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Line Set Dominating Set with Reference to Degree

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Abstract. A set $D^l \subseteq E(G)$ is said to be a Strong Line Set Dominating set (*slsd*-set) of G. If for every set $R \subseteq E - D^l$. There exists an edge $e \in D^l$, such that the sub graph $\langle R \cup \{e\} \rangle$ is induced by $R \cup \{e\}$ is connected and $d(e) \geq d(f)$ for all $f \in R$ where $d(e)$ denote the degree of the edge. The minimum cardinality of a *slsd*-set is called the strong line set dominating number of G and is denote by $\vartheta_{sl}^{'}(G)$. In this paper Strong Line set Dominating set are analyse with respect to the strong domination parameter for separable graphs. The characterization of separable graphs with *slsd* number is derived.

Keywords: Separable graph, line set dominating set, strong line set dominating set.

AMS Mathematics Subject Classification (2010): 05C69

1. Introduction

Domination is an active subject in graph theory. Let $G = (V, E)$ be a graph. A set $D \subseteq V(G)$ of vertices in a graph $G = (V, E)$) is a dominating set. if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of dominating set in G. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set [3, 4].

Let $G = (V, E)$ be a graph. A set $F \subseteq E(G)$ is an edge dominating set of G. if and only if every edge in E-F is adjacent to some edge in F. The edge domination number $\gamma^{'}(G)$ is the minimum of cardinalities of all edge dominating sets of G.[4]

A dominating set S is a strong dominating set if for every vertex u in V-S, There is a vertex v in S with deg(v)≥deg(u) and u is adjacent to v [1, 2].

Let G be a graph. A set $D \subseteq V(G)$ is a point set dominating set (PSD-set) of G. if for each set $S \subseteq V - D$, there exists a vertex $u \in D$ such that the sub graph $\langle S \cup \{u\} \rangle$ induced by $S \cup \{u\}$ is connected. The point set domination number (PSD-number) $\gamma_p^{'}(G)$ of G is the minimum cardinalities of all PSD-Set of G.[6]

Let G be a graph. A set $D \subseteq V(G)$ is said to be a strong point set dominating set (*spsd*-set) of G. if for each set $S \subseteq V-D$, there exists a vertex $u \in D$ such that the sub graph $\langle S \cup \{u\} \rangle$ induced by $S \cup \{u\}$ is connected and d(u)≥ d(s) for all s∈ S where d(u) denote the degree of the vertex u. The Strong point set domination number (*spsd*) $\gamma'_{sp}(G)$ of G is the minimum cardinalities of all spsd-set [7, 8].

Rao and Vijayalakmi introduced the concept of Line set domination set and derived results parallel to those of Sampathkumar and Pushpalatha [5].

Let G be a graph. A set $F \subseteq E(G)$ is a line set dominating set (lsd-set) of G, if for each set $S \subseteq E-F$, there exists an edge $e \in F$ such that the sub graph $\langle S \cup \{e\} \rangle$ is induced by $S \cup \{e\}$is connected. The line set domination number $\upsilon'_l(G)$(lsd-number) is the minimum cardinalities of all lsd-set of G.

Let x,y in E(G) of an isolates free graph G(V,E), then an edge x, e-dominates an edge if y in$\langle N(x) \rangle$. A line graph *L*(*G*) is the graph whose vertices corresponds to the edges of *G* and two vertices in *L*(*G*) are adjacent iff the corresponding edges in *G* are adjacent (V(L(G))=q). For any edge e, let

$$N'(e) = \{e \in F: e \text{ and } f \text{ have a vertex in common}\}$$

and $N'[e] = N'(e) \cup \{x\}$. For a set $F \subseteq E(G)$ Let $N'(F) = \bigcup N'(e)$ .The degree of an edge e=uv of G is defined by deg(e) = deg(u) +deg(v)-2. The maximum and minimum degree among the edge of graph G is denote by $\Delta'(G)$ *and* $\lambda(G)$ (the degree of an edge is the number of edges adjacent to it) A connected graph with at least one cut edge is called a separable graph. That is an edge e such that G-e = {E-{e}} is disconnected [4].

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2. Results and bound

Definition 2.1. A set $D^l \subseteq E(G)$ is said to be a strong line set dominating set (*slsd*-set) of G. If for every set $R \subseteq E - D^l$. There exists an edge $e \in D^l$, such that the sub graph $\langle R \cup \{e\} \rangle$ is induced by $R \cup \{e\}$ is connected and $d(e) \geq d(f)$ for all $f \in R$ where $d(e)$ denote the degree of the edge . The minimum cardinality of a *slsd*-set is called the Strong Line Set Dominating Number of G and is denote by $\vartheta'_{sl}(G)$.

Theorem 2.2. If a connected graph G with n edge, then
$\vartheta'(G) \leq \vartheta'_l(G) \leq \vartheta'_{sl}(G) \leq q - \Delta'(G)$ where $\Delta'(G)$ is the maximum degree of G.

Proof: Since every slsd-set of G is line set dominating set and we known that every line set dominating set of G is a edge dominating set of G and Let e be a edge of maximum degree $\Delta'(G)$. Then e is adjacent to $N'(e)$, such that $\Delta'(G) = N'(e)$. Hence $E - N'(e)$ is a slsd- set. Therefore $\vartheta'_{sl}(G) \leq |E - N'(e)|$. Hence

$\vartheta'(G) \leq \vartheta'_l(G) \leq \vartheta'_{sl}(G) \leq q - \Delta'(G).$

In this next result, we list the exact values of $\vartheta'_{sl}(G)$ for some standard graphs.

Observation 2.3. For any complete graph K_n, then

$$\vartheta_{ls}^{'}(G)=\begin{cases}\frac{n}{2} & for\ any\ positive\ even\ integer\\ \frac{n-1}{2} & for\ any\ positive\ odd\ integer\end{cases}$$

Observation 2.4. For any star K_1, $n-1$, then

$$\vartheta_{sl}^{'}(K_{1,n-1})=1.$$

Observation 2.5. For any path P_n , then

$$\vartheta_{sl}^{'}(P_n)=\begin{cases}1 & n=3,4\\ 3 & n=5\\ n-3 & n\geq 6\end{cases}$$

Observation 2.6. For any cycle C_n, then

$$\vartheta_{sl}^{'}(C_n)=\begin{cases}1 & n=3\\ 2 & n=4,5\\ n-2 & for\ any\ positive\ integer\ n\geq 6\end{cases}.$$

Observation 2.7. If ($K_{n,m}$) is a complete bi-partite graph of m, n>2 vertices, then

$$\vartheta_{sl}^{'}(K_{n,m})=\begin{cases}\frac{m+n}{2} & for\ \ m=n\\ \frac{m+n-1}{2} & for\ \ m<n\end{cases}.$$

In the next result , Strong line set dominating set in graph G is the Strong Point set dominating number of the line graph L(G).

Observation 2.8. For any path P_n, for any positive integer n≥5 vertices

$$\vartheta_{sl}^{'}(P_n)=\gamma_{sl}(L(P_n)=\gamma_{sl}(P_{n-1})=n-3.$$

Observation 2.9. For any cycle C_n , for any positive integer n≥5 vertices

$$\vartheta_{sl}^{'}(C_n)=\gamma_{sp}(L(C_n)=\gamma_{sp}(C_n).$$

Observation 2.10. For any star $K_{1,n}$ for any positive integer with n≥2 vertices

$$\vartheta_{sl}^{'}(L(K_{1,n}))=\vartheta_{sl}^{'}(K_{n-1}).$$

Theorem 2.11. Let G be a connected graph and $D^l \subseteq E(G)$ be a strong line set dominating set of G. then for every subset $R \subseteq E - D^l$ in $\bigcup_{e \in D^l} \langle N^{'}(e) \rangle$. There exists an edge $e \in D^l$, such that $\langle E - N(r) \rangle$ for all $r \in R$ is maximal edge dominating set.

Lemma 2.12. Let $G(V,E)$ be any graph and D^l be any strong line set dominating set. Then $(E - D^l)$ is a proper sub graph of a component $H(G)$.

Proof: Suppose there exists e and f belonging to two different components of G. Since D^l is a strong line set dominating set of G. There must exists $w \in D^l$, such that $\langle e, f, w \rangle$ is connected and $d(w) \geq d(f)$ for all $f \in E - D^l$. Contrary to the assumption, This implies $E - D^l \subseteq E(H)$ for some component H of G . Further, since D^l is a sls-dominating set of $D^l \cap E(H) \in \vartheta_{sl}^{'}(H)$. Hence $D^l \cap E(H) = \phi$, which implies that $(E - F)$ is a proper sub graph of H.

Theorem 2.13. Let G be a finite graph of order n, and C_G denote the set of its components. Then

$$\vartheta_{sl}^{'}(G) = q - \max_{H \in C_G} \{E(H) - \vartheta_{sl}^{'}(H)\}. \qquad (1)$$

Proof: Let D^l be a $\vartheta_{sl}^{'}(G)$ G. By lemma 2.12 it follows that there exists $H \in C_G$. such that $E - D^l \subseteq E(H)$. Clearly $D^l \cap E(H) \in \vartheta_{sl}^{'}(H)$ and since

$$|D^l| = |D^l \cap E(H)| + q - |E(H)| \qquad (2)$$

We have $\vartheta_{sl}^{'}(G) \geq q - |E(H)| + \vartheta_{sl}^{'}(H) \Rightarrow \vartheta_{sl}^{'}(G) \geq q - \max\{E(H) - \vartheta_{sl}^{'}(H)\}$ (3)

On the other hand,

$$\vartheta_{sl}^{'}(G) \leq q - |E(G) - \vartheta_{sl}^{'}(H)| \Rightarrow \vartheta_{sl}^{'}(G) \geq q - \max\{E(H) - \vartheta_{sl}^{'}(H)\} \qquad (4)$$

From inequalities [3] and [4], we have $\vartheta_{sl}^{'}(G) = q - \max_{H \in C_G} \{E(H) - \vartheta_{sl}^{'}(H)\}$

In the remaining discussion of this paper, a graph G always means a separable graph.

Observation 2.15. If G is separable graph with sls-dominating set S. Then $B \cap D^l$ is a sls-dominating set of B for any block $B \in B_G$. (where B_G is the set of all blocks.)

Proof: Let $T \subseteq B - B \cap D^l$. Then that $T \subseteq E - D^l$ and D^l is a *slsd*-set. Therefore there exists $e \in D^l$ such that $T \subseteq N(e)$. Hence e is adjacent to more than one edge in B and $d(e) \geq d(t)$ $\forall t \in T(G)$. i.e. $e \in B \cap D^l$. Therefore $B \cap D^l$ is a *slsd*-set of B.

Observation 2.16. If a block B has a slsd-set $B^{'}$ containing all cut edge belonging to B Then $(E - B) \cup B^{'}$ is a slsd- set .

Remark 2.17. If D^l is an $\vartheta'_{sl}(G)$ set of separable graph, then there are two cases:

i) $\mathcal{D}_{sl}$(G:X) = $\{D^l \in D_{sl}(G): \exists\ a\ B \in B_G\ with\ E - D^l \subseteq E(B)\}$

ii) $\mathcal{D}_{sl}$(G:Z) = $\{D^l \in D_{sl}(G): E - D^l\ contain\ edges\ of\ different\ blocks\}$

Definition 2.18. Let G=(V,E) be any graph with cut vertices, $D^l \in \mathcal{D}_{sl}$ (G) and $E - D^l \subseteq E(B)$,

Define $L(B, D^l) = \{e \in E - D^l : N(e) \cap (D^l \cap E(B)\}$.

Remark 2.19. If $L(B, D^l) \neq \phi$ then $E(B) \cap D^l \in$ $\mathcal{D}_{sl}$ (B). This yields, $|B \cap D^l|$ $< \vartheta'_{sl}(B)$. This, in fact, we have

$$L(B, D^l) \neq \phi \Rightarrow \vartheta'_{sl}(G) = n - \Delta'(G)$$

Theorem 2.20. If $L(B, D^l) \neq \phi$, then $\vartheta'_{sl}(G) = q - k_{sl}$. where $k_{sl} = \max\{E(B) - \vartheta'_{sl}(B)\}$.

Proof: Let $L(B, D^l) \neq \phi$ implies $B \cap D^l$ is a slsd set of B and hence $\vartheta'_{sl}(B) \leq |B \cap D^l|$ Also $\vartheta'_{sl}(B) \geq |B \cap D^l|$. For, if $\vartheta'_{sl}(B) \leq |B \cap D^l|$, then $(E - B) \cup B'$ is a slsd set of G where $|B'| = \vartheta'_{sl}(B)$. Then $|D^l| = (E - B) \cup (B \cap D^l) \geq |(E - B) \cup B'|$. That is, there exists a slsd set $(E - B) \cup B'$ of G with cardinality less than equal to $|D^l|$ which is a contradiction. Hence
$\vartheta'_{sl}(B) \geq |B \cap D^l|$. Therefore, $\vartheta'_{sl}(B) = |B \cap D^l|$. Hence
$\vartheta'_{sl}(G) = |D^l| = |(E - B) \cup (B \cap D^l)| = |(E - B) \cup B'| \geq q - k_{sl}$.
Therefore, $\vartheta'_{sl}(G) = q - k_{sl}$.

Remark 2.21.

i) $\mathcal{D}_{sl}$(G:X_1) denotes the set of all slsd-set F of G with $E - D^l \subseteq E(B)$ and $L(B, D^l) \neq \phi$ for some B∊$\mathcal{B}_G$.

ii) $\mathcal{D}_{sl}$(G:X_1) denotes the set of all slsd-set F of G with $(E - F) - L(B, F) \neq \phi$.

Theorem 2.21. $\mathcal{D}_{sl}$(G;X_1) ≠ Ø if and only if $\Delta'(G) \leq k_{sl} + 1$.

Proof: Let $D^l \in$ $\mathcal{D}_{sl}$(G;X_1) ≠ Ø),Then by definition of $\mathcal{D}_{sl}$(G:X_1) there exist $B \in B_G$ such that $E - D^l \subseteq E(B)$ for some block B of G and $L(B, D^l) \neq \phi$, Also,

$E(B) \cap D^l$ is slsd-set of $B - L(B, D^l)$. By the definition of $L(B, D^l)$, one can easily see that $(E(B) \cap D^l) \cup \{e\}$, $e \in L(B, D^l)$ is a slsd-set for B so that

$$\vartheta'_{sl}(B) \leq \left|B \cap D^l\right| + 1 \quad (1)$$

Also, $\left|B \cap D^l\right| \leq \vartheta'_{sl}(B)$ (2)

For otherwise, $\{E(G) - E(B)\} \cup \vartheta'_{sl}(G)$ would be a slsd-set of G having less than $\left|D^l\right|$ edges contrary to the fact D^l is a $\vartheta'_{sl}(G)$ By (1) and (2) . We get

$$\vartheta'_{sl}(B) - 1 \leq \left|B \cap D^l\right| \leq \vartheta'_{sl}(B) \quad (3)$$

Now, $\left|D^l\right| = n - \left|E(B)\right| + \left|B \cap D^l\right| \geq n - \left|E(B)\right| + \vartheta'_{sl}(B) - 1.$ That is

$n - \Delta'(G) \geq n - \left|E(B)\right| + \vartheta'_{sl}(B) - 1$

Or, equivalently, $\Delta'(G) \leq \left|E(B)\right| - \vartheta'_{sl}(B) + 1 \leq k_{sl} + 1.$ Thus, we have $\mathcal{D}_{sl}(G;X_1) \neq \emptyset$,

$\Rightarrow \Delta' \in \{k_{sl}, k_{sl} + 1\}$.

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REFERENCES

1. B.D.Acharya and P.Gupta, On point-set domination in graphs, in; *Proc. Nat. Seminar on Recent Development in Mathematics,* N.M. Bujurke (Ed.), Karnataka University Press, Dharwad, 1996,106-108.
2. B.D.Acharya and P.Gupta, On point set domination in graphs: IV Separable graphs with unique minimum psd-sets, *Discrete Mathematics*, 195 (1999) 1-13.
3. F.Harary, *Graph theory*, Addison - Wesley, 1969.
4. T.Haynes, S.T.Hedetniemi and P.J.Slater, *Fundamentals of domination in graphs*, Marcel Dekker Inc, New York, 1998.
5. I.H.Nagaraja Rao and B.VijayaLaxami, *Line set domination number of a graph, Indian J. Pure Appl. Math.*, 30(7) (1999) 709-713.
6. E. Sampathkumar and L. Pushpalatha, *Point set domination number of a graph, Indian J. Pure Appl. Math.,* 24(4) (1993) 225 - 229.
7. E.Sampathkumar, *Graph Theory Research Report No:3*, Karnatak University, Dharwad, 1972.
8. E.Sampathkumar and L.Pushpalatha, Strong weak domination and domination balance in a graph, *Discrete Math.*. 161 (1996) 235-242.
9. H.B.Walikar, B.D.Acharya and E.Sampathkumar, *Recent advances in the theory of domination in graphs and its applications*, MRI Lecture Notes in Mathematics, Allahabad, No. 1, 1979.