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Multiplicative Elliptic Sombor and Multiplicative Modified Elliptic Sombor Indices

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Abstract. We define the multiplicative elliptic Sombor index, and multiplicative modified elliptic Sombor index and determine exact formulas for two families of dendrimer nanostars.

Keywords: Multiplicative elliptic Sombor index, multiplicative modified elliptic Sombor index, nanostar

AMS Mathematics Subject Classification (2010): 05C07, 05C09, 05C92

1. Introduction

Throughout this paper, G is a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. For definitions and notations, we refer to the book [1].

Chemical Graph theory has an important effect on the development of Mathematical Chemistry. Topological indices have been considered in Chemistry and have found useful applications, especially in *QSPR*/*QSAR* research see [2, 3].

The elliptic Sombor index [4] of a graph G is defined as

$$ESO(G) = \sum_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}.$$

The modified elliptic Sombor index [5] of a graph G is defined as

$$^{m}ESO(G) = \sum_{uv\in E(G)} \frac{1}{(d_u + d_v)\sqrt{d_u^2 + d_v^2}}.$$

We propose the multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices and they are defined as

$$ESOII(G) = \prod_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}$$

$$^{m}ESOII(G) = \prod_{uv\in E(G)} \frac{1}{(d_u + d_v)\sqrt{d_u^2 + d_v^2}}$$

Recently, some Sombor indices were studied in [6-20] and also some new graph indices were studied in [21-23].

In this paper, we compute the multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices of two families of dendrimer nanostars.

2. Results for dendrimer nanostars $D_1[n]$

We consider a family of dendrimer nanostars with n growth stages, denoted by $D_1[n]$. The molecular graph of $D_1[4]$ with 4 growth stages is depicted in Figure 1.

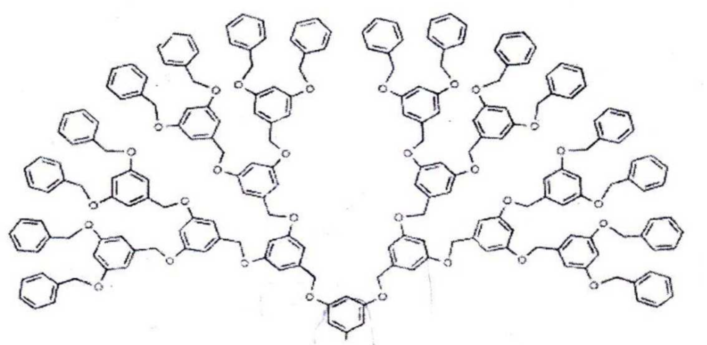

Figure 1: The molecular graph of $D_1[4]$.

Let G be the molecular graph of dendrimer nanostar $D_1[n]$. We obtain that G has $2^{n+4} - 9$ vertices and $18 \times 2^n - 11$ edges. We partition the edge set $E(D_1[n])$ into three sets as follows:

$E_{13} = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}$ $\quad |E_{13}| = 1.$

$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$ $\quad |E_{22}| = 6 \times 2^n - 2.$

$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$ $\quad |E_{23}| = 12 \times 2^n - 10.$

Theorem 1. The multiplicative elliptic Sombor index of a dendrimer nanostar $D_1[n]$ is given by

$$ESOII(G) = \left[4\sqrt{10}\right]^1 \times \left[8\sqrt{2}\right]^{6\times 2^n-2} \times \left[5\sqrt{13}\right]^{12\times 2^n-10}$$

Proof: We have

$$ESOII(G) = \prod_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}$$

$$= \left[(1+3)\sqrt{1^2+3^2}\right]^1 \times \left[(2+2)\sqrt{2^2+2^2}\right]^{6\times 2^n-2} \times \left[(2+3)\sqrt{2^2+3^2}\right]^{12\times 2^n-10}$$

$$= \left[4\sqrt{10}\right]^1 \times \left[8\sqrt{2}\right]^{6\times 2^n-2} \times \left[5\sqrt{13}\right]^{12\times 2^n-10}.$$

Theorem 2. The multiplicative modified elliptic Sombor index of a dendrimer nanostar $D_1[n]$ is given by

$${}^{m}ESOII(G) = \left[\frac{1}{4\sqrt{10}}\right]^1 \times \left[\frac{1}{8\sqrt{2}}\right]^{6\times 2^n-2} \times \left[\frac{1}{5\sqrt{13}}\right]^{12\times 2^n-10}.$$

Proof: We have

$$^{m}ESOII(G) = \prod_{uv\in E(G)} \frac{1}{(d_u + d_v)\sqrt{d_u^2 + d_v^2}}$$

$$= \left[\frac{1}{(1+3)\sqrt{1^2+3^2}}\right]^{1} \times \left[\frac{1}{(2+2)\sqrt{2^2+2^2}}\right]^{6\times 2^n - 2} \times \left[\frac{1}{(2+3)\sqrt{2^2+3^2}}\right]^{12\times 2^n - 10}.$$

After simplification, we obtain the desired result.

3. Results for dendrimer nanostars $D_3[n]$

We consider of dendrimer nanostars with n growth stages, denoted by $D_3[n]$, where $n\geq 0$. The molecular structure of $D_3[n]$ with 3 growth stages is shown in Figure 2.

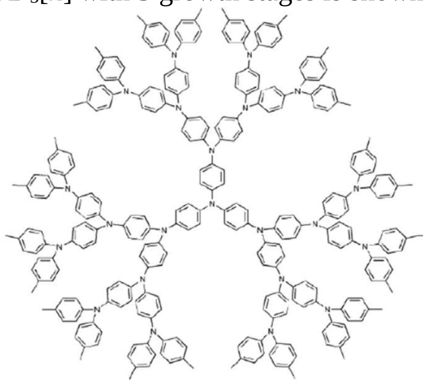

Figure 2: The molecular structure of $D_3[3]$

Let G be the graph of a dendrimer nanostar $D_3[n]$. We obtain that G has $24 \times 2^n - 20$ vertices and $24 \times 2^{n+1} - 24$ edges. The edge set $E(D_3[n])$ can be divided into four partitions:

$E_{13} = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}$	$\lvert E_{13}\rvert = 3\times 2^n$.
$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$	$\lvert E_{22}\rvert = 12\times 2^n - 6$.
$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$	$\lvert E_{23}\rvert = 24\times 2^n - 12$.
$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$	$\lvert E_{33}\rvert = 9\times 2^n - 6$.

Theorem 3. The multiplicative elliptic Sombor index of a dendrimer nanostar $D_3[n]$ is given by

$$ESOII(G) = [4\sqrt{10}]^{3\times 2^n} \times [8\sqrt{2}]^{12\times 2^n - 6} \times [5\sqrt{13}]^{24\times 2^n - 2} \times [18\sqrt{2}]^{9\times 2^n - 6}$$

Proof: We have

$$ESOII(G) = \prod_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}$$

$$= \left[(1+3)\sqrt{1^2+3^2}\right]^{3\times 2^n} \times \left[(2+2)\sqrt{2^2+2^2}\right]^{12\times 2^n - 6}$$

$$\times \left[(2+3)\sqrt{2^2+3^2}\right]^{24\times 2^n - 2} \times \left[(3+3)\sqrt{3^2+3^2}\right]^{9\times 2^n - 6}$$

$$= [4\sqrt{10}]^{3\times 2^n} \times [8\sqrt{2}]^{12\times 2^n - 6} \times [5\sqrt{13}]^{24\times 2^n - 2} \times [18\sqrt{2}]^{9\times 2^n - 6}$$

Theorem 4. The multiplicative modified elliptic Sombor index of a dendrimer nanostar $D_3[n]$ is given by

$$^{m}ESOII(G) = \left[\frac{1}{4\sqrt{10}}\right]^{3\times 2^n} \times \left[\frac{1}{8\sqrt{2}}\right]^{12\times 2^n-6} \times \left[\frac{1}{5\sqrt{13}}\right]^{24\times 2^n-2} \times \left[\frac{1}{18\sqrt{2}}\right]^{9\times 2^n-6}.$$

Proof: We have

$$^{m}ESOII(G) = \prod_{uv\in E(G)} \frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$

$$= \left[\frac{1}{(1+3)\sqrt{1^2+3^2}}\right]^{3\times 2^n} \times \left[\frac{1}{(2+2)\sqrt{2^2+2^2}}\right]^{12\times 2^n-6}$$

$$\times \left[\frac{1}{(2+3)\sqrt{2^2+3^2}}\right]^{24\times 2^n-2} \times \left[\frac{1}{(3+3)\sqrt{3^2+3^2}}\right]^{9\times 2^n-6}$$

gives the desired result after simplification.

4. Conclusion

In this paper, we proposed the multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices of a graph. Also, the multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices of two families of dendrimer nanostars are determined.

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Author's Contributions: This is the authors' sole contribution.

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