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Generalized Exp $(\varphi(\xi))$ -expansion Method for Solving Non-linear Evolution Equations

***Md. Abdus Salam*[1] and *Umme Habiba*[2]**

Department of Mathematics, Mawlana Bhashani Science and Technology University
Tangail-1902, Bangladesh.
[2]E-mail: mahinspr@gmail.com
Corresponding author. [1]E-mail: salam.a.math03@gmail.com

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Abstract. In this work, the generalized $\exp(\varphi(\xi))$ -expansion method is used to find traveling wave solutions of Korteweg-de Varies equation (KdV) and modified Liouville equation. This method gives travelling wave solutions and finally solitary wave solutions with respective graphs. The generalized $\exp(\varphi(\xi))-$expansion method is very powerful and convenient mathematical tool for finding the exact solutions of nonlinear evolution equations arise in science and engineering.

Keywords: Generalized $\exp(\varphi(\xi))-$expansion method; Korteweg-de Varies equation; Modified Liouville equation; Traveling wave solution; Solitary wave solution.

AMS Mathematics Subject Classification (2010): 35QXX

1. Introduction

Nonlinear evolution equations arise in many fields of sciences including physics, mechanics and material science. A lot of methods have been used to handle nonlinear evolution equations with constant coefficients and time dependent coefficients. Recently many effective methods for finding exact solutions of nonlinear evolution equations have been proposed, such as, the extended tan-method [1], enhanced $(G'/G)-$ expansion method [2], modified Kudryashov method [3], exp-function method [4-5], $\exp(\Phi(\xi))$-expansion method [6], F-expansion method [7], the Tanh method [8], Jacobi elliptic function rational expansion method [9], homotopy perturbation method [10], Modified simple equation method [11], the homogenous balance method [12], the variational method [13]. The objectives of this paper is to apply the generalized $\exp(\varphi(\xi))-$expansion method for finding the exact traveling wave solutions of Korteweg-de Varies equation and modified Liouville equation which play an important role in mathematical physics.

This paper is organized as follows: in section 2, we give the description of the generalized $\exp(\varphi(\xi))-$expansion method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 4, we discuss the results. In section 5, conclusion is given.

2. Description of the generalised $\exp(\varphi(\xi))-$ expansion method

Suppose that a nonlinear partial differential equation, say in two independent variables x and t, is given by $P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \cdots\cdots\cdots) = 0,$ (2.1)

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following the main steps of the $\exp(\varphi(\xi))-$expansion method are given:

Step 1: The traveling wave variable $u(x,t) = u(\xi)$ where $\xi = x - ct$ permits us reducing eq. (2.1) to an ODE for $u = u(\xi)$ in the form

$$P(u, -cu', u', c^2u'', -cu'', u'', \cdots\cdots\cdots) = 0, \tag{2.2}$$

Step 2: Suppose the solution of eq. (2.2) can be expressed in the following form

$$u(\xi) = \sum_{i=0}^{m} a_i (\exp(\varphi(\xi)))^i \tag{2.3}$$

where a_i are constants, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in eq. (2.2), and $\varphi = \varphi(\xi)$ satisfies the following equations:

$$\varphi'(\xi) \pm (\exp\varphi(\xi))^n = 0', \quad n \geq 1 \tag{2.4}$$

Equation (2.4) gives the following solutions of the forms:

$$\varphi(\xi) = -(1/n)\ln[\pm n(\xi + c_1)], n \geq 1 \tag{2.5}$$

Step 3: Now we substitute eq. (2.3) into eq. (2.2) and use eq. (2.4). As a result of this substitution, we get a polynomial of $\exp(\varphi(\xi))$ and equate the coefficients of $\exp(\varphi(\xi))$ to zero. This procedure gives a system of algebraic equations which can be solved to find $a_m, a_{m-1}, \ldots\ldots\ldots, a_0$.

Step 4: Putting the values of $a_m, a_{m-1}, \ldots\ldots\ldots, a_0$ into eq. (2.3) along with general solutions of eq. (2.4) complete the determination of the solutions of eq. (2.1).

3. Applications of the generalized $\exp(\varphi(\xi))-$ expansion method

Now, we will apply the generalized $\exp(\varphi(\xi))-$expansion method described in section 2 to find the exact traveling wave solutions of Korteweg-de Varies equation and modified Liouville equation.

Example 1. Solution of Korteweg-de Varies (KdV) equation

We know that the Korteweg-de Varies equation[14] is

$$u_{xxx} - 6uu_x + u_t = 0 \tag{3.1}$$

Suppose the traveling wave transformation is given by

$$u(x,t) = u(\xi),\ \xi = x - ct \tag{3.2}$$

where c is a constant. Using eq. (3.2), eq. (3.1) is reduced to nonlinear ODE in the form

$$u''' + 6uu' - cu' = 0 \tag{3.3}$$

where the prime denotes the differential with respect to ξ. The corresponding exact solutions of eq. (3.1) for different values of *n* are obtained below:

Case 1($n=1$): Considering the homogeneous balance between u''' and uu' rising in eq. (3.3), we get $m=2$. So from eq.(2.3) the solution can be written as

$$u(\xi)=a_0+a_1\exp\varphi(\xi)+a_2\exp\varphi(\xi)^2 \tag{3.4}$$

where a_0,a_1,a_2 are unknown constants to be determined and $\varphi(\xi)$ satisfies the eq. (2.4) and this function is determined from eq. (2.5) by setting $n=1$, Putting eq. (3.4) in the reduced ODE (3.3) and collecting the coefficients of $\exp(\varphi(\xi))$, we get the system of algebraic equations. Solving these equations by using Maple-13, we get the values of constants: $a_0=(1/6)c, a_1=0, a_2=-2$.

Hence, the solution of eq. (3.3) takes the form: $u_1(\xi)=\dfrac{c}{6}-\dfrac{2}{(\pm c_1\pm\xi)^2}$

Finally, putting $\xi=x-ct$, we get the following desired exact solution of eq. (3.1)

$$u_1(x,t)=\frac{c}{6}-\frac{2}{[\pm c_1\pm(x-ct)]^2} \tag{3.5}$$

Similarly, for $n=2,3,4,....$ the corresponding trail solution of the eq. (3.3), values of constants and exact solutions of eq. (3.1) are given below:

Case 2 ($n=2$): The trail solution of eq. (3.3):

$$u(\xi)=a_0+a_1\exp(\varphi(\xi))+a_2\exp(\varphi(\xi))^2+a_3\exp(\varphi(\xi))^3+a_4\exp(\varphi(\xi))^4 \tag{3.6}$$

Values of constants: $a_0=(1/6)c, a_1=0, a_2=0, a_3=0, a_4=-8$

The exact solution of eq.(3.1): $u_2(x,t)=\dfrac{c}{6}-\dfrac{8}{[\pm 2c_1\pm 2(x-ct)]^2}$ (3.7)

Case 3 ($n=3$): The trail solution of eq. (3.3):

$$u(\xi)=a_0+a_1\exp(\varphi(\xi))+a_2\exp(\varphi(\xi))^2+a_3\exp(\varphi(\xi))^3+a_4\exp(\varphi(\xi))^4+a_5\exp(\varphi(\xi))^5+a_6\exp(\varphi(\xi))^6$$

Values of constants: $a_0=(1/6)c, a_1=0, a_2=0, a_3=0, a_4=0, a_5=0, a_6=-18$

The exact solution of eq. (3.1): $u_3(x,t)=\dfrac{c}{6}-\dfrac{18}{[\pm 3c_1\pm 3(x-ct)]^2}$ (3.8)

Case 4 ($n=4$): The trail solution of eq. (3.3):

$$u(\xi)=a_0+a_1\exp(\varphi(\xi))+a_2\exp(\varphi(\xi))^2+a_3\exp(\varphi(\xi))^3+a_4\exp(\varphi(\xi))^4+ \\ a_5\exp(\varphi(\xi))^5+a_6\exp(\varphi(\xi))^6 a_7\exp(\varphi(\xi))^7+a_8\exp(\varphi(\xi))^8 \tag{3.9}$$

Values of constants: $a_0=(1/6)c, a_1=0, a_2=0, a_3=0, a_4=0, c, a_6=0, a_7=0, a_8=-32$

The exact solution of eq. (3.1): $u_4(x,t)=\dfrac{c}{6}-\dfrac{32}{[\pm 4c_1\pm 4(x-ct)]^2}$ (3.10)

In general, the exact solution of eq. (3.1): $u_n(x,t)=\dfrac{c}{6}-\dfrac{2(n-1)^2}{[\pm nc_1 \pm n(x-ct)]^2}$

Velocity profile of $u_1(x,t)$ when $c=1$

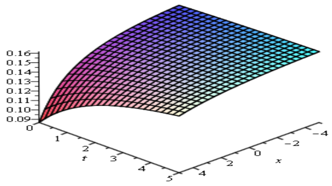

Fig.1(3d plot): profile of (10) for $c_1=-10$

Velocity profile of $u_2(x,t)$ when $c=2$

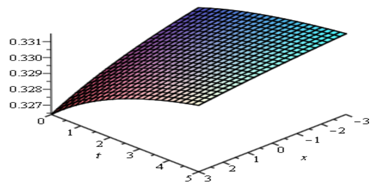

Fig.2(3d plot): profile of (12) for $c_1=-20$

Velocity profile of $u_3(x,t)$ when $c=3$

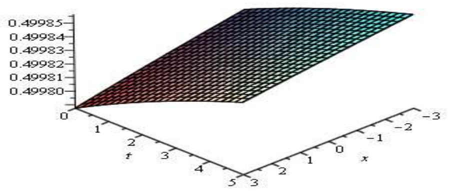

Fig.3(3d plot): profile of (14) for $c_1=-100$

Velocity profile of $u_4(x,t)$ when $c=4$

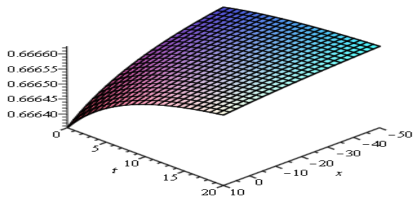

Fig.4(3d plot): profile of (16) for $c_1=-90$

Figure 1-4: Graphical representation of Korteweg-de Varies (KdV) equation

The Figure 1 shows that the waves having high wave speed propagate with more height.

Example2. Solution of modified Liouville equation

We know that the modified Liouville equation[15] is

$$w_{tt}=a^2 w_{xx}+be^{\beta w} \tag{3.11}$$

which is found in hydrodynamics, where $w(x,t)$ is the stream function and a,b,β are nonzero constants. Let us consider that the transformation $u(x,t)=e^{\beta w}$, so that

$$w=(1/\beta)\ln u \tag{3.12}$$

$$ku^3-u'^2+uu''=0 \tag{3.13}$$

where $k=\dfrac{b\beta}{a^2-c^2}$ and $c\neq \pm a$. For different values of n the solutions of eq. (3.11) are:

Case-1 ($n=1$): Solution is $w_1(x,t)=\dfrac{1}{\beta}\ln\left(\dfrac{2(c^2-a^2)}{b\beta[c_1+(x-ct)]^2}\right)$ (3.14)

Case-2 ($n=2$): Solution is $w_2(x,t)=\dfrac{1}{\beta}\ln\left(\dfrac{2(c^2-a^2)}{b\beta[c_1+(x-ct)]^2}\right)$ (3.15)

Case-3 ($n=3$ **)** Solutionis $w_3(x,t)=\dfrac{1}{\beta}\ln\left(\dfrac{2(c^2-a^2)}{b\beta[c_1+(x-ct)]^2}\right)$ (3.16)

Case-4 ($n=4$): Solution is $w_4(x,t)=\dfrac{1}{\beta}\ln\left(\dfrac{2(c^2-a^2)}{b\beta[c_1+(x-ct)]^2}\right)$ (3.17)

In general, the solution of eq. (3.11) is $w_n(x,t)=\dfrac{1}{\beta}\ln\left(\dfrac{2(c^2-a^2)}{b\beta[c_1+(x-ct)]^2}\right)$ (3.18)

From the above it is clear that the exact solutions of eq. (3.11) are same for all cases.

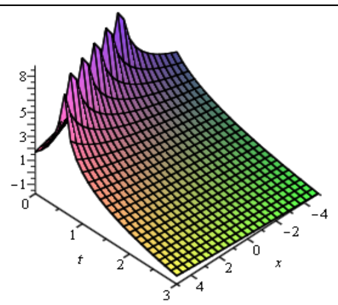

Fig.5(3d plot): Solution profile of $w(x,t)$ for $a=1, b=1, \beta=1, c_1=1$

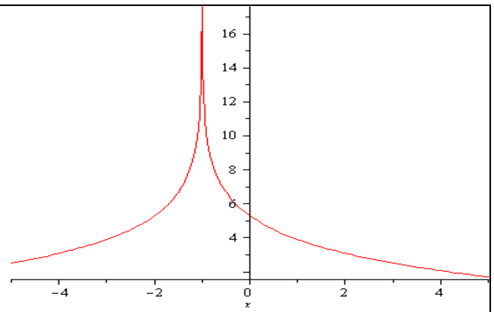

Fig.6(2d plot): Solution profile of $w(x,t)$ for $a=1, b=1, \beta=1, c_1=1$

Figure 5-6: Graphical representation of modified Liouville equation Velocity profile of $w(x,t)$ with wave speed, $c=10$

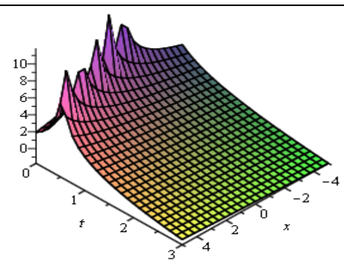

Fig.7(3d plot): Solution profile of $w(x,t)$ for $a=1, b=1, \beta=1, c_1=1$

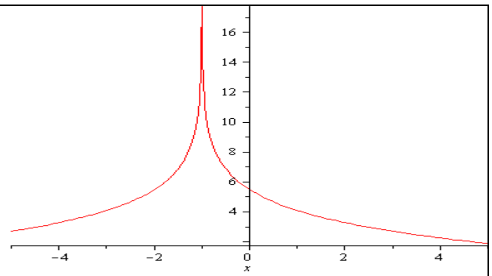

Fig.8(2d plot): Solution profile of $w(x,t)$ for $a=1, b=1, \beta=1, c_1=1$

Figure 7-8: Velocity profile of $w(x,t)$ with wave speed, $c=11$

4. Results and discussions

When the wave speed is 1, Fig. 1(3d plot) shows that the height of the velocity profile is 0.16 unit (approximate). For wave speed 2, Fig.2 (3d plot) yields that the height of the wave is 0.331 unit (approximate). Fig. 3(3d plot) gives that when wave speed is 3, the height of the wave is 0.49985 unit (approximate). When wave speed is 4, the height of the wave is 0.66660 unit (approximate). Again, from Fig. 5(3d plot) it is clear that the height of wave is 8 unit (approximate) for wave speed 10. Also for wave speed 11, from Fig. 7(3d plot) we see that the height of the wave is 10 unit. From the above discussion it is clear that the height of the wave increases with the increase of wave speed.

5. Conclusion

We can say that this method has capacity to minimize the size of computational work compared to other existing techniques. It removes complexity to get new solutions of non-linear evolution equations.

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