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Results on Relatively Prime Dominating Sets in Graphs

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Abstract. In this paper we introduce relatively prime dominating set of a graph G. Let G be a non–trivial graph. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that (deg u, deg v) = 1. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. If there is no such pair exist then $\gamma_{rpd}(G) = 0$. We characterize connected unicyclic graphs with $\gamma_{rpd}(G)$ =2 and also we prove that $\gamma_{rpd}(K_{m,n}) = 2$ iff (m, n) = 1 and $\gamma_{rpd}(\overline{P}_n) = 2$ for $n \geq 4$.

Keywords: Dominating set, relatively prime dominating set, unicyclic graph.

AMS Mathematics Subject Classification (2010): 05C69

1. Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms, we refer to Harary [2] and for terms related to domination we refer to Haynes [3]. A subset S of V is said to be a dominating set in G if every vertex in V- S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G.

Berge and Ore [1,6] formulated the concept of domination in graphs. It was further extended to define many other domination related parameters in graphs.

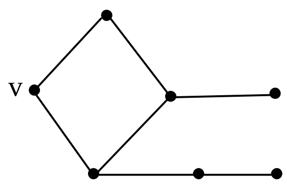
A graph which contains exactly one cycle is called a unicyclic graph. A branch at v in G is a maximal connected subgraph B of G such that the intersection of B with the vertex v is v and B-v is connected [8]. The distance d(u, v) between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. The diameter of a connected graph G is the maximum distance between two vertices of G and is denoted by diam(G). Many other domination parameters in domination theory were studied in [5, 7]. In this paper we define relatively prime dominating set $\gamma_{rpd}(G)$ and initiate a study of this parameter. We obtain $\gamma_{rpd}(G)$ for various classes of graphs. Now consider the following results, which are required in the subsequent section.

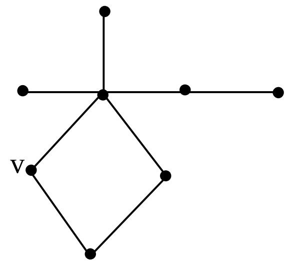
Theorem 1.1. [1] For any G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \le \gamma(G) \le p - \Delta(G)$.

Theorem 1.2. [6] If a graph G has no isolated vertices then $\gamma(G) \le \frac{p}{2}$.

Notations. Consider a cycle $C_r = (v_1, v_2, \ldots, v_r)$ (clock-wise). For our convenience we denote it by $C_{r(v_1)}$. Identifying an end vertex of paths P_m at v_i and P_s at v_j, then $C_{r(v_1)}$ is denoted by $C_{r(v_1)}$ $(0, \ldots, P_m, 0, \ldots, P_s, 0, \ldots, 0)$. Identifying an end vertex of paths P_m and P_s at the vertex v_j, then $C_{r(v_1)}$ is denoted by $C_{r(v_1)}$ $(0, \ldots, P_m \cup P_s, 0, \ldots, 0)$.

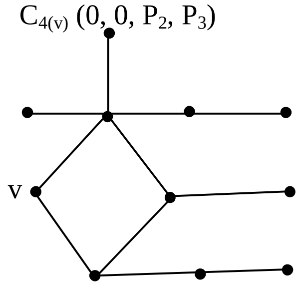
The graphs $C_{4(v)}$ $(0, 0, P_2, P_3)$, $C_{4(v)}$ $(0, 2P_2 \cup P_3, 0, 0)$, $C_{4(v)}$ $(0, 2P_2 \cup P_3, P_2, P_3)$ and $C_{3(v)}$ $(4P_2, 2P_2, 3P_2)$ are given in figure 1.

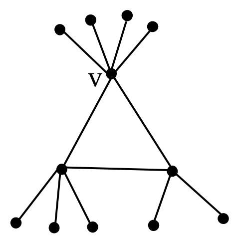




$C_{4(v)}$ $(0, 0, P_2, P_3)$

$C_{4(v)}$ $(0, 2P_2 \cup P_3, 0, 0)$





$C_{4(v)}$ $(0, 2P_2 \cup P_3, P_2, P_3)$

$C_{3(v)}$ $(4P_2, 2P_2, 3P_2)$

Figure 1:

Result 1.3. For $n \ge 2$, $\gamma(\overline{P_n}) = 2$.

2. Definition and example

Definition 2.1. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that (deg u, deg v) = 1. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$.

Example 2.2. Consider the connected graph G given in figure 2. Clearly $\{v_1, v_4, v_6\}$ is a minimal dominating set and $(d(v_1), d(v_4)) = (1, 3) = 1$, $(d(v_1), d(v_6)) = (1, 5) = 1$and $(d(v_4), d(v_6)) = (3, 5) = 1$. By definition, $\{v_1, v_4, v_6\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G) = 3$. Also $\gamma(G) = \gamma_{rpd}(G) = 3$.

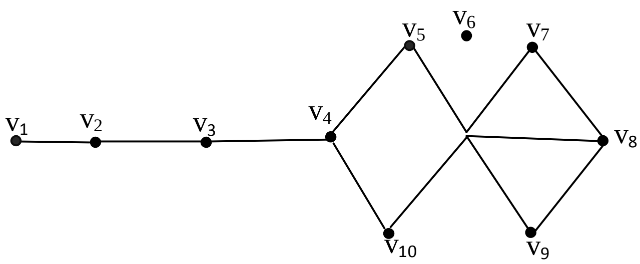


Figure 2: The graph G

Example 2.3. Consider the disconnected graph G given in figure 3. Clearly $\{v_1, v_3\}$ is a dominating set and $(d(v_1), d(v_3)) = (1, 2) = 1$. By definition, $\{v_1, v_3\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(G) = 2$.

Figure 3:

Example 2.4. For n = 2, $\gamma_{rpd}(K_n) = 2$ and for n > 2, $\gamma_{rpd}(K_n) = 0$.

Observations 2. 5.

1. For any graph G of order at least 2, $\gamma_{rpd}(G) \geq 0$ and $\gamma_{rpd}(G) \neq 1$.
2. If $\gamma_{rpd}(G) \neq 0$ then $\gamma(G) \leq \gamma_{rpd}(G)$.
3. For any k-regular graph (k >1) G, $\gamma_{rpd}(G) = 0$.

Example 2.6. Consider the connected graph G given in figure 4. Clearly $\{v_2, v_5\}$ is a minimal dominating set and hence $\gamma(G) = 2$. Also $\{v_1, v_3, v_5\}$ is a minimal relatively prime dominating set and hence $\gamma_{rpd}(G) = 3 > \gamma(G)$.

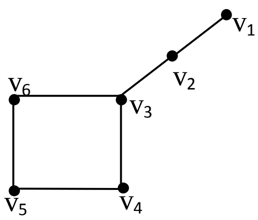
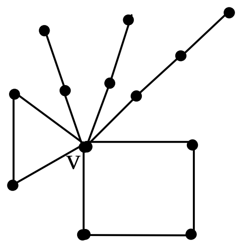


Figure 4:

Definition 2.7. [6] A branch at v in G is a maximal connected subgraph B of G such that the intersection of B with the vertex v is v and B-v is connected.

Notation 2.8. [6] Let v be a cut vertex of a connected graph G. Let $B_1, B_2, \ldots, B_k$ be the branches with $n_1, n_2, \ldots, n_k$ number of copies at v in G, respectively. In this case we denote the graph G by $G(v; n_1B_1, n_2B_2, \ldots, n_kB_k)$.

Example 2.9. Consider the graph G given in figure 5.0. There are four distinct branches B_1, B_2, B_3 and B_4 at v in G and they are given in figures 5.1, 5.2, 5.3 and 5.4, respectively. Therefore, $G = G(v; 2B_1, B_2, B_3, B_4)$.



$G(v; 2B_1, B_2, B_3, B_4)$

Figure 5.0: G

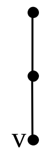


Figure 5.1: B_1

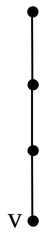


Figure 5.2: B_2

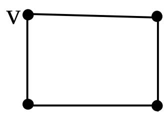


Figure 5.3: B_3

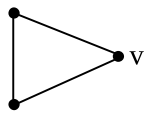


Figure 5.4: B_4

3. Main results

Theorem 3.1. For a connected unicyclic graph G, $\gamma_{rpd}(G) = 2$ iff G is one of the graphs

(1) $C_{3(v_1)}(mP_2, nP_2, 0)$ where $(m+2, n+2) = 1$ and $m, n \geq 1$

(2) $C_{3(v_1)}(K_{1,m}, nP_2, 0)$where$(m, n+2) = 1$ and $m \geq 2, n \geq 1$

(3) $C_{3(v_1)}(mP_2, 0, 0)$ and $m \geq 1$

(4) $C_{3(v_1)}(K_{1,m}, 0, 0)$ where either m is odd or even but not a multiple of 3

(5) $C_{3(v_1)}(mP_2 \cup K_{1,n}, 0, 0)$ where $(m+3, n) = 1$ and $m \geq 1, n \geq 2$

(6) $C_{4(v_1)}$(mP_2, nP_2, 0, 0) where (m+2, n+2)=1 and m, n ≥ 1

(7) $C_{4(v_1)}$(mP_2, 0, nP_2, 0) where (m+2, n+2) = 1 and m, n ≥ 1

(8) $C_{4(v_1)}$(mP_2, 0, $K_{1,n}$, 0) where (m+2, n+1) = 1 m$\geq$1and n$\geq$2

(9) $C_{4(v_1)}$(mP_2, 0, 0, 0) where m is odd

(10) $C_{4(v_1)}$($K_{1,m}$, 0, 0, 0) where m is odd

(11) $C_{5(v_1)}$(mP_2, 0, nP_2, 0 , 0)where (m+2, n+2) = 1 and m, n ≥ 1

(12) $C_{5(v_1)}$(mP_2, 0, 0, 0, 0) where m is odd

(13) $C_{6(v_1)}$(mP_2, 0, 0, nP_2, 0, 0)where(m+2, n+2) =1 and m, n ≥ 1

(14) $C_{6(v_1)}$(mP_2, 0, 0, 0, 0, 0) where (m+2, 2) =1 and m is odd

Proof: Let G be a connected unicyclic graph with cycle C_n. Let $\gamma_{rpd}(G) = 2$. If n $\geq$ 7 then the cardinality of every dominating set is ≥ 3 and hence n $\leq$ 6.

Case 1. n = 3

Let $v_1v_2v_3v_1$ be the cycle C_3. If d(v_i) $\geq$ 3, $1 \leq i \leq 3$, then $\gamma_{rpd}(G) \geq 3$. Hence at most two vertices, say $v_1$and v_2 can have degree $\geq$ 3.

Subcase 1(a) d(v_1) $\geq$ 3 and d(v_2) $\geq$3

In this case, d(v_3) = 2. Since G is unicyclic graph, the branches at v_1 and v_2 are trees other than the branch which contains the cycle C_3. If all branches are P_2 then G is $C_{3(v_1)}$(mP_2, nP_2, 0) where m and n are the number of P_2's at v_1 and v_2, respectively. If (m+2, n+2) = 1 then {v_1,v_2} is a relatively prime dominating set of G. Thus G is $C_{3(v_1)}$(mP_2, nP_2, 0)where (m+2, n+2) = 1and m,n $\geq$ 1.

Clearly at most one branch can be different from P_2, otherwise $\gamma_{rpd}(G) \geq 3$. Let B $\neq P_2$ be the tree branch at v_1. Let u be the vertex adjacent to v_1 in B. Since d(v_2)$\geq$3, any relatively prime dominating set must contain v_2. If d(v_1) $\geq$ 4 then $\gamma_{rpd}(G) \geq 3$. Hence d(v_1) = 3. Since $\gamma_{rpd}(G) = 2$ and d(u)$\geq$ 2, any relatively prime dominating set must contain u. Clearly all branches at u are P_2's other than the branch which contains the cycle C_3. In this case G is $C_{3(v_1)}$ ($K_{1,m}$, nP_2, 0). If (m, n+2) = 1 then {u, v_2} is a relatively prime dominating set of G. Thus G is $C_{3(v_1)}$($K_{1,m}$, nP_2, 0) where (m, n+2) = 1 and m$\geq$ 2, n $\geq$ 1.

Subcase 1(b) d(v_1) $\geq$ 3

In this case, d(v_2) = d(v_3) = 2. If all branches at v_1 other than the cyclic branch are P_2then {u, v_1} is a relatively prime dominating set of G where u is a vertex adjacent to v_1 in P_2. In this case G is $C_{3(v_1)}$(mP_2, 0, 0) where m $\geq$1.

Suppose at least two branches at v_1 other than the cyclic branches are not P_2then $\gamma_{rpd}(G) \geq 3$. Hence exactly one tree branch, say B at v_1 is not P_2. Let u be the vertex adjacent to v_1 in B. If d(u)$\geq$ 2 then each branch at u is P_2 other than the cyclic branch which contains C_3 otherwise $\gamma_{rpd}(G) \geq 3$. In this case G = $C_{3(v_1)}(K_{1,m}, 0, 0)$. Clearly {u, v_1} and {u, v_2} and{u, v_3} are minimal dominating sets. If m is odd then (m, 2)=1 and if m is even and m is not a multiple of 3 then (m, 3) = 1. Thus G = $C_{3(v_1)}(K_{1,m}, 0, 0)$ when m is odd or m is even but not a multiple of 3.

If G has more than one tree branch at v_1 in G then G = $C_{3(v_1)}(mP_2 \cup K_{1,n}, 0, 0)$. If (m+3,n)=1 then {$v_1$,u} is a relatively prime dominating set of G. Thus G = $C_{3(v_1)}(mP_2 \cup K_{1,n}, 0, 0)$ where (m+3, n)=1and m $\geq$1, n$\geq$2.

Case 2. n = 4

Let $v_1v_2v_3v_4v_1$ be the cycle C_4. If $d(v_i) \geq 3$ for any three vertices v_i then $\gamma_{rpd}(G) \geq 3$ and hence at most two vertices can have degree $\geq$ 3.

Subcase 2.1. Two vertices have degree $\geq$3

Here we consider the two sub cases either they are adjacent or non-adjacent.

Subcase 2.1(a) Adjacent

Let the vertices be v_1 and v_2. Then $d(v_1) \geq 3$ and $d(v_2) \geq 3$. In this case $d(v_3) = d(v_4) = 2$. Since G is unicycle graph, the branches at v_1 and v_2 are trees other than the branch which contains the cycle C_4. Suppose B $\neq P_2$ be a branch at $v_1$in G. Then any dominating set has at least one vertex from B other than v_1, the vertex v_2, since $d(v_2) \geq 3$ and v_3. This implies that $\gamma_{rpd}(G) \geq 3$ and hence each branch at v_1 is P_2.Similarly, each branch of v_2 is P_2. Hence G = $C_{4(v_1)}(mP_2, nP_2, 0, 0)$ where m and n are the number of P_2's at v_1 and v_2, respectively. Clearly {v_1, v_2} is aminimal dominating set. If(m+2, n+2) = 1 then {v_1,v_2} is a relatively prime dominating set. Thus G = $C_{4(v_1)}(mP_2, nP_2, 0, 0)$where (m+2, n+2) = 1 and m, n $\geq$ 1.

Subcase 2.1(b) Non–adjacent

Let the vertices be v_1 and v_3. Then $d(v_1) \geq 3$ and $d(v_3) \geq 3$. In this case $d(v_2) = d(v_4) = 2$. Since G is unicycle graph, the branches at v_1 and $v_3$are trees other than the branch which contains the cycle C_4. If all the branches at v_1 and v_3 are P_2's other than the branch which contains the cycle C_4then G = $C_{4(v_1)}(mP_2, 0, nP_2, 0)$. If (m+2, n+2) = 1 then {v_1, v_3} is a relatively prime dominating set. Thus G = $C_{4(v_1)}(mP_2, 0, nP_2, 0)$where (m+2, n+2) = 1 and m, n $\geq$ 1.

Suppose there are two tree branches B_1 and B_2 which are not P_2 either at v_1 or v_2 or both. Then any dominating set has at least one vertex from each of the branches B_1 and B_2 other than v_1 and v_3 and at least two vertices from {v_1, v_2, v_3, v_4} and hence

$\gamma_{rpd}(G) \geq 4$ which is a contradiction. Therefore, at most one tree branch say B $\neq P_2$ at v_3 in G. If there is a branch P_2 other than B at v_3 in G then any dominating set has the vertices v_1, v_3 and at least one vertex from V(B) – v_3 and hence $\gamma_{rpd}(G) \geq 3$ which is a contradiction. Therefore, exactly one tree branch namely B $\neq P_2$ at v_3 in G. Let u be the vertex adjacent to v_3 in G. Since $d(v_1) \geq 3$ and all branches at v_1 are P_2, any minimal dominating set contains the vertex v_1. Since $\gamma_{rpd}(G) = 2$ and d(u) $\geq$ 2 the minimal dominating set is $\{v_1, u\}$ and all the vertices adjacent to u other than v_3 are end vertices and hence B = $K_{1,n}$, n $\geq$ 2. This implies that G = $C_{4(v_1)}$(mP_2, 0, $K_{1,n}$, 0). If (m+2, n+1) = 1 then $\{u, v_1\}$ is a relatively prime dominating set. Thus G = $C_{4(v_1)}$(mP_2, 0, $K_{1,n}$, 0) where (m+2, n+1) = 1 m $\geq$ 1 and n $\geq$ 2.

Subcase 2.2. One vertex has degree ≥ 3

Let the vertex be v_1 with $d(v_1) \geq 3$. Then $d(v_2) = d(v_3) = d(v_4) = 2$. If diam(G) $\geq$ 6 then cardinality of every dominating set is ≥ 3. Therefore, diam(G) $\leq$ 5. Since $d(v_1) \geq 3$, diam(G) $\geq$ 4. Therefore, diam(G) = 4 or 5. If diam(G) = 4 then any branch B at v_1 other than the cyclic branch is P_2. Let m be the number of P_2's at v_1. Then G = $C_{4(v_1)}$(mP_2,0, 0,0). If (m+2, 2) = 1 then m is odd and hence $\{v_1, v_3\}$ is a relatively prime dominating set. Thus G = $C_{4(v_1)}$(mP_2, 0, 0, 0) where m is odd. Suppose diam(G) = 5 and $d(v_1)>3$. Since G is unicyclic, each branch at v_1 other than C_4 is a tree. Since diam(G) = 5, one tree branch at v_1 must be $K_{1,m}$ (m $\geq$ 2) with centre u $\neq v_1$. If $d(v_1)$ >3 then any minimal dominating set has u, v_1 and one of v_2, v_3 and v_4 and hence $\gamma_{rpd}(G)$ >2. This implies that $d(v_1) = 3$ and G = $C_{4(v_1)}$($K_{1,m}$, 0, 0, 0), m $\geq$ 2. Clearly, $\{u, v_3\}$ is a minimal dominating set with d(u) = m and $d(v_3) = 2$. If m is odd then (m, 2) = 1 and hence $\{u, v_3\}$ is a relatively prime dominating set and $\gamma_{rpd}(G) = 2$.

Case 3. n = 5

Let $v_1v_2v_3v_4v_5v_1$ be the cycle C_5. If $d(v_i) \geq 3$ for at least 3 vertices v_i(i =1 to 5) then $\gamma_{rpd}(G) \geq 3$ and hence at most two v_i's can have degree $\geq$ 3.

Subcase 3.1. Two vertices have degree $\geq$ 3

Let v_i and v_j ($i \neq j$) be the vertices such that $d(v_i) \geq 3$ and $d(v_j) \geq 3$, $1 \leq i, j \leq 5$. Since G is unicyclic, each branch at v_i and v_j is a tree. Suppose B $\neq P_2$is a branch at v_i. Then any minimal dominating set has two vertices from $\{v_1, v_2, v_3, v_4, v_5\}$ and one vertex from V(B)–v_i and hence $\gamma_{rpd}(G) \geq 3$ which is a contradiction. Therefore, each branch at v_i is P_2. Similarly each branch at v_j is P_2. If $d(v_i, v_j) = 1$ then $\gamma_{rpd}(G) \geq 3$ which is a contradiction. Hence $d(v_i, v_j)$ is either 2 or 3. Without loss of generality we may assume that $d(v_i, v_j) = 2$ and let $v_i = v_1$ and $v_j = v_3$. This implies that G = $C_{5(v_1)}$ (mP_2, 0, nP_2,0, 0). If (m+2, n+2) =1 then $\{v_1, v_3\}$ is a relatively prime dominating set of G. Thus G = $C_{5(v_1)}$(mP_2, 0, nP_2, 0, 0) where (m+2, n+2) = 1 and m, n $\geq$ 1.

Subcase 3.2. One vertex has degree ≥ 3

Without loss of generality, let $d(v_1) \geq 3$. Then each branch at v_1 other than the branch which contains C_5 is P_2. Then G = $C_{5(v_1)}$ (mP_2, 0, 0, 0, 0). If m is odd then {v_1, v_3} and {v_1, v_4} are relatively prime dominating sets of G. Thus G = $C_{5(v_1)}$ (mP_2, 0, 0, 0, 0) and m is odd.

Case 4. n = 6

Let $v_1v_2v_3v_4v_5v_6v_1$ be the cycle C_6. If $d(v_i) \geq 3$ for at least 3 vertices v_i (i =1 to 6) then $\gamma_{rpd}(G) \geq 3$ and hence at most two v_i's can have degree ≥ 3.

Subcase 4.1. Two vertices have degree ≥ 3

Let v_i and v_j ($i \neq j$) be the vertices such that $d(v_i) \geq 3$ and $d(v_j) \geq 3$, $1 \leq i \neq j \leq 6$. Clearly any minimal dominating set must contain v_i and v_j. If $d(v_i, v_j)$ = 1 or 2, then $\gamma_{rpd}(G) \geq 3$ which is a contradiction. Hence $d(v_i, v_j) = 3$ and let $v_i = v_1$ and $v_j = v_4$.

Clearly all branches at v_1 and v_4 are P_2's other than the cyclic branch otherwise $\gamma_{rpd}(G) \geq 3$ and the only dominating set is {v_1, v_4}. This implies that G = $C_{6(v_1)}$ (mP_2, 0, 0, nP_2, 0, 0). If (m+2, n+2) = 1 then {v_1, v_4} is a relatively prime dominating set of G. Thus G = $C_{6(v_1)}$ (mP_2, 0, 0, nP_2, 0, 0) where (m+2, n+2) = 1 and m, n $\geq$ 1.

Subcase 4.2. One vertex has degree ≥ 3

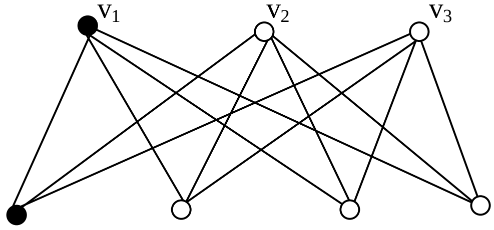
Without loss of generality, let $d(v_1) \geq 3$. Clearly each branch at v_1 other than the branch which contains C_6 is P_2. This implies that G = $C_{6(v_1)}$ (mP_2, 0, 0, 0, 0, 0). If m is odd then {v_1, v_4} is a relatively prime dominating set of G. Thus G = $C_{6(v_1)}$ (mP_2, 0, 0, 0, 0, 0) and m is odd.

Conversely, let G be the graph given in the statement. Then for each graph G, $\gamma_{rpd}(G) = 2$.

Theorem 3.2. For a complete bipartite graph $K_{m,n}$, $\gamma_{rpd}(K_{m,n}) = 2$ iff (m, n) =1.

Proof: Let V_1, V_2 be the bipartition of the vertex set of $K_{m,n}$ with $|V_1|$ = m and $|V_2|$ = n. Clearly, d(u) = n and d(v) = m for $u \in V_1$ and $v \in V_2$. Any minimum dominating set of $K_{m,n}$ has one vertex in V_1 and another vertex in V_2. Hence a minimum dominating set of $K_{m,n}$ becomes a relatively prime dominating set iff (m, n) = 1. Therefore, $\gamma_{rpd}(K_{m,n}) = 2$ iff (m, n) = 1.

Example 3. 3. For $K_{3,4}$, {v_1, u_1} is a relatively prime dominating set.



u_1 u_2 u_3 u_4

Figure 6: $K_{3,4}$

Theorem 3.4. $\gamma_{rpd}(P_n) = \begin{cases} 2 \text{ if } & 2 \le n \le 5 \\ 3 \text{ if } & n = 6, 7 \\ 0 & \text{otherwise} \end{cases}$

Proof: Let $v_1v_2\ldots v_n$ be a path P_n.

Case 1. $2 \le n \le 5$

If n = 2 then $\{v_1, v_2\}$ is the required minimal relatively prime dominating set and hence $\gamma_{rpd}(P_n) = 2$. Let n > 2. In this case $\{v_1, v_{n-1}\}$ is a dominating set. Also $(d(v_1), d(v_{n-1})) = (1, 2) = 1$. Therefore, $\{v_1, v_{n-1}\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(P_n) = 2$.

Case 2. n = 6, 7

In this case $\{v_1, v_4, v_n\}$ is a dominating set. Also $(d(v_1), d(v_4)) = (1, 2) = 1$, $(d(v_1), d(v_n)) = (1, 1) = 1$ and $(d(v_4), d(v_n)) = (2, 1) = 1$. Therefore, $\{v_1, v_4, v_n\}$ is a relatively prime dominating set and hence $\gamma_{rpd}(P_n) = 3$.

Case 3. $n \ge 8$

Clearly any dominating set contains at least two internal vertices v_i, v_j, $2 \le i \ne j \le n-1$ and $(d(v_i), d(v_j)) = 2$ which implies that $\gamma_{rpd}(P_n) = 0$. The theorem follows from above three cases.

Theorem 3.5. $\gamma_{rpd}(\overline{P_n}) = \begin{cases} 2 \text{ if } & n \ge 3 \\ 0 & \text{otherwise.} \end{cases}$

Proof: If n = 2 then $\overline{P_2} = \overline{K_2}$ which is a regular graph of degree 0 and hence $\gamma_{rpd}(\overline{P_2}) = 0$. If n = 3 then $\overline{P_3} = K_2 \cup K_1$ and hence $\gamma_{rpd}(\overline{P_3}) = 2$. Let $n \ge 4$.Let $v_1v_2\ldots v_n$ be a path P_n. In $\overline{P_n}$, $v_1$is adjacent to all vertices except v_2. Clearly $\{v_1, v_2\}$ is a dominating set of $\overline{P_n}$. In $\overline{P_n}$, v_1 has degree n-2 and v_2 has degree n-3. Since (n-2, n-3) = 1, $\{v_1, v_2\}$ is a relatively prime dominating set for $\overline{P_n}$ and hence $\gamma_{rpd}(\overline{P_n}) = 2$ for $n \ge 4$. Thus, the theorem is proved.

Theorem 3.6. If $G_1 \cong G_2$ then $\gamma_{rpd}(G_1) = \gamma_{rpd}(G_2)$.

Proof: Let $G_1 \cong G_2$. Let f be an isomorphism between graphs G_1 and G_2. Let $V(G_1)=\{v_1, v_2,..., v_n\}$. Since $f:V(G_1) \to V(G_2)$ is a bijection, let $V(G_2) =\{f(v_1), f(v_2),...,f(v_n)\}$. Let $\{v_1,v_2,\ldots,v_m\}$be a relatively prime dominating set of G_1. Since f is an isomorphism,

$\{f(v_1), f(v_2),...,f(v_m)\}$ is a dominating set of G_2. Since isomorphism preserves degrees of the vertices, $(d(f(v_i)), d(f(v_j)))=(d(v_i), d(v_j)) = 1$ for $i \neq j$, $1 \leq i \leq j \leq m$. Therefore, $\{f(v_1), f(v_2), ..., f(v_m)\}$ is a relatively prime dominating set of G_2 and hence $\gamma_{rpd}(G_1) = \gamma_{rpd}(G_2)$.

Note 3.7. Converse of theorem 3.7 is not true. For example, consider the graphs G_1 and G_2 given in figure 7 and figure 8, respectively. Here $\gamma_{rpd}(G_1) = \gamma_{rpd}(G_2) = 2$, but the two graphs are not isomorphic.

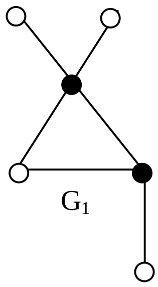


Figure 7:

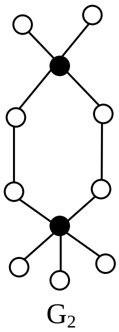


Figure 8:

4. Conclusion

In this paper, we surveyed selected results on relatively prime dominating sets in graphs. These results establish key relationships between the relatively prime numbers and the dominating sets in graphs. Further we characterize connected unicyclic graphs with $\gamma_{rpd}(G) = 2$. In Theorem 3.2, we prove that for a complete bipartite graph $K_{m,n}$, the relatively prime domination number is 2 iff $(m, n) = 1$. We also extend the results for P_n and $\overline{P}_n$. Finally, we have proved that if two graphs are isomorphic then their relatively prime domination numbers must be same.

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