6})n^2 + (\sqrt{3} + 2 - 2\sqrt{6})n.$$ 

3. **Results for hexagonal networks**

Hexagonal network is symbolized by $HX_n$ where $n$ is the number of vertices in each side of hexagon. A hexagonal network of dimension six is shown in Figure 2.
Figure 2: 6-dimensional hexagonal network

In the following theorem, we compute the exact formulas of $R_1(HX_n)$, $R_2(HX_n)$ for hexagonal networks.

**Theorem 3.** Let $HX_n$ be the hexagonal networks. Then

1. $R_1(HX_n) = 54n^2 - 42n - 6$.
2. $R_2(HX_n) = 81n^2 + 33n - 72$.

**Proof:** Let $H$ be the graph of hexagonal network $HX_n$ with $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. From Figure 2, it is easy to see that the vertices of $HX_n$ are either of degree 3, 4 or 6. Thus $\Delta(H) = 6$, $\delta(H) = 3$. In $H$, by algebraic method, there are five types of edges as follows:

$$
E_{34} = \{ uv \in E(H) | d_H(u) = 3, d_H(v) = 4 \} ; |E_{34}| = 12.
$$

$$
E_{36} = \{ uv \in E(H) | d_H(u) = 3, d_H(v) = 6 \} ; |E_{36}| = 6.
$$

$$
E_{44} = \{ uv \in E(H) | d_H(u) = d_H(v) = 4 \} ; |E_{44}| = 6n - 18.
$$

$$
E_{46} = \{ uv \in E(H) | d_H(u) = 4, d_H(v) = 6 \} ; |E_{46}| = 12n - 24.
$$

$$
E_{66} = \{ uv \in E(H) | d_H(u) = d_H(v) = 6 \} ; |E_{66}| = 9n^2 - 33n + 30.
$$

We have $r_G(u) = \Delta(H) + \delta(H) - d_H(u) = 9 - d_H(u)$.

$$
RE_{65} = \{ uv \in E(H) | r_G(u) = 6, r_G(v) = 5 \} ; |RE_{65}| = 12.
$$

$$
RE_{63} = \{ uv \in E(H) | r_G(u) = 6, r_G(v) = 3 \} ; |RE_{63}| = 6.
$$

$$
RE_{55} = \{ uv \in E(H) | r_G(u) = 5, r_G(v) = 5 \} ; |RE_{55}| = 6n - 18.
$$

$$
RE_{53} = \{ uv \in E(H) | r_G(u) = 5, r_G(v) = 3 \} ; |RE_{53}| = 12n - 24.
$$

$$
RE_{33} = \{ uv \in E(H) | r_G(u) = r_G(v) = 3 \} ; |RE_{33}| = 9n^2 - 33n + 30.
$$

(1) To compute $R_1(HX_n)$, we see that

$$
R_1(H) = \sum_{u \in V(H)} [r_G(u) + r_G(v)]
$$

$$
= \sum_{H_{34}} [r_G(u) + r_G(v)] + \sum_{H_{36}} [r_G(u) + r_G(v)] + \sum_{H_{44}} [r_G(u) + r_G(v)]
$$

$$
+ \sum_{H_{46}} [r_G(u) + r_G(v)] + \sum_{H_{66}} [r_G(u) + r_G(v)]
$$

$$
= (6 + 5)12 + (6 + 3)6 + (5 + 5)(6n - 18) + (5 + 3)(12n - 24)
$$

$$
+ (3 + 3)(9n^2 - 33n + 30)
$$

$$
= 54n^2 - 42n - 6.
$$
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(2) To compute $R_2(HX_n)$, we see that

$$R_2(HX_n) = \sum_{u \in E(H)} r_G(u) r_G(v)$$

$$= \sum_{u \in E(H)} r_G(u) r_G(v) + \sum_{u \in E(H)} r_G(u) r_G(v) + \sum_{u \in E(H)} r_G(u) r_G(v) + \sum_{u \in E(H)} r_G(u) r_G(v) =$$

$$(6 \times 5)12 + (6 \times 3)6 + (5 \times 5)(6n-18) + (5 \times 3)(12n-24) + (3 \times 3)(9n^2-33n+30)$$

$$= 81n^2 + 33n - 72.$$  

In the next theorem, we compute the sum connectivity Revan index of $HX_n$.

**Theorem 4.** Let $HX_n$ be the hexagonal networks. Then

$$SR(HX_n) = \frac{9}{\sqrt{6}} n^2 + \left( \frac{6}{\sqrt{10}} + \frac{6}{\sqrt{2}} - \frac{33}{\sqrt{6}} \right) n + \left( \frac{12}{\sqrt{11}} + 2 - \frac{18}{\sqrt{10}} + \frac{30}{\sqrt{6}} \right)$$

**Proof:** Let $H = HX_n$. From equation (2) and by cardinalities of the revan edge partition of hexagonal network $HX_n$, we have

$$SR(HX_n) = \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}}$$

$$= \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}} + \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}} + \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}}$$

$$+ \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}} + \sum_{u \in E(H)} \frac{1}{\sqrt{r_H(u) + r_H(v)}}$$

$$= \left( \frac{1}{\sqrt{6+5}} \right) 12 + \left( \frac{1}{\sqrt{6+3}} \right) 6 + \left( \frac{1}{\sqrt{5+5}} \right) (6n+8)$$

$$+ \left( \frac{1}{\sqrt{5+3}} \right) (12n-24) + \left( \frac{1}{\sqrt{3+3}} \right) (9n^2-33n+30)$$

$$= \frac{9}{\sqrt{6}} n^2 + \left( \frac{6}{\sqrt{10}} + \frac{6}{\sqrt{2}} - \frac{33}{\sqrt{6}} \right) n + \left( \frac{12}{\sqrt{11}} + 2 - \frac{18}{\sqrt{10}} + \frac{30}{\sqrt{6}} \right)$$

**REFERENCES**

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