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On Regular Generalized $B^{\#}$-Closed Sets

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Abstract. The aim of this paper is to introduce a new class of sets called $rgb^{\#}$-closed sets and examine the basic properties of $rgb^{\#}$-closed sets.

Keywords: $b^{\#}$-closed set, $gb^{\#}$-closed set, $rgb^{\#}$-closed set.

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1. Introduction

In topology and related branches of mathematics, a topological space may defined as a set of points, along with a set of neighbourhoods for each point, satisfying a set of axioms relating points and neighbourhoods. The definition of a topological space relies only upon set theory and it is the most general notion of a mathematical space that allows for the definition of concepts such as continuity, connectedness and convergence. Other spaces such as manifolds and metric spaces are specilizations of topolgical spaces with extra structures or constraints. Being so general, topological spaces are a central unifying notions and appear in virtually every branch of modern mathematics. The branch of mathematics that studies topological spaces in their own right is called general topology.

In 1970, Levine [8] introduced the concept of generalized closed set. In 1986 and later in 1996 Andrjivic [2,3] gave new types of open sets in topological spaces called b-open sets. Motivated by these papers, Al-Omari et al. [1] introduced and studied the concept of generalized b-closed sets in topological spaces and thereafter, in various papers Indira et al. [4,5,6,7] introduced different types open and closed sets and analyzed their properties. Recently, Parameswari et al. [9] introduced notions of $b^{\#}$-open sets and $b^{\#}$-closed sets by taking equality in the definitions of b-open sets and b-closed sets. Vithya et al. [10] introduced the notion of generalized $b^{\#}$-closed sets and discussed the properties of such sets. In this paper the notion of regular generalized $b^{\#}$-closed set is introduced and their basic properties are discussed.

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A and int(A) denotes the interior of A in X.

2. Preliminaries

Definition 2.1. A subset A of a space X is said to be

(i) b-open if $A \subseteq cl(int(A)) \cup int(cl(A))$ and b-closed if $cl(int(A)) \cap int(cl(A)) \subseteq A$.
(ii) $b^{\#}$-open if $A = cl(int(A)) \cup int(cl(A))$ and $b^{\#}$-closed if $A = cl(int(A)) \cap int(cl(A))$.
(iii) semi-open if $A \subseteq cl(int(A))$.
(iv) regular open if $A = int(cl(A))$.
(v) a p-set if $cl(int(A)) \subseteq int(cl(A))$.
(vi) π-open if A is a finite union of regular open sets.
(vii) semi pre-open if $A \subseteq cl(int(cl(A)))$.
(viii) pre-open if $A \subseteq int(cl(A))$.

Definition 2.2. A subset A of a space X is called
(i) generalized closed (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(ii) generalized semi-pre-regular-closed (briefly gspr-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
(iii) π-generalized semi-pre-closed (briefly πgsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open.
(iv) π-generalized pre-closed (briefly πgp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open.
(v) generalized pre-regular-closed (briefly gpr-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
(vi) regular weakly generalized closed (briefly rwg-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
(vii) regular generalized b-closed set (briefly rgb-closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
(viii) π-generalized b-closed set (briefly πgb-closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open.
(ix) regular generalized closed set (briefly rg-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
(x) π-generalized closed set (briefly πg-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open.
(xi) generalized $b^{\#}$-closed (briefly $gb^{\#}$-closed) if $b^{\#}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Theorem 2.3. [9] Let A be a subset of a space X. Then
(i) $scl(A) = A \cup int(cl(A))$.
(ii) $pcl(A) = A \cup cl(int(A))$.
(iii) $spcl(A) = A \cup int(cl(int(A)))$.
(iv) $\alpha cl(A) = A \cup cl(int(cl(A)))$.

Theorem 2.4. [9] Let A be a subset of a topological space X. Then $cl(int(A)) \cap int(cl(A)) \subseteq bcl(A) \subseteq b^{\#}cl(A)$.

Theorem 2.5. [9] (i) If A is a p-set, then $cl(int(A)) \subseteq b^{\#}cl(A)$.
(ii) If A is a q-set, then $int(cl(A)) \subseteq b^{\#}cl(A)$.

3. rgb$^{\#}$-closed sets and their properties

Definition 3.1. Let X be a space. A subset A of a space X is called regular generalized b$^{\#}$-closed (briefly rgb$^{\#}$-closed) if b$^{\#}$cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open.

Example 3.2. Let X = {a ,b ,c} with the topology τ = {X , ϕ , {a} ,{b}, {a ,b}}. In this topological space the subset {a, b} is rgb$^{\#}$-closed.

Theorem 3.3. Every closed set is rgb$^{\#}$-closed.

Proof: Let A be any closed set in X such that A $\subseteq$ U where U is regular open. Then b$^{\#}$cl(A) $\subseteq$ cl(A) = A $\subseteq$ U. Therefore b$^{\#}$cl(A) $\subseteq$ U. Hence A is rgb$^{\#}$-closed.

Remark 3.4. The converse of the above theorem is not true as seen from the following example.

Example 3.5. Let X = {a ,b ,c} with the topology τ = {X , ϕ ,{a} ,{b}, {a ,b}}. In this topological space the subset {a ,b} is rgb$^{\#}$-closed but not a closed set.

Theorem 3.6. Every gb$^{\#}$- closed set is rgb$^{\#}$-closed.

Proof: Let A be any gb$^{\#}$- closed set in X such that A $\subseteq$ U where U is regular open. Since every regular open set is open, U is open. Since A is gb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Hence A is rgb$^{\#}$-closed.

Remark 3.7. The converse of the above theorem is not true which is shown in the following example.

Example 3.8. Let X = {a , b , c} with the topology τ = {X , ϕ , {a} , {b}, {a , b}}. In this topological space the subset {a , b} is rgb$^{\#}$-closed but not b$^{\#}$- closed.

Theorem 3.9. If A is rgb$^{\#}$- closed set, then A is rgb- closed.

Proof: Let A be any rgb$^{\#}$ closed set in X. Let A $\subseteq$ U where U is regular open. Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Since bcl(A) $\subseteq$ b$^{\#}$cl(A), bcl(A) $\subseteq$ U. Therefore, bcl(A) $\subseteq$ U and U is regular open. Hence A is rgb- closed.

Theorem 3.10. If A is rgb$^{\#}$-closed set, then A is πgb-closed.

Proof: Let A be any rgb$^{\#}$- closed set in X. Let A $\subseteq$ U where U is π-open. Since every π-open set is regular open, U is regular open. Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Since bcl(A) $\subseteq$ b$^{\#}$cl(A), bcl(A) $\subseteq$ U. Therefore, bcl(A) $\subseteq$ U and U is π-open. Hence A is πgb-closed.

Theorem 3.11. If A is g-closed, then A is rgb$^{\#}$-closed.

Proof: Let A be any g-closed set in X such that A $\subseteq$ U and U is regular open. Since every regular open set is open, U is open. Also b$^{\#}$cl(A) $\subseteq$ cl(A) $\subseteq$ U. Therefore, b$^{\#}$cl(A) $\subseteq$ U and U is regular open. Hence A is rgb$^{\#}$- closed.

The Figure 1 gives the relation between rgb$^{\#}$-closed sets and other sets.

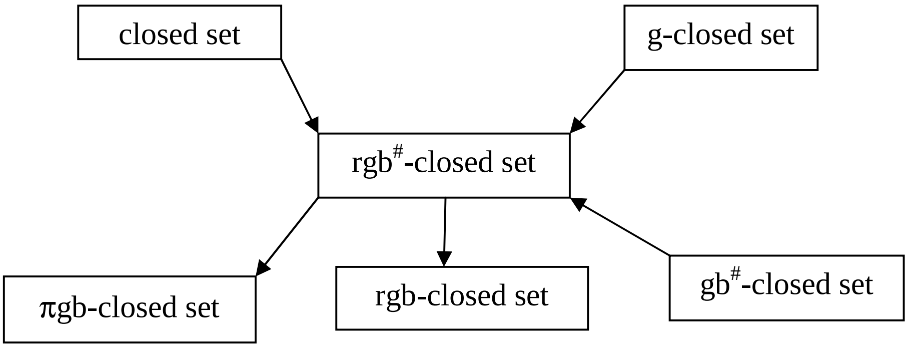

Figure 1: relation between rgb$^{\#}$-closed sets and other sets

Theorem 3.12. Suppose A is a p-set and rgb$^{\#}$-closed. Then A is gpr-closed.
Proof: Let A be any p-set and rgb$^{\#}$-closed. Then cl(int(A)) $\subseteq$ b$^{\#}$cl(A).
Let A $\subseteq$ U. Suppose U is regular open. Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Therefore, cl(int(A)) $\subseteq$ U. This implies that A $\cup$ cl(int(A)) $\subseteq$ U. Therefore pcl(A) $\subseteq$ U and U is regular open. Hence A is gpr-closed.

Theorem 3.13. Suppose A is a p-set and rgb$^{\#}$-closed. Then A is gspr-closed.
Proof: Let A be any p-set and rgb$^{\#}$-closed. Then cl(int(A)) $\subseteq$ b$^{\#}$cl(A). Let A $\subseteq$ U. Suppose U is regular open. Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Therefore, cl(int(A)) $\subseteq$ U. Since every regular open set is open, int(cl(int(A))) $\subseteq$ int(U) $\subseteq$ U,. This implies that A $\cup$ int(cl(int(A))) $\subseteq$ U , because A $\subseteq$ U. Therefore spcl(A) $\subseteq$ U and U is regular open. Hence, A is gspr-closed.

Theorem 3.14. Suppose A is a p-set and rgb$^{\#}$-closed. Then A is rwg-closed.
Proof: Let A be any p-set and rgb$^{\#}$-closed. Then cl(int(A)) $\subseteq$ b$^{\#}$cl(A). Let A $\subseteq$ U. Suppose U is regular open. Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. Therefore, cl(int(A)) $\subseteq$ U and U is regular open. Hence A is rwg-open.

Theorem 3.15. Suppose A is a p-set and rgb$^{\#}$-closed. Then A is πgp-closed.
Proof: Let A be any p-set and rgb$^{\#}$-closed. Let A $\subseteq$ U. Suppose U is π-open. Since every π-open set is regular open, U is regular open.

Since A is a p-set, cl(int(A)) $\subseteq$ b$^{\#}$cl(A). Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. This implies that cl(int(A)) $\subseteq$ U. Hence A $\cup$ cl(int(A)) $\subseteq$ U. Therefore pcl(A) $\subseteq$ U and U is π-open. Hence, A is πgp-closed.

Theorem 3.16. Suppose A is a p-set and rgb$^{\#}$-closed. Then A is πgsp-closed.
Proof: Let A be any p-set and rgb$^{\#}$-closed. Let A $\subseteq$ U. Suppose U is π-open. Since every π-open set is regular open, U is regular open. Since A is a p-set, cl(int(A)) $\subseteq$ b$^{\#}$cl(A). Since A is rgb$^{\#}$-closed, b$^{\#}$cl(A) $\subseteq$ U. This implies that cl(int(A)) $\subseteq$ U. Since every regular open set is open, int(cl(int(A))) $\subseteq$ int(U) $\subseteq$ U. Since A $\subseteq$ U, A $\cup$ int(cl(int(A)) $\subseteq$ U,. Therefore spcl(A) $\subseteq$ U and U is π-open. Hence A is πgsp-closed.

The Figure 2 gives the relation between p-set and rgb$^{\#}$-closed sets with other sets.

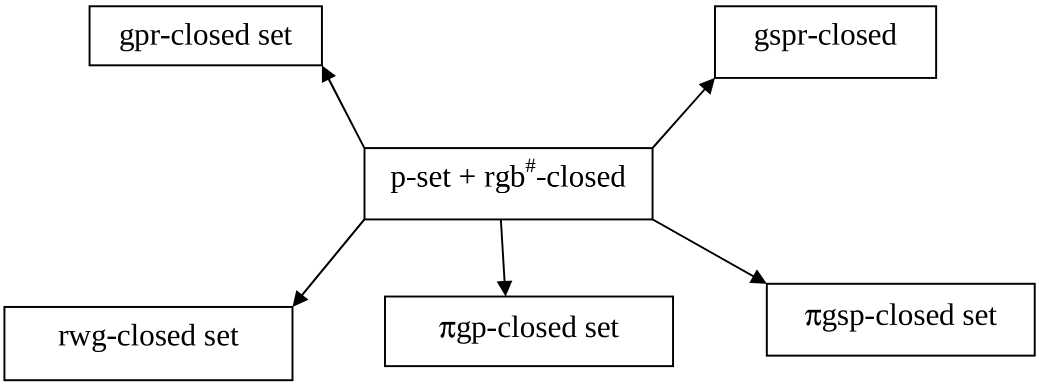

Figure 2: Relation between p-set and rgb$^{\#}$-closed sets with other sets

Theorem 3.17. Suppose A is a p-set that is semi-open and rgb$^{\#}$-closed. Then A is rg-closed.

Proof: Let A be any p-set, semi-open and rgb$^{\#}$-closed. Since A is a p-set, $cl(int(A)) \subseteq b^{\#}cl(A)$. Since A is semi-open, $A \subseteq cl(int(A))$. This implies that $A \subseteq b^{\#}cl(A)$. Since b$^{\#}$-closed sets are closed, $cl(A) \subseteq b^{\#}cl(A)$. Since A is rgb$^{\#}$-closed, $cl(A) \subseteq U$. Therefore A is rg-closed.

Theorem 3.18. Suppose A is a p-set which is semi-open and rgb$^{\#}$-closed. Then A is πg-closed.

Proof: Let A be any p-set, semi-open and rgb$^{\#}$-closed. Let $A \subseteq U$. Suppose U is π-open. Since every π-open set is regular open, U is open. Since A is a p-set, $cl(int(A)) \subseteq b^{\#}cl(A)$. Since A is semi-open, $A \subseteq cl(int(A))$. This implies that $A \subseteq b^{\#}cl(A)$. Since b$^{\#}$-closed sets are closed, $cl(A) \subseteq b^{\#}cl(A)$. Since A is rgb$^{\#}$-closed, $cl(A) \subseteq U$. Therefore A is πg-closed.

The Figure 3 gives the relation between p-set, semi open and rgb$^{\#}$-closed sets with other sets.

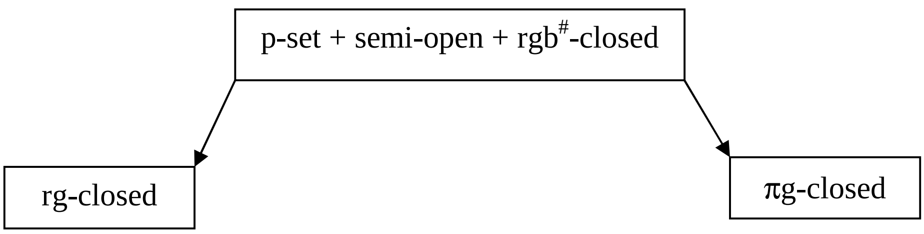

Figure 3: Relation between p-set, semi open and rgb$^{\#}$-closed sets with other sets

Theorem 3.19. If A is rgb$^{\#}$-closed, then $b^{\#}cl(A) - A$ contains no non-empty regular closed set.

Proof: Let F be any regular closed set in X such that $F \subseteq b^{\#}cl(A) - A$. This implies that $F \subseteq b^{\#}cl(A) \cap A^{c}$. This implies that $F \subseteq b^{\#}cl(A)$ and $F \subseteq A^{c}$. Then $A \subseteq X - F$. Since A is rgb$^{\#}$-closed and X – F is regular open, $b^{\#}cl(A) \subseteq X - F$. That is $F \subseteq b^{\#}cl(A)$.
Hence, $F \subseteq (X - b^{\#}cl(A)) \cap (b^{\#}cl(A))$.
Therefore, F is empty.

Theorem 3.20. Let A be a rgb$^{\#}$-closed set in X and B be such that $A \subseteq B \subseteq b^{\#}cl(A)$. Then $b^{\#}cl(B) - B$ contains no non empty regular closed set in X.
Proof: Let A be any rgb$^{\#}$ closed set in X. Since $B \subseteq b^{\#}cl(A)$, $b^{\#}cl(B) \subseteq b^{\#}cl(A)$. Since A is rgb$^{\#}$-closed, $b^{\#}cl(A) = A$. Therefore $b^{\#}cl(B) - B \subseteq b^{\#}cl(A) - A$. Since $b^{\#}cl(A) - A$ contains no non empty regular closed set, $b^{\#}cl(B) - B$ contains no non empty regular closed set.

Theorem 3.21. Let A be a rgb$^{\#}$-closed set. Then $b^{\#}cl(A) = A$ if and only if $b^{\#}cl(A) - A$ is regular closed.
Proof: If $b^{\#}cl(A) = A$, then we have $b^{\#}cl(A) - A = \phi$ which is regular closed.
Conversely, suppose $b^{\#}cl(A) - A$ is regular closed. Then by theorem 3.19, $b^{\#}cl(A) - A$ does not contains any non-empty regular closed set. Since $b^{\#}cl(A) - A$ is regular closed, $b^{\#}cl(A) - A = \phi$.

Since $A \subseteq b^{\#}cl(A)$, it follows that $b^{\#}cl(A) = A$.

4. Conclusion

In this paper, we introduced the regular form of generalized b$^{\#}$-closed sets, studied their properties and established the relationship with other sets.

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