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# Fuzzy Measure: A Fuzzy Set Theoretic Approach

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***Abstract.*** In this paper, the fuzzy measure which is generalized with underlying continuity concept and some examples are discussed.

***Keywords*:** Fuzzy relation, fuzzy measure, membership function

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## 1. Introduction

Classical measure theory [3] is studied based on additive property. But, in many real time applications like fuzzy logic [2], artificial intelligence, decision making, data mining and re-identification problem etc. require the non-additive measure. In this situation, fuzzy measure was introduced by Sugeno [4] which is a non additive measure. Fuzzy measure was discussed by several authors [4, 6, 7, 8, 9,10,12] in different manner.

The fuzziness in the concept of fuzzy measure introduced by Sugeno is abandoning the additive property. However no membership was assigned for the measure as in the fuzzy set [5, 9,11]. Fuzzy measure, defined in [1], was studied with the membership value.

In this paper, we generalize the definition of fuzzy measure which is defined in [1] with underlying continuity conditions and we give some examples.

## 2. Fuzzy relation and fuzzy measure

It is clear that if $R:A\to B$ is a relation then $R$ is a subset of A × B where A and B are any two sets. Hence $R$ can be fuzzified like fuzzification of sets.

**Definition 2.1. (Fuzzy relation)**

If A and B are any two sets, then a relation $R:A\to B$ is said to be a fuzzy relation if

(i) D( $R$ ) = A, where D( $R$ ) is the domain of $R$ and

(ii) there exists a membership function $\mu_R : R\to [0,1]$

**Example 2.2.** Let $A = \{1, 2, 3\}$, $B = \{b, c\}$ and the relation $R$ be given by $R = \{(1, a), (2, a), (2, b), (3, c)\}$

The membership function $\mu_R : R \to [0,1]$ is defined as

$\mu_R[(1,a)] = \mu_R[(2,a)] = 1$, $\mu_R[(2,b)] = 0.3$, $\mu_R[(3,c)] = 0.7$

Then $R$ is a fuzzy relation .

**Remarks 2.3.**

1. For our convenience $\mu_R[(x,y)]$ is simply denotes by $\mu_R(x,y)$.
2. If $(x, y) \in R$, we write it as $R(x) = y$.
3. By $\mu_R(x,y) = a$, we mean that a is the membership value in [ 0, 1 ] of the case $R(x) = y$. That is $\mu(R(x) = y) = a$ is simply represented by $\mu_R(x,y) = a$.

**Definition 2.4. (Fuzzy measure)** Let X be a non empty set, $\Omega$ be a non empty class of subsets of X and ( X, $\Omega$ ) be a measurable space. A fuzzy relation $m : \Omega \to [0, \infty]$ is said to be a fuzzy measure, if the following conditions are satisfied

(i) $\mu_m(\phi, 0) = 1$

(ii) For any two sets A and B in $\Omega$, $A \subseteq B$ and A is a nonempty set implies

$$\sup_{m(A)=x} x \le \sup_{m(B)=y} y \quad \text{and} \quad \mu_m\left(A, \left(\sup_{m(A)=x} x\right)\right) \le \mu_m\left(B, \left(\sup_{m(B)=y} y\right)\right),$$

(iii) For a sequence of non empty sets $\{A_n\} \subset \Omega$, $A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq \ldots\ldots$ and $\bigcup_{n=1}^{\infty} A_n \in \Omega \Rightarrow$

$$\lim_n \left(\sup_{m(A_n)=x} x\right) = \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y \quad \text{and} \quad \lim_n \mu_m\left(A_n, \left(\sup_{m(A_n)=x} x\right)\right) = \mu_m\left(\bigcup_{n=1}^{\infty} A_n, \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y\right)$$

(iv) For a sequence of non empty sets $\{A_n\} \subset \Omega$ ,

$$A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq \ldots\ldots, m(A_1) < \infty \quad \text{and} \quad \bigcap_{n=1}^{\infty} A_n \in \Omega \Rightarrow$$

$$\lim_n \left(\sup_{m(A_n)=x} x\right) = \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y \quad \text{and} \quad \lim_n \mu_m\left(A_n, \left(\sup_{m(A_n)=x} x\right)\right) = \mu_m\left(\bigcap_{n=1}^{\infty} A_n, \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y\right)$$

First we discuss some examples [4] in which the fuzzy relation m is a fuzzy measure.

In the following examples, $P(X)$ denotes the power set of a non empty set X.

**Example 2.5.** Let $X = \{1, 2, 3, ... n\}$, n is a finite value, and $\Omega = P(X)$.

Let the fuzzy relation $m : \Omega \to [0, \infty]$ be defined as

$$m(S) = x \text{ iff } x \le |S| \quad \text{that is} \quad m(S) = \{x \,/\, x \in [0, |S|]\}$$

and the membership function $\mu_m : m \to [0,1]$ be defined as

$$\mu_m(S,x) = \frac{x}{|S|},\ x \le |S|,\quad for\ A \ne \phi \quad \text{and} \quad \mu_m(\phi, 0) = 1$$

In particular case , when n = 3, $X = \{1, 2, 3\}$, and

$\Omega = P(X) = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$, where

$S_1 = \phi,\ S_2 = \{1\},\ S_3 = \{2\},\ S_4 = \{3\},\ S_5 = \{1,2\},$

$S_6 = \{1,3\},\ S_7 = \{2,3\},\ S_8 = X$

Then the values of m and its corresponding membership values for each set in $\Omega$ are

$m(S_1) = 0, \quad \mu_m(S_1, 0) = 1$

$m(S_2) = m(S_3) = m(S_4) = \{x \,/\, x \in [0,1]\}$

$\mu_m(S_2, x) = \mu_m(S_3, x) = \mu_m(S_4, x) = x \quad \text{if} \quad x \le 1$

$m(S_5) = m(S_6) = m(S_7) = \{x \,/\, x \in [0,2]\}$

$$\mu_m(S_5, x) = \mu_m(S_6, x) = \mu_m(S_7, x) = \frac{x}{2} \quad \text{if} \quad x \le 2$$

$$m(S_8) = \{x \,/\, x \in [0, 3]\}, \quad \mu_m(S_8, x) = \frac{x}{3} \quad if \quad x \le 3$$

The conditions in the definition can be verified below

$m(S_1) = 0, \qquad \mu_m(S_1, 0) = 1$

hence condition ( i ) is true

$$S_2 \subseteq S_5 \Rightarrow \sup_{m(S_2)=x} x = 1,\ \sup_{m(S_5)=y} y = 2, \text{ and hence } \sup_{m(S_2)=x} x < \sup_{m(S_5)=y} y$$

$$\text{also} \quad \mu_m\left(S_2, \left(\sup_{m(S_2)=x} x\right)\right) = 1 = \mu_m\left(S_5, \sup_{m(S_5)=y} y\right)$$

$$S_2 \subseteq S_6 \Rightarrow \sup_{m(S_2)=x} x = 1,\ \sup_{m(S_6)=y} y = 2, \text{ and hence } \sup_{m(S_2)=x} x < \sup_{m(S_6)=y} y$$

$$\text{also} \quad \mu_m\left(S_2, \left(\sup_{m(S_2)=x} x\right)\right) = 1 = \mu_m\left(S_6, \sup_{m(S_6)=y} y\right)$$

$$S_2 \subseteq S_8 \Rightarrow \sup_{m(S_2)=x} x = 1,\ \sup_{m(S_8)=y} y = 3, \text{ and hence } \sup_{m(S_2)=x} x < \sup_{m(S_8)=y} y$$

$$\text{also } \mu_m\left(S_2, \left(\sup_{m(S_2)=x} x\right)\right) = 1 = \mu_m\left(S_8, \sup_{m(S_8)=y} y\right)$$

In general, it can be verified that

for i = 3, j = 5, 7 and for i = 4, j = 6, 7

$S_i \subseteq S_j \Rightarrow \sup\limits_{m(S_i)=x} x = 1, \quad \sup\limits_{m(S_j)=y} y = 2$, and hence $\sup\limits_{m(S_i)=x} x \le \sup\limits_{m(S_j)=y} y$

$$\text{also } \mu_m\left(S_i, \left(\sup_{m(S_i)=x} x\right)\right) = 1 = \mu_m\left(S_j, \sup_{m(S_j)=y} y\right)$$

For i = 3, 4

$S_i \subseteq S_8 \Rightarrow \sup\limits_{m(S_i)=x} x = 1, \quad \sup\limits_{m(S_8)=y} y = 3$, and hence $\sup\limits_{m(S_i)=x} x \le \sup\limits_{m(S_8)=y} y$

$$\text{Also } \mu_m\left(S_i, \left(\sup_{m(S_i)=x} x\right)\right) = 1 = \mu_m\left(S_8, \sup_{m(S_8)=y} y\right)$$

For i = 5, 6, 7

$S_i \subseteq S_8 \Rightarrow \sup\limits_{m(S_i)=x} x = 2, \quad \sup\limits_{m(S_8)=y} y = 3$, and hence $\sup\limits_{m(S_i)=x} x \le \sup\limits_{m(S_8)=y} y$

$$\text{Also } \mu_m\left(S_i, \left(\sup_{m(S_i)=x} x\right)\right) = 1 = \mu_m\left(S_8, \sup_{m(S_8)=y} y\right)$$

Hence condition (ii) is true.

Now, consider the sequence of non empty sets such that

$A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq A_5$ ..... and $\bigcup\limits_{n=1}^{\infty} A_n = S_8 \in \Omega$, where

$A_1 = S_2,\ A_2 = S_5,\ A_n = S_8,\ n = 3,4,5, .....$

When we consider $\sup\limits_{m(A_n)=x} x$, for n = 1, 2, 3, 4, 5,… ,

we get the sequence 1, 2, 3, 3, 3, 3,..... and

$$\lim_n\left(\sup_{m(A_n)=x} x\right) = \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y = 3$$

When we consider $\mu_m\left(A_n, \left(\sup\limits_{m(A_n)=x} x\right)\right)$, for n = 1, 2, 3, 4, 5,… ,

we get the sequence 1, 1,1,1, 1, ...... and

$$\lim_{n} \mu_m \left( A_n, \left( \sup_{m(A_n)=x} x \right) \right) = \mu_m \left( \bigcup_{n=1}^{\infty} A_n , \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y \right) = 1$$

Now, consider another the sequence of non empty sets such that

$A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq A_5$ ..... and $\bigcup_{n=1}^{\infty} A_n = S_8 \in \Omega$ , where

$A_1 = S_5$, $A_n = S_8$, $n =$ 2, 3, 4, 5, .....

When we consider $\sup_{m(A_n)=x} x$ , for n = 1, 2, 3, 4, 5,… ,

we get the sequence 2, 3, 3, 3, 3,..... and $\lim_{n} \left( \sup_{m(A_n)=x} x \right) = \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y = 3$

When we consider $\mu_m \left( A_n, \left( \sup_{m(A_n)=x} x \right) \right)$ , for n = 1, 2, 3, 4, 5,… ,

we get the sequence 1,1,1,1,1, ...... and

$$\lim_{n} \mu_m \left( A_n, \left( \sup_{m(A_n)=x} x \right) \right) = \mu_m \left( \bigcup_{n=1}^{\infty} A_n , \sup_{m\left(\bigcup_{n=1}^{\infty} A_n\right)=y} y \right) = 1$$

Similarly for all other possible cases, the condition ( iii ) can be verified .
Now , consider the sequence of non empty sets such that

$A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq A_5 \supseteq$...... and $\bigcap_{n=1}^{\infty} A_n = S_2 \in \Omega$, where

$A_1 = S_8$, $A_2 = S_5$, $A_n = S_2$, $n = 3, 4, 5, .....$

When we consider $\sup_{m(A_n)=x} x$ , for n = 1, 2, 3, 4, …. ,

We get the sequence 3, 2, 1, 1, 1, 1,..... and $\lim_{n} \left( \sup_{m(A_n)=x} x \right) = \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y = 1$

When we consider $\mu_m \left( A_n, \left( \sup_{m(A_n)=x} x \right) \right)$ , for n = 1, 2, 3, 4, …. , we get the sequence 1, 1,1,1,1, ..... and

$$\lim_n \mu_m\left(A_n, \left(\sup_{m(A_n)=x} x\right)\right) = \mu_m\left(\bigcap_{n=1}^{\infty} A_n, \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y\right) = 1$$

Now, consider the sequence of non empty sets such that

$A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq A_5 \supseteq \ldots\ldots$ and $\bigcap_{n=1}^{\infty} A_n = S_4 \in \Omega$ , where

$A_1 = S_6, \quad A_n = S_4, \; n = 2, 3, 4, 5, \ldots..$

When we consider $\sup_{m(A_n)=x} x$ , for n = 1, 2, 3, 4, ……, we get the sequence

2, 1, 1, 1, 1,..... and

$$\lim_n \left(\sup_{m(A_n)=x} x\right) = \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y = 1$$

When we consider $\mu_m\left(A_n, \left(\sup_{m(A_n)=x} x\right)\right)$ , for n = 1, 2, 3, 4, …..we get the sequence

1,1,1,1,1, ..... and

$$\lim_n \mu_m\left(A_n, \left(\sup_{m(A_n)=x} x\right)\right) = \mu_m\left(\bigcap_{n=1}^{\infty} A_n, \sup_{m\left(\bigcap_{n=1}^{\infty} A_n\right)=y} y\right) = 1$$

Similarly, for all other possible cases, the condition ( iv ) can be verified.

Thus all the conditions which have been stated in the definition 2.4 are verified for the given fuzzy function m and its membership function $\mu_m$ and hence the given m is a fuzzy measure.
Hence for any finite value of n, the m is a fuzzy measure.

**Example 2.6.** Let $X = \{a, b, c\}$ , and
$\Omega = P(X) = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$ ,
where $S_1 = \phi, S_2 = \{a\}, S_3 = \{b\}, S_4 = \{c\}, S_5 = \{a, b\},$
$S_6 = \{a, c\}, S_7 = \{b, c\}, S_8 = X$
Let the fuzzy relation $m : \Omega \to [0, \infty]$ be defined as

$$m(S)=\left\{ x \,/\, 0\le x\le \left(\frac{|S|}{3}\right)^2+0.2 \right\}, \quad \text{and} \qquad m(\phi)=0$$

and the membership function $\mu_m : m \to [0,1]$ be defined as

$$\mu_m(S,x)=\begin{cases} \dfrac{x}{\left(\frac{|S|}{3}\right)^2} & if \quad 0\le x \le \left(\frac{|S|}{3}\right)^2 \\ \dfrac{\left(\frac{|S|}{3}\right)^2+0.2-x}{0.2} & if \quad \left(\frac{|S|}{3}\right)^2 \le x \le \left(\frac{|S|}{3}\right)^2+0.2 \end{cases}$$

$for\ S \ne \phi$ and $\qquad \mu_m(\phi,0)=1$

Then the values of m and its corresponding membership values for each set in $\Omega$ are

$m(S_1)=0, \qquad \mu_m(S_1,0)=1$

$m(S_2)=m(S_3)=m(S_4)=\{x \,/\, 0\le x \le 0.31\}$

$$\mu_m(S_2,x)=\mu_m(S_3,x)=\mu_m(S_4,x)=\begin{cases} \dfrac{x}{0.11} & if \quad 0\le x\le 0.11 \\ \dfrac{0.31-x}{0.2} & if \quad 0.11\le x\le 0.31 \end{cases}$$

$m(S_5)=m(S_6)=m(S_7)=\{x \,/\, 0\le x \le 0.64\}$

$$\mu_m(S_5,x)=\mu_m(S_6,x)=\mu_m(S_7,x)=\begin{cases} \dfrac{x}{0.44} & if \quad 0\le x\le 0.44 \\ \dfrac{0.64-x}{0.2} & if \quad 0.44\le x\le 0.64 \end{cases}$$

$$m(S_8)=\{x \,/\, 0\le x\le 1.2\}, \qquad \mu_m(S_8,x)=\begin{cases} x & if \quad 0\le x\le 1 \\ \dfrac{1.2-x}{0.2} & if \quad 1\le x\le 1.2 \end{cases}$$

Conditions (i) - (iv) are satisfied for the fuzzy relation m and its membership function $\mu_m$. Hence the fuzzy relation m is a fuzzy measure.

**Example 2.7.** Let $X=\{a_1, a_2, a_3\}$, and

$\Omega=P(X)=\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$, where

$S_1=\phi, S_2=\{a_1\}, S_3=\{a_2\}, S_4=\{a_3\}, S_5=\{a_1, a_2\},$
$S_6=\{a_1, a_3\}, S_7=\{a_2, a_3\}, S_8=X$

Let the fuzzy relation $m : \Omega \to [0,\infty]$ be defined as

$$m(S)=\begin{cases} 1 \; if \;\; x_0 \in S_i \\ 0 \; if \;\; x_0 \notin S_i \;, \end{cases} \qquad \forall S_i \in \Omega \quad \text{and} \quad x_0 \text{ is a fixed point in } X$$

and the membership function $\mu_m : m \to [0,1]$ be defined as

$$\mu_m(S,x) = 1 - \frac{1}{|S|+x}, \quad for \;\; S \neq \phi \quad \text{and} \quad \mu_m(\phi,0) = 1$$

As a particular case, we take $x_0 = a_1$.

Then the values of m and its corresponding membership values for each set in $\Omega$ are
$m(S_1) = 0, \; m(S_2)=1, m(S_3)=0, \; m(S_4)=0,$
$m(S_5)=1, \; m(S_6)=1, m(S_7)=0, \; m(S_8)=1$
$\mu_m(S_1, 0)=1, \mu_m(S_2, 1)=0.5, \; \mu_m(S_3, 0)=0, \mu_m(S_4, 0)=0,$
$\mu_m(S_5, 1)=0.67, \quad \mu_m(S_6, 0)=0.67, \quad \mu_m(S_7, 0)=0.5, \; \mu_m(S_8, 0)=0.75$

Conditions (i) - (iv) in the definition are satisfied for the fuzzy relation m and its membership function $\mu_m$.

In general, for any $x_0$ in $X$, the conditions are satisfied. Hence the fuzzy relation m is a fuzzy measure.

**Example 2.8.** Let $X = \{1, 2, 3, ... n\}$, and $\Omega = P(X)$. Let the fuzzy relation $m : \Omega \to [0,\infty]$ be defined as $m(S) = \max S$, for $S \neq \phi$ and $m(\phi)=0$
and the membership function $\mu_m : m \to [0,1]$ be defined as

$$\mu_m(S,x) = 1 - \frac{1}{x}, \quad for \;\; A \neq \phi \quad \text{and} \quad \mu_m(\phi,0) = 1$$

In particular case, when n = 3, $X = \{1, 2, 3\}$, and
$\Omega = P(X) = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\},$
where $S_1=\phi, S_2=\{1\}, S_3=\{2\}, S_4=\{3\}, S_5=\{1,2\},$
$S_6=\{1,3\}, S_7=\{2,3\}, S_8=X$
Then the values of m and its corresponding membership values for each set in $\Omega$ are
$m(S_1) = 0, \quad m(S_2)=1, \; m(S_3)=2, m(S_4)=3, \; m(S_5)=2, m(S_6)=3,$
$m(S_7)=3, \; m(S_8)=3$

$\mu_m(S_1,0)=1,\ \mu_m(S_2,1)=0,\ \mu_m(S_3,2)=0.5,$

$\mu_m(S_4,3)=0.7, \mu_m(S_5,2)=0.5,$

$\mu_m(S_6,3)=0.7, \mu_m(S_7,3)=0.7,\ \mu_m(S_8,3)=0.7$

Conditions (i) - (iv) in the definition are satisfied for the fuzzy relation m and its membership function $\mu_m$.

In general, for any value finite of n, the conditions are satisfied. Hence the fuzzy relation m is a fuzzy measure.

**Remark:** In the first two examples, measure takes values from an interval and the next two examples measure is a non negative finite real number. In general, fuzzy measure need not be an interval.

Secondly, we discuss some examples [4] which are not a fuzzy measure.

**Example 2.9.** Let $X_0=\{1,2,3,......\}$, $X=X_0\times X_0=\{(x, y)/ x, y\in X_0\}$ and, $\Omega = P(X)$.

Let the fuzzy relation $m:\Omega\to[0,\infty]$ be defined as

$$m(S)=|\Pr oj\, S\,|,\text{for } S\neq\phi \text{ and } \forall S\in P(X)$$

where $\Pr oj\, S=\{x/(x, y)\in S\}$ and $m(\phi)=0$

and the membership function $\mu_m: m\to[0,1]$ be defined as

$$\mu_m(S,x)=1-\frac{1}{1+x^2},\quad for\ S\neq\phi \quad \text{and} \quad \mu_m(\phi,0)=1$$

Then m satisfies the conditions (i), (ii) and (iii) but it does not satisfy the condition (iv).

For if we take $S_n=\{1\}\times\{n,n+1,n+2,....\}$, then

$S_1\supseteq S_2\supseteq S_3\supseteq S_4\supseteq....$ and $m(S_n)=1$ for any $n$

Therefore $\lim\limits_{n}\left(\sup\limits_{m(S_n)=x} x\right) = 1$ but $\bigcap\limits_{n=1}^{\infty} S_n=\phi$ and $\sup\limits_{m\left(\bigcap\limits_{n=1}^{\infty} S_n\right)=y} y = 0$

Thus $\lim\limits_{n}\left(\sup\limits_{m(S_n)=x} x\right) \neq \sup\limits_{m\left(\bigcap\limits_{n=1}^{\infty} S_n\right)=y} y$. Hence the fuzzy relation m is not a fuzzy measure.

**Example 2.10.** Let $X=\{a_1, a_2, a_3\}$ and

$\Omega=P(X)=\{S_1,S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$, where

$S_1 = \phi,\ S_2 = \{a_1\}, S_3 = \{a_2\}, S_4 = \{a_3\}, S_5 = \{a_1, a_2\}, S_6 = \{a_1, a_3\},$
$S_7 = \{a_2, a_3\},\ S_8 = X$

Let the fuzzy relation $m : \Omega \to [0, \infty]$ be defined as

$$m(S) = \begin{cases} 1 \ if \ x_0 \in S_i \\ 0 \ if \ x_0 \notin S_i \ , \end{cases} \quad \forall S_i \in \Omega \quad \text{and} \quad x_0 \text{ is a fixed point in } X$$

and the membership function $\mu_m : m \to [0,1]$ be defined as

$$\mu_m(S, x) = \frac{1}{1 + (x - |S|)^2} \quad \forall S_i \in \Omega$$

As a particular case, we take $x_0 = a_1$

Then the values of m and its corresponding membership values for each set in $\Omega$ are

$m(S_1) = 0,\ m(S_2) = 1,\ m(S_3) = 0,\ m(S_4) = 0,\ m(S_5) = 1,\ m(S_6) = 1,$
$m(S_7) = 0,\ m(S_8) = 1$

$\mu_m(S_1, 0) = 1,\ \mu_m(S_2, 1) = 1, \mu_m(S_3, 0) = 0.5,$
$\mu_m(S_4, 0) = 0.5, \mu_m(S_5, 1) = 1,\ \mu_m(S_6, 1) = 1, \mu_m(S_7, 0) = 0.2,$
$\mu_m(S_8, 1) = 0.5$

This is not a fuzzy measure. For if

$S_3 \subset S_7 \Rightarrow m(S_3) = m(S_7) = 0$ but $\mu_m(S_3) = 0.5 > \mu_m(S_7) = 0.2.$

**3. Conclusion**

In this paper, we have attempted to give the definition of fuzzy measure, using the fuzzy set approach and we cited some examples in which m is fuzzy measure and some examples in which m is not a fuzzy measure . A detailed investigation on the structure of fuzzy measures and fuzzy integrals can be then made using the present definition.

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