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On Exploring the Exponential Diophantine Equation $(3^k - 1)^x + (3^k)^y = z^2$ with k is a Nonnegative Integer

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Abstract. Let k, x, y, and z be non-negative integers. Consider the Exponential Diophantine equation $(3^k - 1)^x + (3^k)^y = z^2$. This is a non-linear Diophantine equation in four variables k, x, y and z. This paper investigates the Exponential Diophantine equation $(3^k - 1)^x + (3^k)^y = z^2$ for all the non-negative integer solutions. This paper uses Catalan's Conjecture known as Mihailescue's theorem as a special tool in addition to the factorization methods, modular arithmetic and elementary mathematical concepts in the construction of the proofs. Moreover in some special cases it agrees with results obtained by the previous papers also. Some important consequences are drawn through the observation of the solutions to the problem.

Keywords: Exponential Diophantine equation; Integer Solution; Catalan's Conjecture; Non-linear equation; Congruence; Modular arithmetic

AMS Mathematics Subject Classification (2020): 11D61, 11D72

1. Introduction

Eugene Charles Catalan conjectured a theorem known as Catalan's Conjecture [2] (or Mihailescu's theorem [3]) in 1844. It was proved by Mihailescu in 2002. Let a, b, x, y, z be non-negative integers. The Exponential Diophantine $a^x + b^y = z^2$ is studied by so many mathematicians [2, 3, 4, 5 , 6, 7, 8, 9,10,11,12,13,14, 15, 16,17, 18,19,20, 21]. Suvarnamani [4] studied the Diophantine equation $2^x + p^y = z^2$ where p is a prime number. Banyat Sroysang [7] presented that the Diophantine equation $2^x + 3^y = z^2$ has exactly three nonnegative integer solutions (3,0,3), (0,1,2) and (4,2,5). Simtrakankul [10] solved the Diophantine equation $(2^k - 1)^x + (2^k)^y = z^2$ with k as even positive whole number. Hoque [11] studied the Diophantine equation $M_{pq}{}^x + (M_{pq} + 1)^y = z^2$, where M_{pq} is a Mersenne prime. Rehmawati, Sugandha, Trippena and Prabhavo [12] solved the Non linear Diophantine Equation $(7^k - 1)^x + (7^k)^y = z^2$ with k is the positive even whole number. In 2021 Gayo and Bacani [14] investigated for the non-negative integer solutions of Diophantine Equation$M_p{}^x + (M_q + 1)^y = z^2$. In the same year Sandhya [13] investigated for the non-negative integer solutions of the exponential Diophantine equation $p^x + (p+1)^y = z^2$, p is a prime number. In 2023, Kulprapa Srimud and Suton Tadee [15] investigated for the nonnegative integer solutions of the Exponential Diophantine equation $3^x + b^y = z^2$, where b is a positive integer such that $b \equiv 5 mod 20$ or $b \equiv 5 mod 30$.There is an open problem in the remaining cases as stated in their conclusion part [15]. In 2023, Rao [16] proved that the exponential Diophantine equation $23^x + 233^y = z^2$ has a unique solution (1,1,16).In 2018, Rao [17] proved that the Diophantine equation $3^x + 7^y = z^2$ has exactly two non-negative integer solutions (1,0,2) and (2,1,4).In 2025, Suton Tadee [18] proved that the Diophantine equation $7^x + 147^y = z^2$ has exactly two non-negative integer solutions (2, 1, 14) and (5, 2, 196).In 2019, Burshtien [19] proved that the Diophantine equation $8^x + 9^y = z^2$ has no solutions in positive integers x, y, z. In 2024, Biswas [20] investigated the Diophantine equation $3^x + 63^y = z^2$ and found that it has exactly two solutions $(1,0,2), (0,1,8)$. In 2022, Rao [21] investigated the Diophantine equation $23^x - 19^y = z^2$ and proved that it has exactly two solutions$(0,0,0), (1,1,2)$.

In this paper the exponential Diophantine equation $(3^k - 1)^x + (3^k)^y = z^2$, where k is a nonnegative integer, is solved for non-negative integer solutions by using Catalan's Conjecture [2] and modular arithmetic and some elementary methods.

2. Preliminaries

Catalan's conjecture [2] plays an important role in solving Exponential Diophantine equations. This conjecture was proved by Preda Mihailescu [3] in 2002.

Lemma 2.1. *(Catalan's Conjecture [2] or Mihailescu's Theorem [3]):*
The quadruple $(a, x, b, y) = (3,2,2,3)$ is the only integer solution for the Diophantine equation $a^x - b^y = 1$, where a, x, b, y are integers with $min\{a,\ x,\ b,\ y\} > 1$.

Lemma 2.2. The exponential Diophantine equation $1 + (3^k)^y = z^2$ has an integer solution $(k, y, z) = (1, 1, 2)$, where k is a non-negative integer.
Proof: Let x, y, z, k be non-negative integers such that

$$1 + (3^k)^y = z^2 \tag{1}$$

The proof is presented in proof by three cases.

Case-1 Put $y = 0$ in (1), we get

$$z^2 = 2$$

This is not solvable for integer solution.

Case-2 Put $y = 1$ in (1), we get

$$1 + (3^k)^1 = z^2,$$
$$3^k = (z-1)(z+1)$$

So that,

$$3^{k-u} = (z+1),$$

where $3^u = (z-1)$.
Thus

$$2z = 3^{k-u} + 3^u = 3^u(3^{k-2u} + 1).$$

Hence,

$$3^u = 1 \text{ and } \left(3^{k-2u} + 1\right) = 2z.$$

From $3^u = 1$, we get $u = 0$.

Thus from $3^u = (z-1)$, we get $z = 2$.

Therefore $(k, y, z) = (1, 1, 2)$ is an integer solution of the Diophantine equation (1).

Case-3 when $y > 1$.
From (1)

$$z^2 - (3^k)^y = 1$$

This has no integer solution by lemma 2.1 (Catalan's Conjecture).

Lemma 2.3. The exponential Diophantine equation $(3^k - 1)^x + 1 = z^2$ has integer solutions $(k, x, z) = (1,3,3)$ and $(k, x, z) = (2n, 1, 3^n)$ for any non-negative integer n.

Proof: Let k, x, z, k be non-negative integers such that

$$(3^k - 1)^x + 1 = z^2 \tag{2}$$

Consider the proof by three cases.
Case-1: When $x = 0$ equation (2) gives $z^2 = 2$ which is not solvable for integer solution.

Case-2: When $x = 1$, from (2) we get $(3^k - 1)^1 + 1 = z^2$
Then $3^k = z^2$ which gives $3^n = z$ when $k = 2n$ where n is a non-negative integer.
Therefore $(k, x, z) = (2n, 1, 3^n)$ is a solution for any non-negative integer n.

Case-3: when $x > 1$, from (2) we can write $z^2 - \left(3^k - 1\right)^x = 1$.
Using Catalan's conjecture the solution is $z = x = 3 \; and \; 3^k - 1 = 2$, then $3^k = 3$, so that $k = 1$.
Therefore $(k, x, z) = (1, 3, 3)$ is a solution of (2).

Lemma 2.4. Let x, y, z, k be non-negative integers. Then the value of z is always an odd integer in the solution of the Exponential Diophantine equation $(3^k - 1)^x + (3^k)^y = z^2$.

Proof: Let x, y, z, k be non-negative integers.
Consider

$$(3^k - 1)^x + (3^k)^y = z^2.$$

Case-1: When k is an even integer,

$$3^k \equiv 1 \; mod \; 4$$
$$3^{ky} \equiv 1 \; mod \; 4$$

But, for $x \geq 1$,

$$\left(3^k - 1\right)^x \equiv 0 \; mod \; 4$$

Hence,

$$z^2 .= (3^k - 1)^x + (3^k)^y \equiv 1 \; mod \; 4$$

Therefore z is an odd integer.

Case-2: When k is an odd integer,

$$3^k \equiv 3 \; mod \; 4$$
$$3^{ky} \equiv 3^y \; mod \; 4$$
$$3^{ky} \equiv 3^y \; mod \; 4 \equiv \begin{cases} 3 \; mod \; 4, & when \; y \; is \; an \; odd \; integer \\ 1 \; mod \; 4, & when \; y \; is \; an \; even \; integer \end{cases}$$

When $x \geq 1$

$$\left(3^k - 1\right)^x \equiv 2^x \; mod \; 4 \equiv \begin{cases} 2 \; mod \; 4 & when \; x = 1 \\ 0 \; mod \; 4, & when \; x > 1 \end{cases}$$
$$3^{ky} \equiv 3^y \; mod \; 4 \; \equiv \begin{cases} 3 \; mod \; 4, & when \; y \; is \; an \; odd \; ineger \\ 1 \; mod \; 4, & when \; y \; is \; even \; integer \end{cases}$$

$$z^2. = (3^k - 1)^x + (3^k)^y \equiv \begin{cases} 1 \; mod \; 4, & when \; x = 1 \; and \; y \; is \; odd \\ 3 \; mod \; 4, & when \; x = 1 \; and \; y \; is \; even \\ 3 \; mod \; 4, & when \; x > 1 \; and \; y \; is \; odd \\ 1 \; mod \; 4, & when \; x > 1 \; and \; y \; is \; even \end{cases}$$

$$z^2. \equiv \begin{cases} 1 \; mod \; 4, & & when \; x = 1 \; and \; y \; is \; odd \\ 3 \; mod \; 4, & impossible & when \; x = 1 \; and \; y \; is \; even \\ 3 \; mod \; 4, & impossible & when \; x > 1 \; and \; y \; is \; odd \\ 1 \; mod \; 4, & & when \; x > 1 \; and \; y \; is \; even \end{cases}$$

Hence $z^2. \equiv 1 \; mod \; 4$

Therefore z is an odd positive integer only. (3)

Thus we may search for the solutions in two cases only.

3. Main result

Theorem 3.1. Let x, y, z, k be non-negative integers. The Exponential Diophantine equation $(3^k - 1)^x + (3^k)^y = z^2$ has the following non-negative integer solutions.

1. The quadruple $(k, x, y, z) = (0, c_1, c_2, 1)$ is a solution, where c_1, c_2 are non-negative integers.
2. $(k, x, \; y, \; z) = (1, 0, 1, 2)$, $(1,3,0,3), (1,4,2,5)$.
3. $(k, x, y, z) = (2n, 1,0, 3^n)$, where n is a non- negative integer.

Proof: Let k be a non-negative integer and x, y, z be non-negative integers such that (k, x, y, z) be an integer solution of the exponential Diophantine equation

$$(3^k - 1)^x + (3^k)^y = z^2 \qquad (4)$$

Case-1: when $y = 0$, equation (4) becomes equation (2)

$$(3^k - 1)^x + 1 = z^2$$

By lemma 2.3 (2) has integer solutions, for any non-negative integer n

$$(k, x, z) = (1,3,3) \text{ and } (k, x, z) = (2n, 1, 3^n)$$

Therefore $(k, x, y, z) = (1,3,0,3)$ and $(k, x, y, z) = (2n, 1,0, 3^n)$ are solutions of (4) for any non-negative integer n.

Case-2: When $y \geq 1, k \geq 0$

Subcase-2.1 when x = 0 **,** (4) becomes

$$1 + (3^k)^y = z^2$$

By lemma 2.2 it has an integer solution $(k, y, z) = (1, 1, 2)$, where k is a non-negative integer.
So that (4) has an integer solution $(k, x, y, z) = (1, 0,1, 2)$.

Subcase-2.2 When $x \geq 1$.
Now there are two cases namely x is even and x is odd.

Subcase-2.2.1: Suppose that x is an even number.
For some integer m, $x = 2m$.
Using the following factorization in (4)

$$\begin{aligned}(3^k)^y &= z^2 - \left(3^k - 1\right)^{2m} \\ &= \left[z - \left(3^k - 1\right)^m\right]\left[z + \left(3^k - 1\right)^m\right] \\ &= 3^{\beta+\alpha}\end{aligned} \tag{5}$$

Here, α *and* β are chosen such that $\alpha > \beta$, such that

$$\left[z - \left(3^k - 1\right)^m\right] = 3^\beta \, , \left[z + \left(3^k - 1\right)^m\right] = 3^\alpha. \tag{6}$$

From (5) we get,

$$3^{ky} = 3^{\beta+\alpha}$$
$$ky = \beta + \alpha. \tag{7}$$

From (6) we get,

$$3^\alpha - 3^\beta = 2\left(3^k - 1\right)^m$$
$$3^\beta(3^{\alpha-\beta} - 1) = 2\left(3^k - 1\right)^m \tag{8}$$

Applying the factorization method for (8), we get (9) and (10)

$$\left.\begin{aligned}(3^{\alpha-\beta} - 1) &= 2 \\ 3^\beta &= \left(3^k - 1\right)^m\end{aligned}\right\} \tag{9}$$

or

$$\left.\begin{aligned}(3^{\alpha-\beta} - 1) &= 3^\beta \\ 2 &= \left(3^k - 1\right)^m\end{aligned}\right\} \tag{10}$$

Subcase-2.2.1.1 From (9) ,

t $\alpha - \beta = 1$ and $\beta = m = 0$.

Therefore $\alpha = 1, \beta = 0$, $m = 0$
From (7),

$$ky = \beta + \alpha = 1,$$

This gives $k = y = 1$ only.
From (6),

$$z = 2.$$

Therefore $(k, x, y, z) = (1,0,1,2)$ is an integer solution of (4).

Subcase-2.2.1.2: From (10),

$$2 = \left(3^k - 1\right)^m$$

As a result $m = k = 1$, So that $x = 2$.
Put $x = 2$ in (4) we get $2^2 + 3^y = z^2$.
Also from (10),

$$(3^{\alpha-\beta} - 1) = 3^{\beta}$$

Then by the factorization method, we get

$$(3^{\alpha-\beta} - 1) = 1 \text{ and } 3^{\beta} = 1.$$

Thus $\beta = 0$ and$(3^{\alpha} - 1) = 1$, Then

$$(3^{\alpha}) = 2$$

This is not solvable for the integer solution.

Using another factorization in (4)
So using $\left[z - \left(3^k - 1\right)^m\right] = 3^{\beta}$ in (5)
We get,

$$\left[z + \left(3^k - 1\right)^m\right] = 3^{ky-\beta}$$

Adding these two

$$2z = 3^{\beta} + 3^{ky-\beta}$$

As $\beta = 0$, it reduces to $2z = 1 + 3^{ky}$
By lemma 2.4 z is an odd integer
For some integer a

$$z = 2a + 1$$
$$2\,z = 2(2a + 1) = 1 + 3^{ky}$$
$$2(2a + 1) - 1 = 3^{ky}$$
$$(4a + 1) = 3^{ky} \tag{11}$$

From (11) we get $a = 0\,, 2$

Put $a = 0$ in (11), we get

$$1 = 3^{ky}$$

Hence we get $ky = 0$, and hence either $k = 0$ or $y = 0$
For $k = 0$,

$(k, x, y, z) = (0,2m, c, 1)$, this is a solution, for any values of the non-negative integers m, c. (12)

For $y = 0$, $x = 2m$, in (1) it becomes

$$(3^k - 1)^{2m} + 1 = z^2$$

This has no solution by lemma 2.1.

Put $a = 2$ we get $z = 5\,, ky = 2$,

i.e. $(k, x, y, z) = (1,2m, 2,5)$ or $(k, x, y, z) = (2,2m, 1,5)$

When $k = 1\,, y = 2\,, x = 2m$, in (4) we get,

$$2^x + 3^2 = 5^2$$

Solving this we get $x = 4$.

Therefore we get $(k, x, y, z) = (1,4,2,5)$ is a solution. (13)

For $k = 2\,, y = 1$ and $x = 2m$, in (4) we get

$$8^{2m}+9=25$$
$$2^{6m}=2^4$$
$$6m=4,$$

Therefore m is not an integer and hence $x=2m$ is not an integer.
Therefore there is no integer solution for $k=2\,,y=1$

Subcase-2.2.2*:* Suppose that x is an odd positive number.
Then, $x=2m+1$, for some integer m.
From (4),

$$(3^k-1)^{2m+1}+(3^k)^y=z^2. \tag{14}$$

Subcase-2.2.2.1 Select two non-negative integers a, b such that $3^k-1=a^2+b$.
Then from (14),

$$(3^k)^y=z^2-\ (3^k-1)^{2m}(3^k-1)=z^2-\ (3^k-1)^{2m}(a^2+b)$$
$$(3^k)^y+b(3^k-1)^{2m}=z^2-\ (3^k-1)^{2m}(a^2)$$
$$=\left[z-a(3^k-1)^m\right]\left[z+a(3^k-1)^m\right] \tag{15}$$

Let $\left[z-a(3^k-1)^m\right]=(3^k-1)^u$ for some non-negative integer u. (16)

From (15) and (16),

$$\left[z+a(3^k-1)^m\right]=\left[(3^k)^y+b(3^k-1)^{2m}\right](3^k-1)^{-u}$$
$$=\left[(3^k)^y(3^k-1)^{-u}+b(3^k-1)^{2m-u}\right] \qquad 17)$$

Subtracting (16) from (17) we get

$$2a(3^k-1)^m=\left[(3^k)^y(3^k-1)^{-u}+b(3^k-1)^{2m-u}\right]-(3^k-1)^u$$
$$=(3^k-1)^u\left[3^{ky}(3^k-1)^{-2u}+b(3^k-1)^{2m-2u}-1\right]$$

Then we must have

$$\left.\begin{aligned}(3^k-1)^m&=(3^k-1)^u\\ 2a&=\left[3^{ky}(3^k-1)^{-2u}+b(3^k-1)^{2m-2u}-1\right]\end{aligned}\right\} \tag{18}$$

Hence

$$\left.\begin{aligned}m&=u\\ 2a&=\left[3^{ky}(3^k-1)^{-2m}+b-1\right]\end{aligned}\right\} \tag{19}$$

Thus $2a=\left[3^{ky}(3^k-1)^{-2m}+b-1\right]$

Then $(2a-b+1)(3^k-1)^{2m}=[3^{ky}]$ (20)

When k is an even integer,

$$3^{ky}\equiv 1\ mod\ 4\ \text{ and }\ (3^k-1)^{2m}\equiv 0\ mod\ 4$$

Then from (20) we get $0\equiv 1\ mod\ 4$ which is impossible, so no solution.

When k is an odd integer and $m>1$,

$$(3^k-1)^{2m}\equiv 0\ mod\ 4\ \text{and}\ \ 3^{ky}\equiv 3\ mod\ 4$$

Then from (20) we get $0 \equiv 3 \, mod \, 4$ which is impossible, so no solution.

Thus, for $k > 1 \text{ and } x > 1, y > 1$, there is no integer solution for (4) in this case.

Subcase-2.2.2.2

Select two integers a, b such that $3^k - 1 = a^2 - b$.
Then from (12),

$$(3^k)^y = z^2 - \left(3^k - 1\right)^{2m}\left(3^k - 1\right) = z^2 - \left(3^k - 1\right)^{2m}(a^2 - b).$$

$$\left(3^k\right)^y - b\left(3^k - 1\right)^{2m} = z^2 - \left(3^k - 1\right)^{2m}(a^2)$$

$$= \left[z - a\left(3^k - 1\right)^m\right]\left[z + a\left(3^k - 1\right)^m\right] \tag{21}$$

Let $\left[z - a\left(3^k - 1\right)^m\right] = \left(3^k - 1\right)^u$ (22)

Let $\left[z + a\left(3^k - 1\right)^m\right] = \left(3^k - 1\right)^v$ (23)

where u , v are non-negative integers such that $u \leq v$.
Adding and subtracting (22) and (23), we get

$$2z = \left(3^k - 1\right)^u + \left(3^k - 1\right)^v = \left(3^k - 1\right)^u\left[\left(3^k - 1\right)^{v-u} + 1\right] \tag{24}$$

$$2a\left(3^k - 1\right)^m = \left(3^k - 1\right)^v - \left(3^k - 1\right)^u = \left(3^k - 1\right)^u\left[\left(3^k - 1\right)^{v-u} - 1\right] \tag{25}$$

From (24), $$\left.\begin{aligned} 2 &= \left(3^k - 1\right)^u \\ z &= \left[\left(3^k - 1\right)^{v-u} + 1\right] \end{aligned}\right\} \tag{26}$$

Or $$\begin{aligned} z &= \left(3^k - 1\right)^u \\ 2 &= \left[\left(3^k - 1\right)^{v-u} + 1\right] \end{aligned} \tag{27}$$

From (26), we get $k = 1, u = 1$ so that $z = 2^{v-1} + 1$.
From (25), we get $2a2^m = 2[2^{v-1} - 1]$ then we must have $m = 0, v = 2, a = 1$ and then $x = 1, z = 3$
Using $k = 1, x = 1, z = 3$ in (14), we get
$2 + 3^y = 9$.
This has no integer solution.
Thus there is no solution in this case.

Suppose (27) holds
Then $2 = \left[\left(3^k - 1\right)^{v-u} + 1\right]$ which holds for $u = v$ and using $z = \left(3^k - 1\right)^u$ in (25), we get

$$\left[-a\left(3^k - 1\right)^m\right] = 0, \text{ this implies } k = 0 \text{ only.}$$

Thus for $k = 0$, we get $(k, x, y, z) = (0, 2m + 1, c_2, 1)$ is a solution of (4), for any values of the non-negative integers m, c_2. (28)

From (12), (29) the quadruple $(k, x, y, z) = (0, c_1, c_2, 1)$ is a solution of (4), where c_1, c_2 are non-negative integers.

4. Open problem

In 2013, Chotchaisthit [6] investigated for the solutions of the Diophantine equation $p^x + (p+1)^y = z^2$, where x, y, z are nonnegative integers and p is a Mersenne prime. But there the case when p is not a Mersenne prime number is still an open problem. Hence the title of this paper is an open problem.

5. Conclusion

In this work the Exponential Diophantine equation $(3^{\mathrm{k}} - 1)^{\mathrm{x}} + (3^{\mathrm{k}})^{\mathrm{y}} = \mathrm{z}^2$ is investigated for the non-negative integer solutions and it takes the following integer solutions and some interesting observations agreeing the previous works are provided at the end.

1. The quadruple $(k, x, y, z) = (0, c_1, c_2, 1)$ is a solution, where c_1, c_2 are non-negative integers.
2. $(k, x, y, z) = (1, 0, 1, 2)$, $(1,3,0,3)$, $(1,4,2,5)$.
3. $(k, x, y, z) = (2n, 1, 0, 3^n)$, where n is a non- negative integer.

It is observed that k had never taken the odd positive integers such that $k > 1$ in the solutions of (4).
So when k is an odd positive integer and k>1, the Exponential Diophantine equation (4) has no solutions.

For $k = 0$, the equation (4) has infinitely many integer solutions.
For $k = 1$,the solutions found for (4) agree with (Suvarnamani, 2011) [4] and with (Sroysang, 2013) [7].

For $k = 2$, (1,0,3) is the unique nonnegative integer solution for (4), so there is no solution to (4) in positive integers x, y, z .This conclusion agrees with Burshtien [19].
For each even positive integer value of k, the Diophantine equation (4) has the unique solution in nonnegative integers x, y, z and hence (4) has no solutions in positive integers x, y, z.

This paper finds the solutions in the particular case when $k = 2n$ in (4) which act as part of solutions for the open problem suggested by Kulprapa Srimud and Suton Tadee [15] in their Conclusion part given at the end.

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Author's Contribution: This is author's sole contribution.

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