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An Acceptable Payment Delay Policy Method to Address the Deteriorating Inventory Model with Price-Dependent Demand

Md Abdul Hakim

Department of Mathematics, Comilla University, Cumilla-3506, Bangladesh
Email: mahakim1972@gmail.com

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Abstract. We have created an inventory model for degrading goods in this model with price-dependent demand under a credit policy approach. Any shortages are permitted and partially backlogged with a variable rate based on the amount of time until the arrival of the following lot. The associated models have been developed and solved. A numerical example has been considered to explain the findings, and the key aspects of the findings are discussed. Finally, using these examples as a basis, sensitivity studies have been used to study the effects of various parameters on the initial stock level, shortage level, cycle length, as well as ideal profit. Each parameter was evaluated separately, with the other values remaining constant.

***Keywords*:** Inventory, deteriorating item, shortage level, variable demand, credit policy.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

According to the existing literature of inventory control system, most of the inventory models have been developed under the assumption that the life time of an item is infinite while it is in storage i.e., an item once in stock remains unchanged and fully usable for satisfying future demand. In real life situation, this assumption is not always true due to the effect of deterioration in the preservation of commonly used physical goods like wheat, paddy or any other type of food grains, vegetables, fruits, drugs, pharmaceuticals, etc. A certain fraction of these goods are either damaged or decayed or vaporized or affected by some other factors, etc. and are not in a perfect condition to satisfy the demand. As a result, the loss due to this natural phenomenon (i.e., the deterioration effect) can’t be ignored in the analysis of the inventory system. Ghare and Schrader [1] first developed an inventory model for exponentially decaying inventory. Then Emmons [2] proposed this type of model with variable deterioration which follows two-parameter Weibull distribution. These models were extended and improved by several researchers, viz. Covert and Philip[3] Giri et al[4], Ghosh and Chaudhari [5]. On the other hand, Chakrabarty et al. [6], Giriet al[4], Sana et al. [7], Sana and Chaudhari [8] and others developed inventory models for deteriorating items with there-parameter Weibull distributed deterioration. Misra [9] developed an EOQ model with Weibull deterioration rate for perishable product without considering shortages. These investigations were followed by several researchers like, Deb and Chaudhari [10] , Giri et. al. [4], Goswami and Chaudhari [11], Mandal and Phaujdar [12], Padmanabhan and Vrat [13], Pal et al.[14], Mandal and Maiti [15], Goyal and Gunasekaran [16], Bhunia et al. [17], etc., where a time-proportional deterioration rate was considered.

In the present competitive market, the effect of marketing policies and conditions such as the price variations and the advertisement of an item change its demand pattern amongst the public. The propaganda and canvassing of an item by advertisement in the well-known media such as Newspaper, Magazine, Radio, T. V., Cinema, etc. and also through the sales representatives have a motivational effect on the people to buy more. Also, the selling price of an item is one of the decisive factors in selecting an item for use. It is commonly observed that lower selling price causes increase in demand whereas higher selling price has the reverse effect. Hence, it can be concluded that the demand of an item is a function of displayed inventory in a show-room, selling price of an item and the advertisement expenditures frequency of advertisement, Very few OR researchers and practitioners studied the effects of price variations and the advertisement on the demand rate of items. Kotler [13] incorporated marketing policies into inventory decisions and discussed the relationship between economic order quantity and pricing decisions. Ladany and Sternleib [13] studied the effect of price variation on selling and consequently on EOQ.

On the other hand, in the present competitive marketing situation, permissible delay in payment is one of the most important factors in the business world. Therefore wholesalers/suppliers offer different types of facilities to their retailers. One such facility is to offer to sell a large volume of goods on credit within a certain time period. During this period, no interest is charged from the retailer. However, after this period, a higher rate of interest is charged by the supplier under certain terms and conditions. This type of inventory problem is known as an inventory problem with permissible delay in payments. This type of problem was first introduced by Haley and Higgins [14]. After Haley and Higgins [15], a number of works have been done by several researchers. These works include different types of considerations regarding demand, shortage policy, deterioration, two or multiple warehousing, etc.

2. Assumptions and notations

Assumptions

The following assumptions and notations are used to develop the proposed model:

(i) The entire lot is delivered in one batch.

(ii) The demand rate $D(p)$ is dependent on the selling price (p) of an item and it is denoted by $D(p) = a\,p^{-\alpha}, a, \alpha > 0$,

(iii) The inventory system involves only one item and one stocking point and the inventory planning horizon is infinite

(iv) Replenishments are instantaneous with a known and lead time is constant.

(v) The replenishment cost (ordering cost) is constant and the transportation cost does not include replenishing the item.

(vi) The deteriorated units ware neither repaired nor refunded.

(vii) Shortages, if any, are allowed and unsatisfied demands are partially backlogged. During the stock-out period, the backlogging rate is dependent on the length of the waiting time up to the arrival of the next lot. In this situation, the rate is defined as

$$\frac{1}{[1+\delta(T-t)]}, (\delta > 0).$$

Notations:

$q(t)$	Inventory level at time t
S	Highest stock level at the beginning of stock-in period
R	Highest shortage level
α	Positive constant
θ	Deterioration rate ($0<\theta<<1$)
C_o	Replenishment cost per order
δ	Backlogging parameter
C_p	Purchasing cost per unit
p	The selling price per unit of the item is denoted by $p = m\,C_p$
$D(p)$	Price dependent demand
C_1	Holding cost per unit per unit time
C_b	Shortage cost per unit per unit time
C_{ls}	Unit opportunity cost due to lost sale
t_2	The time at which the stock level reaches zero
T	The time at which the highest shortage level reaches the lowest point
M	The credit period offered by the supplier
$\boldsymbol{I_e}$	Interest earned by the retailer
$\boldsymbol{I_p}$	Interest charged by the suppliers to the retailers
$Z(.)$	The average profit

3. Inventory model with shortages

In this model, shortages, if any, are allowed and partially backlogged. During the shortage period, the backlogging rate is dependent on the length of the waiting time up to the arrival of fresh lot. Considering this situation, the rate is defined as $[1+\delta(T-t)]^{-1}, \delta > 0$.

In this model, it is assumed that after fulfilling the backorder quantity, the on-hand inventory level is S at $t=0$ and it declines continuously up to the time $t = t_2$ when it reaches the zero level. The decline in inventory during the closed time interval $t_1 \leq t \leq t_2$ occurs due to the customer's demand and deterioration of the item. After the time $t = t_2$ shortage occurs and it accumulates at the rate $[1+\delta(T-t)]^{-1}$, $(\delta > 0)$ up to the time $t = T$, when the next lot arrives. At time $t = T$, the maximum shortage level is R. This entire cycle then repeats itself after the cycle length T.

Let $q(t)$ be the instantaneous inventory level at any time $t \geq 0$. Then the inventory level $q(t)$ at any time t satisfies the differential equations as follows:

$$\frac{dq(t)}{dt} = -D(p), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dq(t)}{dt} + \theta q(t) = -D(p), \qquad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dq(t)}{dt} = \frac{-D(p)}{1+\delta(T-t)}, \quad t_2 < t \le T \tag{3}$$

with the boundary conditions

$q(t) = S$ at $t = 0$, $\quad q(t) = 0$ at $t = t_2$. (4)

And $q(\mathrm{t}) = -\mathrm{R}$ at $\mathrm{t} = \mathrm{T}$. (5)

Also, $q(\mathrm{t})$ is continuous at $t = t_1$ and t_2.

Using the conditions (3) and (.4), the solutions of the differential equations (1)-(.2) are given by

$$q(t) = S - D(p)t \qquad\qquad 0 \le t \le t_1$$

$$= \frac{D(p)}{\theta}\left\{e^{\theta(t_2-t)} - 1\right\} \qquad t_1 \le t \le t_2$$

$$= \frac{D(p)}{\delta}\log\left|1+\delta(T-t)\right| - R, \quad t_2 < t \le T$$

Using continuity condition we have

$$S = D(p)t_1 + \frac{D(p)}{\theta}\left\{e^{\theta(t_2-t_1)} - 1\right\} \tag{6}$$

From the continuity condition, we have

$$R = \frac{D(p)}{\delta}\log\left|1+\delta(T-t_1)\right| \tag{7}$$

The total number of deteriorated units is given by

Now the total inventory holding cost for the entire cycle is given by

$$C_{hol} = C_1\left\{\int_0^{t_1} q(t)\,dt + \int_{t_1}^{t_2} q(t)dt\right\}$$

Again, the total shortage cost C_{Sho} over the entire cycle is given by

$$C_{sho} = C_b\int_{t_2}^{T}\left\{-q(t)\right\}dt$$

Cost of lost sale OCLS over the entire cycle is given by

$$OCLS = C_{ls}\int_{t_1}^{t_2}\left\{1-\frac{1}{1+\delta(T-t)}\right\}D(p)dt$$

Total sales revenue during the entire cycle is

$$SR = p\int_{0}^{t_2} D(p)dt + p\,R$$

The supplier offered the trade credit period of his retailers is M and there may arise three scenarios as follows:

Case 1 $\boldsymbol{M} < \mathrm{t}_1$

Case 2 $\boldsymbol{t_1} < \boldsymbol{M} < \boldsymbol{t_2}$

Case 3 $\boldsymbol{t_2} < \boldsymbol{M}$

Now, we shall discuss all the cases in details.

Case 1: $M < t_1$
In this scenario, the total interest earned during the period $[0, M]$ is given by

$$IE1 = p\int_{0}^{M} D(p)dt + pRI_eM$$

Again, the interest paid during the period $[M, t_2]$ is given by

$$IP1 = pI_p\int_{M}^{t_1} q(t)dt + pI_p\int_{t_1}^{t_2} q(t)dt$$

Hence, in this case, the average profit $Z^{(1)}(t_1, T)$ is given by

$$Z^{(1)}(t_1, T) = \frac{X_1}{T}$$

where X_1=*SR+IE1-IP1-TC*

And *TC* = *<ordering cost>* + *<purchasing cost>* + *<inventory holding cost>*
+ *<cost of lost sale>*+*<inventory shortage cost>*

$$= C_4 + C_p(S+R) + C_{hol} + OCLS + C_{sho}$$

Case 2 $\boldsymbol{t_1 < M < t_2}$

In this scenario, the total interest earned during the period $[0, M]$ is given by

$$IE2 = p\int_{0}^{M} D(p)dt + pRI_e M$$

Again, the interest paid during the period $[M, t_2]$ is given by

$$IP2 = pI_p \int_{M}^{t_2} q(t)dt$$

Hence, in this case, the average profit $Z^{(2)}(t_1, T)$ is given by

$$Z^{(2)}(t_1, T) = \frac{X_2}{T}$$

where $X_2 = SR + IE1 - IP2 - TC$

And *TC* = <*ordering cost*> + <*purchasing cost*> + <*inventory holding cost*>

+ <*cost of lost sale*>+<*inventory shortage cost*>

$$= C_4 + C_p(S+R) + C_{hol} + OCLS + C_{sho}$$

Case 3 $\boldsymbol{t_2 < M}$

In this scenario, the total interest earned during the period $[0, M]$ is given by

$$IE3 = \frac{pI_e D(p) M^2}{2} + pRI_e M$$

Hence, in this case, the average profit $Z^{(3)}(t_1, T)$ is given by

$$Z^{(3)}(t_1, T) = \frac{X_3}{T}$$

where $X_3 = SR + IE1 - TC$

and *TC* = <*ordering cost*> + <*purchasing cost*> + <*inventory holding cost*>
+ <*cost of lost sale*>+<*inventory shortage cost*>

$$= C_4 + C_p(S+R) + C_{hol} + OCLS + C_{sho}$$

4. Numerical example

To illustrate the model with partially backlogged shortages, a numerical example with the following data has been considered.

$C_1 = \$1.5$ per unit per unit time, C_b=$12 per unit per unit time, $C_p = \$22$ per unit,

$C_o = \$250,$ per order,

$\theta = 0.05$, a = 250, b = 0.3, M=0.3,

$\delta = 1.5,\ I_e = 0.09, I_p = 0.11,$

$C_{ls} = 15, \alpha = 0.2.$

According to the solution procedure, the optimal solution has been obtained with the help of LINGO software for different values of m. The optimum values of t_1, t_2, T, S and R along with the maximum average profit are displayed in Table 1.

Table 1: Optimal solution for different values of mark-up rate m

m	*Cases*	S	R	t_1	t_2	T	Z	*Remark*
1.25	Case 1	114.2779	12.1205	0.6879	0.8869	0.9879	308.4783	Case 1
	Case 2	99.4807	12.9577	M	0.7666	0.8751	276.7441	
	Case 3	38.6540	18.4605	M	M	0.4598	74.3889	
1.30	Case 1	112.4419	11.7605	0.6875	0.8795	0.9781	441.9943	Case 1
	Case 2	98.2310	12.5486	M	0.7629	0.8687	410.8646	
	Case 3	38.3520	17.8451	M	M	0.4552	207.6092	
1.32	Case 1	111.7296	11.6237	0.6596	0.8766	0.9743	494.8845	Case 1
	Case 2	97.7429	12.3933	M	0.7615	0.8662	463.9918	
	Case 3	38.2350	17.6109	M	M	0.4535	260.4167	

5. Sensitivity analysis

For the given example mentioned earlier, sensitivity analysis has been performed to study the effect of changes (under or over-estimation) of different parameters like demand, deterioration, inventory cost parameters and mark-up rate on maximum initial stock level, shortage level, cycle length, frequency of advertisement along with the maximum profit of the system. This analysis has been carried out by changing (increasing and decreasing) the parameters from – 20% to + 20%, taking one or more parameters at a time making the other parameters at their more parameters at a time and making the other parameters at their original values. The results of this analysis are shown in Table 2.

Table 2: Sensitivity analysis with respect to different parameters with respect

Parameter	% changes of parameters	% changes in Z^*	% changes in R^*	S^*	T^*
	– 20	3.33	-3.14	3.54	2.84

	– 10	1.73	-1.64	1.82	1.46
C_1	10	-1.70	1.61	-1.74	-1.38
	20	-3.36	3.19	-3.39	-2.70
	– 20	0.33	5.65	-0.28	0.92
C_b	– 10	0.16	2.75	-0.14	0.18
	10	-0.15	-2.61	0.13	-0.16
	20	-0.30	-5.08	0.25	-0.31
	– 20	---	---	---	---
C_p	– 10	-14.93	0.28	4.85	2.17
	10	14.67	-0.34	-4.26	-2.00
	20	29.09	-0.74	-8.08	-3.86
	– 20	12.21	-11.47	-10.33	-10.51
C_o	– 10	5.93	-5.60	-5.02	-5.11
	10	-5.64	5.37	4.78	4.88
	20	-11.04	10.53	9.34	9.55
	– 20	-30.92	-9.57	-10.76	11.81
a	– 10	-15.63	-4.65	-5.24	5.42
	10	15.94	4.42	5.00	-4.64
	20	32.09	8.42	9.77	-8.68
	– 20	---	---	---	---
α	– 10	11.01	3.08	3.49	-3.28
	10	-10.17	-2.98	-3.37	3.41
	20	---	---	---	---

This model includes a credit policy approach and a deterministic inventory model for degrading goods with changing demand based on selling prices $D(p) = a\,p^{-\alpha}$.

6. Concluding remarks

The demand rate is used in this model as $D(p) = a\,p^{-\alpha}$. Price demand is a well-known phenomenon. The current model can be used to solve issues where the demand for products is influenced by both their selling prices and their advertising. It also applies to items that are in style.

A relevant area of study is the issue of interconnections between inventory and transportation. Based on what has been shown in this model, there is a lot of room for future research in the area.

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