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Some Features of Pairwise α-T_1 Spaces in Supra Fuzzy Bitopology

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Abstract. Four concepts of supra fuzzy pairwise T_1 bitopological spaces are introduced and studied in this paper. We also establish some relationships among them and study some other properties of these spaces.

Keywords: Fuzzy set, fuzzy bitopological space, supra fuzzy bitopological space, good extension

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1. Introduction

American Mathematician Zadeh in his historical paper [16] has introduced the concept of fuzzy sets. This historic paper of Zadeh enthralled Mathematicians all over the world and they started to study almost all the Mathematical concepts based on Cantor's set theory in terms of fuzzy set theory. In this way a new branch of Mathematics started to emerge which is today known as fuzzy Mathematics. Chang [3] and Lowen [7] developed the theory of fuzzy topological space using fuzzy sets. Next time much research have been done to extend the theory of fuzzy topological spaces in various directions. Lowen [7], Wong [14], Srivastava and Ali [13] have developed the fuzzy topological spaces as well as fuzzy subspace topology. Hossain and Ali [4] worked on T_1 -fuzzy topological spaces.

The research for fuzzy bitopological spaces started in early nineties. The fuzzy bitopological spaces with separation axioms has become attractive as these spaces possesses many desirable properties and can be found throughout various areas in fuzzy topologies. Recent progress has been made constructing separation axioms on fuzzy bitopological space in [5, 6, 11]. Ruhul Amin et al. [12] have also developed T_1 concepts in fuzzy bitopological spaces in quasi coincidence sense.

In this paper, we study, some features of $\alpha - T_1$-spaces in supra fuzzy bitopological spaces. Significant results have been obtained. We also establish relationship among them. As usual I = [0, 1] and $I_1 = [0, 1)$.

2. Preliminaries

In this section, we review some concepts, which will be needed in the sequel. Through the present paper X and Y are always presented non -empty sets.

Definition 2.1. [16] For a set X, a function $u: X \to [0, 1]$ is called a fuzzy set in X. For every $x \in X, u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say u is a fuzzy subset of X. Thus a usual subset of X, is a special type of a fuzzy set in which the ranges of the function is {0, 1}. The class of all fuzzy sets from X into the closed unit interval I will be denoted by I^X .

Definition 2.2. [16] Let X be a nonempty set and A be a subset of X. The function $I_A: X \to [0, 1]\{0, 1\}$ defined by $I_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ \ x \notin A \end{cases}$ is called the characterstic function of A. The present authors also write 1_x for the characterstic function of {x}. The characteristic functions of subsets of a set X are referred to as the crisp sets in X.

Definition 2.3. [3] Let X and Y be two sets and $f: X \to Y$ be a function. For a fuzzy subset u in X, we define a fuzzy subset v in Y by

$v(y) = \sup\{u(x)\}\, if\ f^{-1}[\{y\}] \neq \varphi, x \in X.$

=0; otherwise

Definition 2.4. [3] Let X and Y be two sets and $f: X \to Y$ be a function . For a fuzzy subset v of Y, the inverse image of v under f is the fuzzy subset $f^{-1}(v) = v \circ f$ in X and is defined by $f^{-1}(v)(x) = v\big(f(x)\big)$, for $x \in X$.

Definition 2.5. [3] Let X be a non empty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1,\ 0 \in t$

(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t$.

(iii) If $u_1,\ u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space(fts, in short) and members of t are called t-open(or simply open) fuzzy sets. If u is open fuzzy set, then the fuzzy sets of the form 1-u are called t-closed (or simply closed) fuzzy sets.

Definition 2.6. [7] Let X be a non empty set and t be the collection of fuzzy sets in I^X such that

(i) $1,\ 0 \in t$

(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i \in \Lambda} u_i \in t$.

(iii) If $u_1,\ u_2 \in t$ then $u_1 \cap u_2 \in t$.

(iv) All constants fuzzy sets in X belongs to t.

Then t is called a fuzzy topology on X.

Definition 2.7. [8] Let X be a non empty set. A subfamily t^* of I^X is said to be a supra fuzzy topology on X if and only if

(i) $1,\ 0 \in t^*$

(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological spaces. The elements of t^* are called supra open fuzzy sets in (X, t^*) and complement of a supra fuzzy open set is called supra closed fuzzy set

Definition 2.8. [8] Let (X, t) and (Y, s) be two topological spaces. Let s^* and t^* are associated supra fuzzy topologies with s and t respectively and $f:(X, s^*) \to (Y, t^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in t^*, f^{-1}(v) \in s^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous.

Definition 2.9. [2] Let (X, s^*) and (X, t^*) be two supra fuzzy topological spaces. If u_1 and u_2 are supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min\,[u_1(x), u_2(y)]$, for each pair $(x, y) \in X \times Y$.

Definition 2.10. [15] Suppose$\{X_i, i \in \Lambda\}$, be any collection of sets and X denoted the Cartesian product of these sets, i.e., $X = \prod_{i\in\Lambda} X_i$. Here X consists of all points $p =< a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_0 \in \Lambda$, the authors defined the projection π_{j_0} by $\pi_{j_0}(a_i : i \in \Lambda) = a_{j_0}$. These projections are used to define the product supra fuzzy topology.

Definition 2.11. [15] Let $\{X_\alpha\}_{\alpha\in\Lambda}$ be a family of nonempty sets. Let $X = \prod_{\alpha\in\Lambda} X_\alpha$ be the usual products of X_α's and let $\pi_\alpha : X \to X_\alpha$ be the projection. Further, assume that each X_α is a supra fuzzy topological space with supra fuzzy topology t_α^*. Now the supra fuzzy topology generated by $\{\pi_\alpha^{-1}(b) : b_\alpha \in t_\alpha^*, \alpha \in \Lambda\}$ as a sub basis, is called the product supra fuzzy topology on X. Thus if w is a basis element in the product, then there exists $\alpha_1, \alpha_2, \dots\dots\alpha_n \in \Lambda$ such that $w(x) = \min\{b_\alpha(x_\alpha) : \alpha = 1, 2, 3, \dots, n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$.

Definition 2.12. [1] Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f: X \to R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\in R$.

Definition 2.13. [9] Let (X, T) be a topological space and T^* be associated supra supra topology with T. Then the lower semi continuous topology on X associated with T^* is $\omega(T^*) = \{\mu : X \to [0, 1], \mu$ is supra lsc$\}$. If $\omega(T^*) : (X, T^*) \to [0, 1]$ be the set of all lower semi continuous (lsc) functions. We can easily show that $\omega(T^*)$ is a supra fuzzy topology on X.

Definition 2.14. [10] Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two supra fuzzy bitopological spaces and $f:(X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a supra pairwise fuzzy continuous if both the function $f:(X, s_1^*) \to (Y, s_2^*)$ and $f:(X, t_1^*) \to (Y, t_2^*)$ are supra fuzzy continuous.

Definition 2.15. [10] Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) are two supra fuzzy bitopological spaces and $f:(X, s_1^*, t_1^*) \to (Y, s_2^*, t_2^*)$ be a function. Then the function f is a supra pairwise fuzzy open if both the function $f:(X, s_1^*) \to (Y, s_2^*)$ and $f:(X, t_1^*) \to (Y, t_2^*)$ are supra fuzzy open. i.e., for every open set $u \in s_1^*$, $f(u) \in s_2^*$ and for every $v \in t_1^*, f(v) \in t_2^*$.

Definition 2.16. [15] Let $\{(X_i, s_i, t_i) : i \in \Lambda\}$ is a family of fuzzy bitopological spaces. Then the space $(\prod X_i, \prod s_i, \prod t_i)$ is called the product fuzzy bitopological space of the family $\{(X_i, s_i, t_i) : i \in \Lambda\}$, where $\prod s_i$ *and* $\prod t_i$ denote the usual product fuzzy topologies of the families $\{\prod s_i : i \in \Lambda\}$ and $\{\prod t_i : i \in \Lambda\}$ of the fuzzy topologies respectively on X .

Let S^* and T^* be two supra topologies associated with two topologies S and T respectively. Let P be the property of a supra bitopological space (X, S^*, T^*) and FP be its supra fuzzy topological analogue. Then FP is called a 'good extension' of P 'if and only if the statement (X, S^*, T^*) has P if and only if $(X, \omega(S^*), \omega(T^*))$ has FP" holds good for every supra topological space (X, S^*, T^*).

3. $\alpha - T_1(I), \alpha - T_1(II), \alpha - T_1(\mathrm{III})$ and $T_1(IV)$ spaces in supra fuzzy bitopological space

In this section, we have given some new notions of $\alpha - T_1$ such as $\alpha - T_1(i), \alpha - T_1(ii), \alpha - T_1(\mathrm{iii})$ and $T_1(iv)$ spaces in supra fuzzy bitopological spaces. We also discuss some properties of them and establish relationships among them by using these concepts.

Definition 3.1. Let (X, s^*, t^*) be a supra fuzzy bitopological space and $\alpha \in I_1$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$ and there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) = 0, u(y) > \alpha$ and there exists $v \in t^*$ such that $v(x) > \alpha, v(y) = 0$.

(c) (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ and there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$.

(d) (X, s^*, t^*) is a pairwise $T_1(iv)$ space if and only if for all distinct elements $x, y \in X$, there exists $u \in s^*$ such that $u(x) \neq u(y)$ and $v \in t^*$ such that $v(x) \neq v(y)$.

Lemma 3.1. Suppose (X, s^*, t^*) is a supra fuzzy bitopological space and $\alpha \in I_1$. Then the following implications are true:

(a) (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ implies (X, s^*, t^*) is a pairwise $T_1(iv)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_1(iv)$.

Proof: Suppose that (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$. We have to prove that (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$. Let x and y be two distinct elements in X. Since (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$, for $\alpha \in I_1$,by definition there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$ and there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$, which shows that there exists $u \in s^*$ such that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ and there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$. Hence by definition (c), (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$.

Suppose (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ .i.e., $u(x) \neq u(y)$and there exists $v \in t^*$

such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. i.e., $v(x) \neq v(y)$. Hence by definition (X, s^*, t^*) is a pairwise $T_1(iv)$.

Let (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ such that $u(x) = 0, u(y) > \alpha$ which implies that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ and there exists $v \in t^*$ such that $v(x) > \alpha, v(y) = 0$ which implies that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ and hence (X, s^*, t^*) a is pairwise $T_1(iv)$. Therefore the proof is complete.

The non-implications among pairwise $\alpha - T_1(i), \alpha - T_1(ii), \alpha -$ T_1(iii) and $T_1(iv)$ are shown in the following examples:

Example 3.1. Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.45, u(y) = 0.82$ and $v(x) = 0.82, v(y) = 0.45$. The supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ \text{constants}\}$ and $\{0,\ v,\ 1,\ \text{constants}\}$ respectively. For $\alpha = 0.85$, we can easily show that (X, s^*, t^*) is pair wise $T_1(iv)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_1(iii)$ it can also be shown that (X, s^*, t^*) is not a pairwise $\alpha - T_1(i)$ and pairwise $\alpha - T_1(ii)$.

Example 3.2. Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.42, u(y) = 0.85$ and $v(x) = 0.85, v(y) = 0.42$. The supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ \text{constants}\}$ and $\{0,\ v,\ 1,\ \text{constants}\}$ respectively. For $\alpha = 0.81$, we have $0 \leq u(x) \leq 0.81 < u(y) \leq 1$ and $0 \leq v(y) \leq 0.81 < v(x) \leq 1$. This according to the definition (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_1(i)$. Also it can be easily shown that (X, s^*, t^*) is not a pairwise $\alpha - T_1(ii)$.

Example 3.3. Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.63$ and $v(x) = 0.63, v(y) = 1$. Consider the supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ \text{constants}\}$ and $\{0,\ v,\ 1,\ \text{constants}\}$ respectively. For $\alpha = 0.75$, we have $(x) = 1\ , u(y) \leq 0.75$ and $(x) \leq 0.75\ , v(y) = 1$. This according to the definition (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ but (X, s^*, t^*) not a pairwise $\alpha - T_1(ii)$.

Example 3.4. Let X= $\{x,\ y\}$ and $u, v \in I^X$ are defined by $u(x) = 0, u(y) = 0.73$ and $v(x) = 0.73, v(y) = 0$. Consider the supra fuzzy topologies s^* and t^* on X are generated by $\{0,\ u,\ 1,\ \text{constants}\}$ and $\{0,\ v,\ 1,\ \text{constants}\}$ respectively. For $\alpha = 0.35$ it can be easily shown that (X, s^*, t^*) is a pair wise $\alpha - T_1(ii)$ but (X, s^*, t^*) not a pair wise $\alpha - T_1(i)$. This completes the proof.

Lemma 3.2. Let (X, s^*, t^*) is a supra fuzzy bitopological space and $\alpha, \beta \in I_1$ with $0 \leq \alpha \leq \beta < 1$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ implies (X, s^*, t^*) is a pairwise $\beta - T_1(i)$.

(b) (X, s^*, t^*) is a pairwise $\beta - T_1(ii)$ implies (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$.

$(c)$$(X, s^*, t^*)$ is a pairwise $0 - T_1(ii)$ implies (X, s^*, t^*) is a pairwise $0 - T_1(iii)$.

Proof: Suppose (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$. We have to show that (X, s^*, t^*) is a pairwise $\beta - T_1(i)$. Let any two distinct points $x, y \in X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$, for $\in I_1$, there exists $u \in s^*$ such that $(x) = , u(y) \leq \alpha$. This implies that $(x) = 1\ , u(y) \leq \beta$, since $0 \leq \alpha \leq \beta < 1$ and there exists exists $v \in t^*$ such that

$v(x) \leq \alpha, v(y) = 1$. This implies that $v(x) \leq \beta, v(y) = 1$, since $0 \leq \alpha \leq \beta < 1$. Hence by definition (X, s^*, t^*) is a pairwise $\beta - T_1(i)$.

Suppose (X, s^*, t^*) is a pairwise $\beta - T_1(ii)$. Then for $x, y \in X$, $x \neq y$ there exists $u \in s^*$ such that $u(x) = 0$,$u(y) > \beta$, which implies that $u(x) = 0$,$u(y) > \alpha$, since $0 \leq \alpha \leq \beta < 1$. And there exists $v \in t^*$ such that $v(x) > \beta, v(y) = 0$,which implies that $v(x) > \alpha, v(y) = 0$, since $0 \leq \alpha \leq \beta < 1$.
Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$.

Example 3.5. Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.72$ and $v(x) = 0.72, v(y) = 1$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, \text{constants}\}$ and $\{0, v, 1, \text{constants}\}$ respectively. Then by definition for $\alpha = 0.4$ and $\beta = 0.9$; (X, s^*, t^*) is a pairwise $\beta - T_1(i)$ but (X, s^*, t^*) is not a pairwise $\alpha - T_1(i)$.

Example 3.6. Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0, u(y) = 0.55$ and $v(x) = 0.45, v(y) = 0$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, \text{constants}\}$ and $\{0, v, 1, \text{constants}\}$ respectively. Then by definition for $\alpha = 0.40$ and $\beta = 0.65$; (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$ but (X, s^*, t^*) is not a pairwise $\beta - T_1(ii)$.

Theorem 3.1. Suppose (X, S^*, T^*) is a supra fuzzy bitopological space and $\alpha \in I_1$. Suppose the following statements:
(1) (X, S^*, T^*) be a pairwise T_1 space.
(2) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_1(i)$ space.
(3) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_1(ii)$ space.
(4) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_1(iii)$ space.
(5) $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_1(iv)$ space.
The following implications are true:
(a) (1) $\Rightarrow$ (2) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (1).
(b) (1) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (1).

Proof: Suppose (X, S^*, T^*) be a T_1 bitopological space. We have to prove that $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_1(i)$ space. Suppose x and y are two distinct elements in X. Since (X, S^*, T^*) be a pairwise T_1 space, there exists $U \in S^*$ such that $x \in U, y \notin U$ and there exists $V \in T^*$ such that $\in V, x \notin V$. By the definition of lsc, we have $I_U \in \omega(S^*)$ and $I_U(x) = 1, I_U(y) = 0$ and $I_V \in \omega(T^*)$and $I_V(x) = 0, I_V(y) = 1$.

Hence we have $(X, \omega(S^*), \omega(T^*))$ be a pairwise $\alpha - T_1(i)$. Further it is easy to show that (2) $\Rightarrow$ (3), (3) $\Rightarrow$ (4) and (4) $\Rightarrow$ (5).

We therefore prove that (5) $\Rightarrow$ (1). Suppose $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_1(iv)$ space. We have to prove that (X, S^*, T^*) be a pairwise T_1 space. Let $x, y \in X$, and $x \neq y$. Since $(X, \omega(S^*), \omega(T^*))$ be a pairwise $T_1(iv)$, there exists $u \in \omega(S^*)$ such that $u(x) < u(y)$ or $u(x) > u(y)$ and there exists $v \in \omega(T^*)$ such that $v(x) < v(y)$ or $v(x) > v(y)$. Suppose $u(x) < u(y)$ for $r_1 \in I_1$ such that $u(x) < r_1 < u(y)$. We observe that $x \notin u^{-1}(r_1, 1)$ and $y \in u^{-1}(r_1, 1)$. By definition of lsc $u^{-1}(r_1, 1) \in S^*$. Suppose $v(y) < v(x)$ and for $r_2 \in I_1$, such that $v(y) < r_2 < v(x)$. We observe that $y \notin v^{-1}(r_2, 1)$ and $x \in v^{-1}(r_2, 1)$ and by the definition of lsc $v^{-1}(r_2, 1) \in$

t^*. Hence (X, S^*, T^*) be a pairwise T_1 space. Thus it seen that pair wise $\alpha - T_1(p)$ is a good extension of its bitopological counterpart (p=I, ii, iii, iv).

Theorem 3.2. Let (X, s^*, t^*) be a supra fuzzy bitopological space, $\alpha \in I_1$ and let $I_\alpha(s^*) = \{u^{-1}(\alpha, 1): u \in s^*\}$ and $I_\alpha(t^*) = \{v^{-1}(\alpha, 1): v \in t^*\}$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$ implies $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1.

(c) (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ if and only if $\left(X, I_\alpha(s^*), I_\alpha(t^*)\right)$ is a pairwise T_1.

Proof: (a) Let (X, s^*, t^*) be a supra fuzzy bitopological space and (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$. Suppose x and y be two distinct elements in X. Then for $\alpha \in I_1$, there exists $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$. Since $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $y \notin u^{-1}(\alpha, 1)$, $x \in u^{-1}(\alpha, 1)$ and there exists $v \in t^*$ such that $v(x) \leq \alpha$, $v(y) = 1$. Since $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$, $x \notin v^{-1}(\alpha, 1)$, $y \in v^{-1}(\alpha, 1)$.
So this implies that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1.

Similarly (b) can be proved.

(c) Suppose (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$. Let $x, y \in X, x \neq y$, then for $\in I_1$, there exists $u \in s^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$. Since $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$, $x \notin u^{-1}(\alpha, 1), y \in u^{-1}(\alpha, 1)$ and there exists $v \in t^*$ such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$. Since $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$, $y \notin v^{-1}(\alpha, 1), x \in v^{-1}(\alpha, 1)$. So this implies that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1 space.

Conversely, suppose that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1 space. Let $x, y \in X, x \neq y$. Then there exists $u^{-1}(\alpha, 1) \in I_\alpha(s^*)$ such that $x \in u^{-1}(\alpha, 1)$ and $y \notin u^{-1}(\alpha, 1)$, where $u \in s^*$. Thus, we have $u(x) > \alpha, u(y) \leq \alpha$. i.e., $0 \leq u(y) \leq \alpha < u(x) \leq 1$ and there exists $v^{-1}(\alpha, 1) \in I_\alpha(t^*)$ such that $x \notin v^{-1}(\alpha, 1)$ and $y \in v^{-1}(\alpha, 1)$, where $v \in s^*$. Thus, we have $v(x) \leq \alpha, v(y) > \alpha$. i.e. $0 \leq v(x) \leq \alpha < v(y) \leq 1$. This implies that (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$

Example 3.7. Let X= $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0.6$ and $v(x) = 0..4, v(y) = 0.8$. . Let the supra fuzzy topologies s^* and t^* on X are generated by $\{0, u, 1, \text{constants}\}$ and $\{0, v, 1, \text{constants}\}$ respectively. Then by definition for $\alpha = 0.5$ (X, s^*, t^*) is not a pairwise $\alpha - T_1(ii)$. Now let $I_\alpha(s^*) = \{X, \varphi, \{x\}\}$ and let $I_\alpha(t^*) = \{X, \varphi, \{y\}\}$. Then we see that $I_\alpha(s^*)$ and $I_\alpha(t^*)$ are supra topology on X and $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise T_1 space. This completes the proof.

Theorem 3.3. Let (X, s^*, t^*) be a supra fuzzy bitopological space. $A \subseteq X$ and $s_A^* = \{u/A: u \in s^*\}$ and $t_A^* = \{v/A: v \in t^*\}$, then

(a) (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_1(i)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_1(ii)$

(c) (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_1(iii)$.

(d) (X, s^*, t^*) is a pairwise $\alpha - T_1(iv)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - T_1(iv)$.

Proof: Suppose that (X, s^*, t^*) is a supra fuzzy bitopological space and (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$ space. Let $x, y \in A$ with $x \neq y$. So that $x, y \in X$ as $A \subseteq X$. Since (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$, for $\alpha \in I_1$, there exists $u \in s^*$ such that $(x) = 1, u(y) \leq \alpha$. For $A \subseteq X$, we have $u/A \in s_A^*$ and $(u/A)(x) = 1$,then $(u/A)(x) = 1, (u/A)(y) \leq \alpha$. And there exists $v \in t^*$ such that $v(x) \leq \alpha, v(y) = 1$. For $A \subseteq X$, we have $v/A \in t_A^*$ and $(v/A)(y) = 1$, then $(v/A)(x) \leq \alpha, (v/A)(y) = 1$. as $x, y \in A$. Hence by definition(A, s_A^*, t_A^*) is a pairwise $\alpha - T_1(i)$.
Similarly (b), (c) and (d) can be proved.

Theorem 3.4. Suppose $\{ (X_i, s_i^*, t_i^*), i \in \Lambda\}$ is a family of supra fuzzy bitopological spaces and $(\prod X_i, \prod s_i^*, \prod t_i^*) = (X, s^*, t^*)$ be the product topological space on X, then

(a) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_1(i)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$
(b) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_1(ii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_1(ii)$.
(c) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_1(iii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_1(iii)$.
(d) $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_1(iv)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - T_1(iv)$.

Proof: Suppose $\forall\, i \in \Lambda$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - T_1(i)$. Let $x, y \in X$ with $x \neq y$, then $x_i \neq y_i$, for some $i \in \Lambda$. Since (X_i, s_i^*, t_i^*) is a pair wise $\alpha - T_1(i)$, for $\alpha \in I_1$,there exists $u_i \in s_i^*$, $i \in \Lambda$ such that $u_i(x_i) = 1, u_i(y_i) \leq \alpha$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $u_i(\pi_i(x)) = 1$and $u_i(\pi_i(y)) \leq \alpha$ i.e., $(u_i o \pi_i)(x) = 1, (u_i o \pi_i)(y) \leq \alpha$ and there exists $v_i \in t_i^*$, $i \in \Lambda$ such that $v_i(x_i) \leq \alpha$ and $v_i(y_i) = 1$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $v_i(\pi_i(x)) \leq \alpha$ and $v_i(\pi_i(y)) = 1$ i.e., $(v_i o \pi_i)(x) \leq \alpha$, $(v_i o \pi_i)(y) = 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - T_1(i)$.

Conversely, suppose that (X, s^*, t^*) is pairwise $\alpha - T_1(i)$. We have to show that (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is a pairwise $\alpha - T_1(i)$. Let a_i be a fixed point in X_i and $A_i = \{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_{i\,,}$ for some $i \neq j\}$. Thus A_i is a subset of X and hence $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is pairwise $\alpha - T_1(i)$, $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a pairwise $\alpha - T_1(i)$. Now we have A_i is homeomorphic image of X_i. Thus (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is a pairwise $\alpha - T_1(i)$.
Similarly (b), (c) and (d) can be proved.

Theorem 3.5. Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) be two supra fuzzy bitopological spaces. $f: X \to Y$ be one-one, onto and open map, then

(a) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(i)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(i)$.
(b) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(ii)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(ii)$.
(c) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(iii)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(iii)$.
(d) (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(iv)$ implies (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(iv)$.

Proof: (a) Suppose (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(i)$. We have to prove that (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_1(i)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$, there exist $x_1, x_2 \in X$ with $f(x_1) = y_1, f(x_2) = y_2$, since f is onto and $x_1 \neq x_2$ as f is one-one. Again since (X, s_1^*, t_1^*) is pair wise $\alpha - T_1(i)$, $\alpha \in I_1$, there exists $u \in s_1^*$ such that $u(x_1) = 1, u(y_1) \leq \alpha$ and there exists $v \in t_1^*$ such that $v(x_1) \leq \alpha, v(y_1) = 1$.

Now $f(u)(y_1) = \{\sup u(x_1) : f(x_1) = y_1\}$
$=1$.
and $f(u)(y_2) = \{\sup u(x_2) : f(x_2) = y_2\}$
$\leq \alpha$,
Now $f(v)(y_1) = \{\sup v(x_1) : f(x_1) = y_1\} \leq \alpha$.
and $f(v)(y_2) = \{\sup v(x_2) : f(x_2) = y_2\} = 1$.

Since f is open, $f(u) \in s_2^*$ as $u \in s_1^*$. We observe that there exists $f(u) \in s_2^*$ such that $f(u)(y_1) = 1, f(u)(y_2) \leq \alpha$ and there exists $f(v) \in t_2^*$,as $v \in t_1^*$ and f is open such that $f(v)(y_1) \leq \alpha, f(v)(y_2) = 1$. Hence by definition (Y, s_2^*, t_2^*) is a pair wise $\alpha - T_1(ii)$. Similarly (b), (c), (d) can be proved.

Theorem 3.6. Let (X, s_1^*, t_1^*) and (Y, s_2^*, t_2^*) be two supra fuzzy bitopological spaces. $f: X \to Y$ be continuous and one-one map, then

(a) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(i)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(i)$.
(b) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(ii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(ii)$.
(c) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(iii)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(iii)$.
(d) (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(iv)$ implies (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(iv)$.

Proof: (a) Let (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(i)$. We have to prove that (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(i)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ in Y, since f is one-one. Also since (Y, s_2^*, t_2^*) is a pairwise $\alpha - T_1(i)$, $\alpha \in I_1$, there exists $u \in s_2^*$ such that $u(f(x_1)) = 1, u(f(x_2)) \leq \alpha$. This implies that $f^{-1}(u)(x_1) = 1, f^{-1}(u)(x_2) \leq \alpha$, since $u \in s_2^*$ and f is continuous, then $f^{-1}(u) \in s_1^*$ such that $f^{-1}(u)(x_1) = 1, f^{-1}(u)(x_2) \leq \alpha$ and there exists $v \in t_2^*$ such that $v(f(x_1)) \leq \alpha, v(f(x_2)) = 1$, since $v \in t_2^*$ and f is continuous, then $f^{-1}(v) \in t_1^*$ such that $f^{-1}(v)(x_1) \leq \alpha, f^{-1}(v)(x_2) = 1$. Hence (X, s_1^*, t_1^*) is a pairwise $\alpha - T_1(i)$.

Similarly (b), (c) and (d) can be proved.

4. Conclusion

One of the main results of this paper is introducing some new definitions of supra fuzzy pairwise $\alpha - T_1$ bitopological spaces. We represent their good extension, hereditary, productive and projective properties.This concepts would be interesting in supra fuzzy bitopological spaces.

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