Annals of Pure and Applied Mathematics
Vol. 21, No. 2, 2020, 119-124
ISSN: 2279-087X (P), 2279-0888(online)
Published on 6 May 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v21n2a6666

Annals of Pure and Applied Mathematics

Theory of Fuzzy Sets: An Introduction of the Concept of Negative Partial Presence

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Received 23 March 2020; accepted 4 May 2020

Abstract. In this article, we are going to introduce the concept of negative partial presence of elements of a set with special reference to discrete fuzzy numbers. We shall show with numerical examples how the necessity of introducing this concept arises. We shall attempt further to explain the physical significance of negative level of presence of an element in a set. Finally, using the idea of negative partial presence we shall show that the normal fuzzy numbers do conform to the requirements needed to define a Group in the classical group theoretic sense with respect to the operation of addition of fuzzy numbers.

Keywords: Discrete fuzzy number, fuzzy membership value, normal fuzzy number, fuzzy membership function.

AMS Mathematics Subject Classification (2010): 03E72, 08A72

1. Introduction

When the elements are partially present in a set, we say that the set is fuzzy. It is well known that the range of the membership function of a fuzzy set (Zadeh [1], Klir *et al.* [2]) is the interval [0, 1].

The notion of fuzziness is rooted at the idea that the level of presence of an element in a set can be a number lying between 0 and 1. Going one step further, we have discussed (Baruah [3]) about fractional presence of real numbers, the word fractional meaning that the level of presence could be equal to any real number. Using this concept, Baruah [4] proposed a new formula for addition of discrete fuzzy numbers because the existing formula for addition of discrete fuzzy numbers does not quite look logical ([5, 6, 7, 8, 9]). The new formula introduced in [4] was for addition of discrete fuzzy numbers. The formula returns expected results that were shown numerically in that article. In this article, our objective is to discuss about subtraction of one discrete fuzzy number from another, and when we proceed to do that it becomes necessary to define negative partial presence of discrete fuzzy numbers. In what follows, we are going to show how this concept can be of use with reference to the operation of subtraction of discrete fuzzy numbers. The physical significance of negative presence of an element in a set would also be made clear. We would like to mention at this point that introducing a new concept, be it in mathematics or in any other field of knowledge for that matter, is tough.

It is tough for the simple reason that anything that happens to be new is always viewed with skepticism. We hope, the definition of negative partial presence would be found acceptable by the fuzzy mathematics fraternity.

We would like to reiterate that in this article we are going to introduce the concept of negative partial presence in the theory of fuzzy sets and hence there is no article by any other author to refer to as far as negative presence of an element in a set is concerned. Since the beginning of the theory of fuzzy sets, level of presence of an element in a fuzzy set has been considered to be between 0 and 1. No other author has as yet considered the concept of negative partial presence; in this regard, we would like to cite some recent works on theory and applications of fuzziness (see for example [10, 11, 12]).

It has been accepted since the beginning that the sum of fuzzy numbers A and $(-A)$ is not equal to 0, and therefore the normal fuzzy numbers do not form a group with respect to the operation of addition of fuzzy numbers in the classical group theoretic sense (see for example Mordeson *et al.*[13]). In this article, we are going to show that the normal fuzzy numbers do form a Group in the classical sense with respect to addition if we consider the matters from our standpoint.

2. A discussion on fractional presence

In the theory of sets it is naturally assumed that in a set an element is either fully present or fully absent. For example, in the interval [10, 20] the number 15 is fully present while the number 25 is fully absent. In other words, in this interval, the level of presence of 15 is 100%, or 1, and in the same way, the level of presence of 25 is 0%, or 0. In the theory of fuzzy sets, this natural notion was extended to study the situations in which the level of presence of an element in a set can be anything in the interval [0, 1]. In [3], we have made an attempt to introduce the idea of fractional presence of an element in a set, where we used the word fractional to mean any real number. The idea is a very fundamental one, and indeed it might actually look very trivial. However, we have shown in [4] that this simple idea can be of use in describing the operation of addition of discrete fuzzy numbers. We had started with the notion that for example the number 32 with level of presence 100% is numerically the same as the number 40 with level of presence 80%, which is again numerically same as the number 80 with level of presence 40%.

Let $a^{(\nu)}$ represent the real number a with level of presence ν. It can be seen that numerically $a^{(\nu)} = \nu a, \nu \in R,$ and $a \in R.$

For example,

$$80^{(0.9)} = 72.$$

Accordingly, for real numbers a and ν, $a^{(\nu)}$ can represent every real number. Therefore, for any given real number a and for all real numbers ν, the fractional number $a^{(\nu)}$ represents the entire set of the real numbers.

Now, if $x^{(\alpha)}$ is a discrete fuzzy number where *x* is a real number and $\alpha \in [0, 1]$ is the presence level of *x*, we get

$$x^{(\alpha)} + y^{(\beta)}$$

$$= \alpha x + \beta y = (x + y).\frac{(\alpha x + \beta y)}{(x + y)}$$

$$= (x + y)^{(\frac{\alpha x + \beta y}{(x+y)})}, x \neq 0, y \neq 0,$$

where

$$\frac{\alpha x + \beta y}{x + y}$$

is the weighted average of α and β. For $x = 0$, y = 0,

$$x^{(\alpha)} + y^{(\beta)}$$
$$= \alpha x + \beta y = 0$$

because both αx and βy are equal to 0 already. Indeed, using the relation $x^{(\alpha)} = \alpha x$, the formula for addition of discrete fuzzy numbers comes out automatically. In [4], we have explained how to find the sum of discrete fuzzy numbers with the help of numerical examples.

3. Negative partial presence

Now we are coming to our main objective. We would like to show how the question of negative presence comes up. Consider the following numerical examples of subtraction of discrete fuzzy numbers. Applying the formula discussed in the earlier Section, we get

$$100^{(0.74)} - 20^{(0.1)} = 80^{(\frac{74-2}{80})} = 80^{(0.9)}.$$
$$20^{(0.1)} - 100^{(0.74)} = (-80)^{(\frac{2-74}{-80})} = (-80)^{(0.9)}$$

Here we can see that $(100^{(074)} - 20^{(0.1)})$ has to be positive because 74% of 100 is larger than 10% of 20, and for the same reason $(20^{(0.1)} - 100^{(0.74)})$ has to be negative.

Consider now the following example. 10% of 100 is smaller than 90% of 20. Therefore the value of $(100^{(0.1)} - 20^{(0.9)})$ must be negative. Application of the formula in this case shows why we need to define the concept of negative presence:

$$100^{(0.1)} - 20^{(0.9)} = 80^{(\frac{10-18}{80})} = 80^{(-0.1)}$$

It is another matter that $x^{(\alpha)} = \alpha x$ leads to:

$$80^{(-0.1)} = (-80)^{(0.1)},$$

and this is why

$$100^{(0.1)} - 20^{(0.9)} = (-80)^{(0.1)}$$

is negative as expected. However $80^{(-0.1)}$ came out logically as equal to $(100^{(0.1)} - 20^{(0.9)})$ and this is how negative presence of a number happens to be a reality. Here then is an example of negative partial presence of a positive number, which can anyway be transferred to a negative number with positive partial presence.

4. An application of the concept of negative partial presence in normal fuzzy numbers

Thus far we have discussed about negative partial presence in a discrete fuzzy set. We now proceed to discuss the matter with reference to normal fuzzy numbers. Let $A = [3, 5, 6]$ be a normal fuzzy number defined by the membership function

$$\mu_A^{(x)} = \frac{x-3}{2}, \text{ for} 3 \le x \le 5,$$
$$= 6 - x, \text{ for } 5 \le x \le 6,$$
$$= 0, \text{ otherwise.}$$

Then the fuzzy membership function of $B = (-A) = [-6, -5, -3]$ is

$$\mu_B^{(x)} = 6 + x\,, -6 \le x \le -5\,,$$
$$= \frac{3+x}{(-2)}\,, -5 \le x \le -3\,,$$
$$= 0, \text{ otherwise.}$$

We have taken a very simple case used in a numerical example in [14]. Even with any other normal fuzzy numbers, what we are now going to show would be valid. It may be observed that the function

$$\mu_B^{(x)} = \frac{3+x}{(-2)}\,, -5 \le x \le -3\,,$$

is the mirror image of the function

$$\mu_A^{(x)} = \frac{x-3}{2}, \text{ for } 3 \le x \le 5,$$

and similarly, the function

$$\mu_B^{(x)} = 6 + x\,, -6 \le x \le -5\,,$$

is the mirror image of the function

$$\mu_A^{(x)} = 6 - x\,, \text{ for } 5 \le x \le 6\,,$$

with reference to the Y - axis as the mirror. It can be seen that

$$\mu_A^{(3.5)} = 0.25,\ \mu_A^{(4.0)} = 0.5,\ \mu_A^{(5.25)} = 0.75,\ \mu_A^{(5.5)} = 0.5,$$

and

$$\mu_B^{(-3.5)} = 0.25,\ \mu_B^{(-4.0)} = 0.5,\ \mu_B^{(-5.25)} = 0.75,\ \mu_B^{(-5.5)} = 0.5.$$

We are aiming at showing that the fuzzy number $B = (-A) = [-6, -5, -3]$ with membership

$$\mu_B^{(x)} = 6 + x\,, -6 \le x \le -5\,,$$
$$= \frac{3+x}{(-2)}\,, -5 \le x \le -3\,,$$
$$= 0, \text{ otherwise,}$$

and the fuzzy number $C = [3, 5, 6]$ with negative membership

$$\mu_C^{(x)} = -\left(\frac{x-3}{2}\right), \text{ for } 3 \le x \le 5,$$
$$= -(6 - x)\,, \text{ for } 5 \le x \le 6\,,$$
$$= 0, \text{ otherwise,}$$

are numerically the same. Observe that geometrically $\mu_A^{(x)}$ for $A = [3, 5, 6]$ and $\mu_C^{(x)}$ for $C = [3, 5, 6]$ are mirror images of each other with reference to the X – axis. We have

$$\mu_C^{(3.5)} = -0.25,\ \mu_C^{(4.0)} = -0.5,\ \mu_C^{(5.25)} = -0.75,\ \mu_C^{(5.5)} = -0.5.$$

Indeed,

$$\mu_A^{(x)} = -\mu_C^{(x)}$$

for $A = C = [3, 5, 6]$. Therefore, with the help of this numerical example, we can see that

$$A + (-A) = A + B = A + C = 0.$$

This is of course based on our definition of negative partial presence in a fuzzy number. If we however look into the matters from the classical standpoint $A + (-A) \neq 0$.

We have mentioned about this in [3], a detailed discussion of which has been made in the present article. Suppose, $A = [a, b, c]$ is a normal fuzzy number with membership function $\mu_A^{(x)}, x \in [a, b, c]$. Let $B = (-A) = [-c, -b, -a]$. The membership function of B can be found from $\mu_A^{(x)}, x \in [a, b, c]$. We have mentioned earlier in the example that the function $\mu_B^{(x)}$, $x \in [-c, -b, -a]$, would be the mirror image of $\mu_A^{(x)}, x \in [a, b, c]$ with reference to the Y – axis as the mirror. Our concept of negative presence of a real number would lead to observe that the fuzzy number $B = [-c, -b, -a]$ with the membership function $\mu_B^{(x)}$ would numerically be the same as the fuzzy number $C = [a, b, c]$ with the membership function $(-\mu_A^{(x)}), x \in [a, b, c]$. In this numerical equivalence, the interval $[-c, -b]$ in B corresponds to the interval $[b, c]$ in C, and the interval $[-b, -a]$ in B corresponds to the interval $[a, b]$ in C.

From our definition of fractional presence, we have

$$x^{\mu_A^{(x)}} + x^{(-\mu_A^{(x)})} = x^{(0)} = 0,$$

for all $x \in [a, b, c]$. Therefore, the fuzzy number $C = [a, b, c]$ with the membership function $\left(-\mu_A^{(x)}\right), x \in [a, b, c]$, added to the fuzzy number $A = [a, b, c]$ with the membership function $\mu_A^{(x)}, x \in [a, b, c]$, added point for point for every x, would give us the fuzzy number $[a, b, c]$ with membership equal to 0 for all x concerned. Note that the number A with level of presence 0 for all x concerned is nothing but 0 itself.

At this point it may be noted that because for a fuzzy number A we get $A + (-A) \neq 0$, the fuzzy numbers do not form a Group in the classical group theoretic sense. However, if we accept the notion of negarive partial presence as explained in this article, it becomes clear that fuzzy numbers do conform to the postulates that define a Group. It is the classical definition of a Group that is followed by fuzzy numbers. What we mean is that we do not need to define a *Fuzzy* Group in this case, because from what we have seen, the fuzzy numbers with respect to addition follow the classical definition of a Group already.

5. Conclusions

The very idea of negative presence of an element in a set might not look meaningful initially. However, we have shown in this article how this concept can help to describe the operation of subtraction of discrete fuzzy numbers. Indeed, in a particular type of situation the concept of negative partial presence occurs automatically.

We have seen that negative partial presence of a real number does have a physical significance of its own. From what we have so far discussed, we can conclude that if we look into the matters from our standpoint, the fuzzy sets do form a Group with respect to addition with 0 as the additive identity. This conclusion is however contrary to what has been accepted till now by the fraternity of fuzzy mathematics that the fuzzy sets do not conform to the structure of Group in the classical group theoretic sense because in the theory of fuzzy sets it has been accepted that the sum of fuzzy sets A and $(-A)$ is not equal to 0.

We would like to mention at last that the question of defining negative presence has arisen while going to describe the operation of subtraction of discrete fuzzy numbers. In the numerical example concerned, it has been clearly shown how a positive number with negative presence can be numerically equal to a negative number with positive presence. Therefore as far as usage is concerned, negative presence may remain just as a theoretical concept. However, that a real number with negative partial presence can actually be shown to exist is an undeniable reality.

Acknowledgement. We are very grateful to the reviewer for the comments and the suggestions towards improving this article.

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