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An Inventory Model with Deteriorating Items having Price Dependent Demand and Time Dependent Holding Cost under Influence of Inflation

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Abstract. This study presents a mathematical inventory model where the demand is considered a function of the selling price, indicating its dependence on the selling price. Also, the holding cost is assumed to be a linear function of time. The main objective is to determine the maximum total profit through the establishment of this model. To achieve this, we analyze the effects of varying parameter values used in our model. To illustrate the sensitivity analysis, a numerical example is employed. Graphs are generated to depict the relationships between the model parameters, the economic order quantity (EOQ), the optimal time, and the total profit.

Keywords: Price dependent demand, time dependent holding cost, deteriorating items, inflation

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

Inventory encompasses various types of goods, including physical resources, raw materials, and finished products. Inventory theory studies optimal resource allocation and employs information technology for decision-making. Costs associated with storage and maintenance are key considerations. Challenges such as product damage arise in inventory management. Over the past 20 years, numerous research papers have focused on inventory models. These studies explore various aspects such as demand patterns, including ramp, time, selling price, and combinations of time and demand, as well as linear and quadratic demand functions. In [2], the authors divided the time cycle into two parts and introduced a convex function with time. An inventory model was created for deteriorating items considering inflation in [18] and incorporated partial backlogging, time-varying replacement cycles, and time-varying shortage intervals. In [10], a mathematical inventory model was presented for managing deteriorating items by considering demand as a function of time with a ramp-type pattern, and the deterioration rate as a Weibull density function. In [11], the demand was taken time-dependent and quadratic, while the deterioration rate was assumed to be constant and maximized the overall profit. A stochastic deteriorating inventory (SDI) model was recommended in [5]. In [9], the authors explored a joint optimization model for multi-unit systems, specifically focusing on block replacement and periodic review inventory policies. In [3], an inventory model where the deterioration cost is assumed to be constant and the demand follows an exponential decline was presented. In [17], a mathematical inventory model was introduced and developed with an infinite replenishment rate, zero lead time, constant deterioration rate, and a holding cost that varies linearly over time. In [4] an innovative fuzzy inventory model was designed that addresses the challenges posed by deteriorating items with price and time-dependent demand under influence of inflation.

An EOQ model was developed considering two-stage deterministic demand in [14]. An inventory model introduced in [7] with deterioration of stochastic nature and exponential distribution. Various factors such as unreliability function and hazard rate function were derived in [12]. An inventory model developed in [1], in which deterioration rate taken as Weibull function with three parameter. And cost of transportation taken as depending on lot size. In [6], an inventory model was considered for a perishable rate modeled as a Pareto distribution and linear holding cost. In [16], the concept of inventory level was defined within three distinct intervals and incorporated the term "net discount rate of inflation" into the analysis. [8] extended an inventory model by dividing the inventory level into three parts. The model incorporated a demand rate that depends on the selling price and a Weibull deterioration rate. In [15], an EPQ model was presented that considered the carrying cost as a linear function of time and incorporated a price-dependent demand.

In our present article, we aimed to develop a novel mathematical model for inventory management. This model incorporates the concept of item deterioration over time, with the demand rate of items being dependent on the selling price. The holding cost is considered as a function of time. It is important to note that all these conditions are considered while considering the influence of inflation.

This paper is divided into several parts: Section 2 presents the notations and assumptions. The mathematical development of our model is presented in Section 3, along with its solution. Section 4, provides a numerical example where specific values of parameters are used. A table is included in Section 5, showing the sensitivity of the model. Section 6, presents the observations and results. The conclusion of the paper is presented in Section 7.

2. Assumption and notations

The demand rate is depending on selling price or function of selling price, i.e.,

$$D = a - bp^m, \quad 0 \le p \le \left(\frac{a}{b}\right)^m$$

where $a > 0$ and a is initial demand units/year. And b , m > 0

p(t): selling price/unit at time t.

r: inflation rate.

h(t): h.t, is time dependent holding cost, where h is holding cost parameter and is greater than zero.

A: ordering Cost

C: unit purchase costs

T: cycle time.

θ: deterioration rate and $0 < \theta < 1$.

Q: lot size.

TP: total profit/cycle time.

3. Mathematical formulation of the model

In this model initial at t=0, inventory level is Q. Then inventory level diminishes due to demand and deterioration. It becomes zero at time t=T. Figure 1 describe our mathematical model.

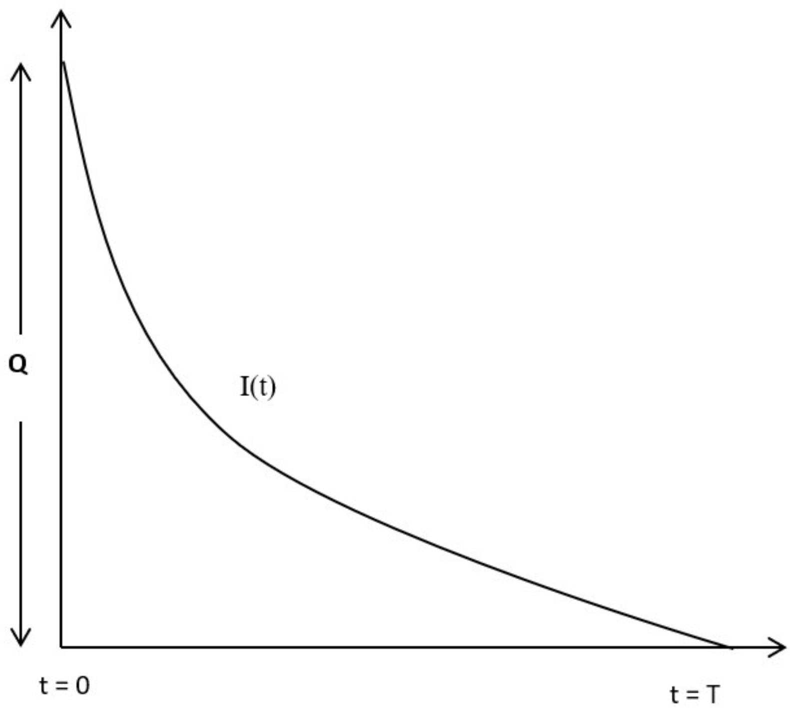

Figure 1:

Governing Differential equation of the model: -

$$\frac{d}{dt}\left(I\left(t\right)\right)+\theta I\left(t\right)=-D$$

With condition $I(T)=0$

We have

$$\frac{d}{dt}\left(I(t)\right)+\theta I(t)=-(a-bp^{m})=bp^{m}-a \qquad \ldots(1)$$

Integrating Factor of equation (1)

$$e^{\int \theta dt}=e^{\theta t}$$

Solution of equation (1)

$I(t)e^{\theta t}=(\mathrm{bp}^{m}-a)\int e^{\theta t}dt+c_1$ with taking $I(T)=0$

We get $I(t)=\left(\frac{a-bp^{m}}{\theta}\right)\left[e^{\theta(T-t)}-1\right]$ ...(2)

using $I(0)=Q$, level get initial inventory level

$$Q=\frac{a-bp^{m}}{\theta}\left(e^{\theta T}-1\right) \qquad \ldots.(3)$$

Different costs associated with this model are calculated as follows:

(i) Ordering Cost (O.C.) $=$A

(ii) The Sales revenue (S.R.) $=\int_0^t p.Ddt=\int_0^t p.\left(a-bp^{m}\right)dt=p.\left(a-bp^{m}\right)t$ .....(4)

(iii) The deterioration cost (D.C.) $=C\left[Q-\int_0^t De^{-rt}dt\right]$

$$=C\left(a-bp^{m}\right)\left[\left(\frac{e^{\theta T}-1}{\theta}\right)+\left(\frac{e^{-rt}-1}{r}\right)\right] \qquad \ldots..(5)$$

(iv) The holding cost (H.C.) $=\int_0^t h.t.e^{-rt}.I(t)dt$

$$=\frac{h\left(a-bp^{m}\right)}{\theta}\left[-\frac{1}{r^2}+\frac{e^{\theta T}}{(r+\theta)^2}+e^{-t(r+\theta)}\left\{e^{t\theta}\frac{(1+rt)}{r^2}-e^{T\theta}\frac{\left(1+t\left(r+\theta\right)\right)}{(r+\theta)^2}\right\}\right] \qquad \ldots(6)$$

(v) Total Cost (T.C.) $=$O.C. + D.C. +H.C

$$=A+C\left(a-bp^{m}\right)\left[\left(\frac{e^{\theta T}-1}{\theta}\right)+\left(\frac{e^{-rt}-1}{r}\right)\right]+\frac{h\left(a-bp^{m}\right)}{\theta}\left[-\frac{1}{r^2}+\frac{e^{\theta T}}{(r+\theta)^2}+e^{-t(r+\theta)}\left\{e^{t\theta}\frac{(1+rt)}{r^2}-e^{T\theta}\frac{\left(1+t\left(r+\theta\right)\right)}{(r+\theta)^2}\right\}\right]$$

.......(7)

(vi) Total Profit $=$S.R. – (O.C. + D.C. +H.C.)

$$= p.\left(a-bp^m\right)t - A - C\left(a-bp^m\right)\left[\left(\frac{e^{\theta T}-1}{\theta}\right)+\left(\frac{e^{-rt}-1}{r}\right)\right] - \frac{h\left(a-bp^m\right)}{\theta}\left[-\frac{1}{r^2}+\frac{e^{\theta T}}{(r+\theta)^2}+e^{-t(r+\theta)}\left\{e^{t\theta}\frac{(1+rt)}{r^2}-e^{T\theta}\frac{(1+t(r+\theta))}{(r+\theta)^2}\right\}\right] \quad \ldots\ldots(8)$$

(vii) Total Profit per cycle $= \frac{1}{T}\left[S.R. - \left(O.C. + D.C. + H.C.\right)\right]$

$$= \frac{p.\left(a-bp^m\right)t}{T} - \frac{A}{T} - \frac{C\left(a-bp^m\right)}{T}\left[\left(\frac{e^{\theta T}-1}{\theta}\right)+\left(\frac{e^{-rt}-1}{r}\right)\right] - \frac{h\left(a-bp^m\right)}{T\theta}\left[-\frac{1}{r^2}+\frac{e^{\theta T}}{(r+\theta)^2}+e^{-t(r+\theta)}\left\{e^{t\theta}\frac{(1+rt)}{r^2}-e^{T\theta}\frac{(1+t(r+\theta))}{(r+\theta)^2}\right\}\right] \quad \ldots\ldots\ldots(9)$$

4. Numerical example

Values of parameters used in our Inventory Model as:
a = 100 units/year, b= 0.2, θ = 0.05, C = 20 units/year, h = 80/year, p = 25/unit, m = 2, T=3, r = 0.003, A=1000

We put these values in equation (3) and (9), and solved this problem by MATLAB Software. We obtained the following optimum values:

t* = 2.7991
Q* =2832.1
TAP=89775

5. Sensitivity analysis

By changing values of parameters used in our model and read out the effects on t*, Q* and TAP. Rate of changes in values of parameters are taken -20 %, -10%, +10% and +20%. (see the table 1)

Table 1 Variation of t*, Q* and TAP w.r.t. a, b, p, m, T, r, C, θ, h, and A

Parameter	Change in Parameter	t*	Q*	TAP
	-20%	2.7991	2184.8	69331
	-10%	2.7991	2508.4	79553
a =1000	0%	2.7991	2832.1	89775
	10%	2.7991	3155.8	99996
	20%	2.7991	3479.4	110220
	-20%	2.7991	2913	92330
	-10%	2.7991	2872.6	91052
b =0.2	0%	2.7991	2832.1	89775
	10%	2.7991	2791.6	88497
	20%	2.7991	2751.2	87219
	-20%	2.8229	2977.8	98685
	-10%	2.8111	2909	94303
p = 25	0%	2.7991	2832.1	89775
	10%	2.7871	2747.1	85116

	20%	2.7749	2654.1	80344
m =2	-20%	2.7991	3125	99026
	-10%	2.7991	3024.2	95840
	0%	2.7991	2832.1	89775
	10%	2.7991	2466.5	78228
	20%	2.7991	1770.5	56248
T=3	-20%	2.1377	2231.2	50779
	-10%	2.4729	2529.4	69052
	0%	2.7991	2832.1	89775
	10%	3.1196	3139.4	112910
	20%	3.4360	3451.3	138460
r = 0.003	-20%	2.7993	2832.1	89856
	-10%	2.7992	2832.1	89815
	0%	2.7991	2832.1	89775
	10%	2.7990	2832.1	89734
	20%	2.7989	2832.1	89694
C=20	-20%	2.8181	2832.1	89261
	-10%	2.8086	2832.1	89515
	0%	2.7991	2832.1	89775
	10%	2.7896	2832.1	90039
	20%	2.7800	2832.1	90310
θ =0.055	-20%	2.7989	2789	88660
	-10%	2.7990	2810.4	89215
	0%	2.7991	2832.1	89775
	10%	2.7992	2854	90338
	20%	2.7993	2876.1	90906
h = 80	-20%	2.7442	2832.1	68400
	-10%	2.7750	2832.1	79067
	0%	2.7991	2832.1	89775
	10%	2.8186	2832.1	100510
	20%	2.8346	2832.1	111270
A=1000	-20%	2.7991	2832.1	89708
	-10%	2.7991	2832.1	89741
	0%	2.7991	2832.1	89775
	10%	2.7991	2832.1	89808
	20%	2.7991	2832.1	89841

6. Observations and results

From above table we find results as following:
The values of t* (time), TP (total production), and Q* (quantity) show different patterns when varying certain parameters. Here's a summary of the trends:

(i) With increasing parameter a, the values of t* and TP increase linearly, while Q* remains unchanged.

(ii) With increasing parameter A, the values of t* and Q* remain unchanged, while TP increases linearly.
(iii) With increasing parameter b, the value of t* remains unchanged, while both Q* and TP decrease linearly.
(iv) With increasing parameter c, Q* remains unchanged, t* decreases linearly, and TP increases linearly.
(v) When increasing parameter h, Q* remains unchanged, t* initially increases and then remains constant, and TP increases linearly.
(vi) When increasing parameter m, Q* and TP initially decrease linearly and then become constant. The values of t* remain unchanged.
(vii) When increasing parameter p, Q* initially decreases linearly and then becomes constant. Both t* and TP decrease linearly.
(viii) When increasing parameter r, Q* remains unchanged, while both t* and TP decrease linearly.
(ix) When increasing parameter T, Q* and t* initially increase linearly and then become constant, while TP(T) increases linearly.
From the above results, it can be observed that Q*, t*, and TP all increase linearly with increasing values of the parameter Theta.

7. Conclusion

We tried to develop an inventory model while also aiming to maximize total profit and minimize total cost. The value of total profit increases as the values of parameters a, T, C, theta, h, and A increase. The value of total profit decreases as the values of parameters b, p, m, and r increase. Therefore, this model proves to be very helpful for inventory holders who have similar conditions in their business, profession, or companies. By incorporating additional considerations, conditions, and special assumptions, we have enhanced the benefits and realism of this model for future applications.

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