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On the Class of the Diophantine Equations $5^x + (10K + M)^y = z^2$ and $5^x + 5^y = z^2$ with Positive Integers x, y, z when $M = 1, 3, 7, 9$

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Abstract. In this article, we consider the equations $5^x + 5^y = z^2$ and $5^x + (10K + M)^y = z^2$ in which $K \geq 0$ is an integer, $M = 1, 3, 7, 9$, and x, y, z are positive integers. We establish that $5^x + 5^y = z^2$ and the cases $M = 3, 7$ yield no solutions. When $M = 1, 9$, we show for all values $x \geq 1$ with $y = 1$, that infinitely many solutions exist. Several solutions are also exhibited.

Keywords: Diophantine equations

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1. Introduction

The field of Diophantine equations is ancient, vast, and no general method exists to decide whether a given Diophantine equation has any solutions, or how many solutions. In most cases, we are reduced to study individual equations, rather than classes of equations.

The famous general equation

$$p^x + q^y = z^2$$

has many forms. The literature contains a very large number of articles on non-linear such individual equations involving particular primes and powers of all kinds.

In this article we consider the equation $5^x + 5^y = z^2$, and the class of Diophantine equations $5^x + (10K + M)^y = z^2$ in which x, y, z are positive integers, $K \geq 0$ is an integer, and $M = 1, 3, 7, 9$. We shall prove that $5^x + 5^y = z^2$, and the cases $M = 3, 7$ have no solutions. For solutions when $M = 1, 9$, it is shown that K and y must be odd values. Then, for all values $x \geq 1$ with $y = 1$, it is established that infinitely many solutions exist.

The results are obtained by our new technique which uses the last digits of the powers involved, and applies to primes and composites as well with no distinction.

The process of finding the solutions to the equations when $M = 3, 7$ (Sections 2, 4), and also when $M = 1, 9$ (Sections 5, 6) is quite identical and has many similarities. Nevertheless, for the sake of simplicity, clarity and completeness, and also for the convenience of the readers interested in particular equations, each value M is considered separately, and all theorems are self-contained.

2. On $5^x + (10K + 3)^y = z^2$

In $5^x + (10K + 3)^y = z^2$, $K \geq 0$ is an integer. For all values $y \geq 1$, the power $(10K + 3)^y$ has a last digit which respectively equals one of values 3, 9, 7, 1. For all values $x \geq 1$, the power 5^x has a last digit which is equal to 5. Therefore, the sum $5^x + (10K + 3)^y$ has a last digit which is respectively equal to 8, 4, 2, 6, and the sum is even. If $5^x + (10K + 3)^y = z^2$ has a solution for some values x, K, y and z, then the even value z^2 does not end in 2 nor does it end in 8. Hence, z^2 has a last digit which is equal to 4 or equal to 6. We shall now examine $5^x + (10K + 3)^y = z^2$ for solutions in the two cases when z^2 ends in 4 and when z^2 ends in 6. This is done in the following respective Theorems 2.1, 2.2.

Theorem 2.1. Suppose that $K \geq 0$ is an integer, and x, y, z are positive integers. If z^2 ends in the digit 4, then no value K satisfies $5^x + (10K + 3)^y = z^2$.

Proof: We shall assume that for some value K, there exist positive integers x, y, z where z^2 ends in the digit 4 and reach a contradiction.

The square z^2 has a last digit equal to 4 when $(10K + 3)^y$ has a last digit equal to 9. Thus $y = 2 + 4n$ where $n \geq 0$ is an integer. Then, by our assumption $5^x + (10K + 3)^y = 5^x + (10K + 3)^{2+4n} = z^2$ yields

$$5^x = z^2 - (10K + 3)^{2+4n} = z^2 - (10K + 3)^{2(2n+1)} = (z - (10K + 3)^{2n+1})(z + (10K + 3)^{2n+1}).$$

Denote

$$z - (10K + 3)^{2n+1} = 5^A, \qquad z + (10K + 3)^{2n+1} = 5^B, \qquad A < B, \quad A + B = x,$$

where A, B are non-negative integers. Then $5^B - 5^A$ implies

$$2(10K + 3)^{2n+1} = 5^A(5^{B-A} - 1). \tag{1}$$

It follows from (1) that $A > 0$ is impossible. Hence $A = 0$. When $A = 0$, then $B = x$, and from (1) we have

$$2(10K + 3)^{2n+1} = 5^x - 1. \tag{2}$$

Since $5^x - 1 = 5^x - 1^x$, we rewrite (2) as

$$2(10K + 3)^{2n+1} = 5^x - 1^x = (5 - 1)(5^{x-1} + 5^{x-2} + \cdots + 5^1 + 1).$$

The product $2(10K + 3)^{2n+1}$ is a multiple of 2 only, whereas $5^x - 1^x$ is a multiple of 4. This is a contradiction implying that our assumption is false.

When z^2 has a last digit which is equal to 4, then $5^x + (10K + 3)^y = z^2$ has no solutions.

This concludes the proof of Theorem 2.1. □

Theorem 2.2. Suppose that $K \geq 0$ is an integer, and x, y, z are positive integers. If z^2 ends in the digit 6, then no value K satisfies $5^x + (10K + 3)^y = z^2$.

Proof: We shall assume that for some value K, there exist positive integers x, y, z such that z^2 ends in the digit 6 and reach a contradiction.

The square z^2 has a last digit equal to 6 when $(10K + 3)^y$ has a last digit equal to 1. Hence $y = 4n$ where $n \geq 1$ is an integer. Then, by our assumption $5^x + (10K + 3)^y = 5^x + (10K + 3)^{4n} = z^2$ yields

$$5^x = z^2 - (10K + 3)^{4n} = z^2 - (10K + 3)^{2(2n)} = \big(z - (10K + 3)^{2n}\big)\big(z + (10K + 3)^{2n}\big).$$

Denote

$$z - (10K + 3)^{2n} = 5^E, \qquad z + (10K + 3)^{2n} = 5^F, \qquad E < F, \qquad E + F = x,$$

where E, F are non-negative integers. Then $5^F - 5^E$ implies

$$2(10K + 3)^{2n} = 5^E(5^{F-E} - 1). \tag{3}$$

It follows from (3) that $E > 0$ is impossible. Hence $E = 0$. When $E = 0$, then $F = x$, and from (3) we obtain

$$2(10K + 3)^{2n} = 5^x - 1. \tag{4}$$

Since $5^x - 1 = 5^x - 1^x$, we can write (4) as

$$2(10K + 3)^{2n} = 5^x - 1^x = (5 - 1)(5^{x-1} + 5^{x-2} + \cdots + 5^1 + 1).$$

The product $2(10K + 3)^{2n}$ is a multiple of 2 only, whereas $5^x - 1^x$ is a multiple of 4. This contradiction implies that our assumption is false.

When z^2 ends in the digit 6, then $5^x + (10K + 3)^y = z^2$ has no solutions.

This completes the proof of Theorem 2.2. □

Remark 2.1. In accordance with the preface of this section, and as a direct consequence of both Theorems 2.1 and 2.2, it now follows for all integers $K \geq 0$, that the equation $5^x + (10K + 3)^y = z^2$ has no solutions.

3. On $5^x + 5^y = z^2$

In the following Theorem 3.1, we show that $5^x + 5^y = z^2$ has no solutions.

Theorem 3.1. The equation $5^x + 5^y = z^2$ has no solutions in positive integers x, y, z.

Proof: For all values x, y, the sum $5^x + 5^y$ is even. If $5^x + 5^y = z^2$ has a solution, then z^2 is even, and $z = 2T$ where T is an integer. Thus $z^2 = 4T^2$.
We shall assume that for some values x, y, z, the equation $5^x + 5^y = z^2$ has a solution and reach a contradiction.

If $x = y$, then by our assumption we have that

$$z^2 = 5^x + 5^y = 2 \cdot 5^x \neq 4T^2$$

a contradiction. Hence $x \neq y$.

If x, y are distinct, then without loss of generality let $x > y$.
Denote $x - y = R$. By our assumption we obtain

$$z^2 = 5^x + 5^y = 5^y(5^{x-y} + 1) = 5^y(5^R + 1) = 4T^2. \qquad (5)$$

From (5) it follows that $4 \mid (5^R + 1)$. Since $5 = 4N + 1$ ($N = 1$), therefore for all values R the power 5^R is of the form $4V + 1$. Thus

$$5^R + 1 = (4V + 1) + 1 = 4V + 2 = 2(2V + 1).$$

Hence in (5) $4 \nmid (5^R + 1)$. The contradiction derived implies that our assumption is false.

The equation $5^x + 5^y = z^2$ has no solutions. □

Corollary 3.1. As a consequence of Theorem 3.1, it clearly follows that if 5 is replaced by any prime $p > 5$ where $p = 4N + 1$ ($N > 1$), then the equation $p^x + p^y = z^2$ has no solutions in positive integers x, y, z.

Proof: The proof is the same proof as that of Theorem 3.1 when 5 is replaced by p. □

4. On $5^x + (10K + 7)^y = z^2$

In $5^x + (10K + 7)^y = z^2$, $K \geq 0$ is an integer. For all values $y \geq 1$, the power $(10K + 7)^y$ has a last digit which is respectively equal to one of the values 7, 9, 3, 1. For all values $x \geq 1$, the power 5^x has a last digit which is equal to 5. Therefore, the sum $5^x + (10K + 7)^y$ has a last digit which is respectively equal to 2, 4, 8, 6, and the sum is even. If $5^x + (10K + 7)^y = z^2$ has a solution for some values x, K, y and z, then the even value z^2 does not have a last digit which is equal to 2 and also equal to 8. Hence, z^2 has a last digit equal to 4 or equal to 6. We therefore investigate the two cases when z^2 ends in 4 and also ends in 6. This is done in the following respective two theorems namely Theorem 4.1 and Theorem 4.2.

Theorem 4.1. Suppose that $K \geq 0$ is an integer, and x, y, z are positive integers. If z^2 ends in the digit 4, then no value K satisfies $5^x + (10K + 7)^y = z^2$.

Proof: We shall assume that for some value K, there exist positive integers x, y, z such that z^2 ends in the digit 4 and reach a contradiction.

The square z^2 has a last digit equal to 4 when $(10K + 7)^y$ has a last digit equal to 9. Hence $y = 2 + 4m$ where $m \geq 0$ is an integer. Then, by our assumption $5^x + (10K + 7)^y = 5^x + (10K + 7)^{2+4m} = z^2$ yields

$$5^x = z^2 - (10K + 7)^{2+4m} = z^2 - (10K + 7)^{2(2m+1)} = (z - (10K + 7)^{2m+1})(z + (10K + 7)^{2m+1}).$$

Denote

$$z - (10K + 7)^{2m+1} = 5^A, \qquad z + (10K + 7)^{2m+1} = 5^B, \qquad A < B, \qquad A + B = x,$$

where A, B are non-negative integers. Then $5^B - 5^A$ yields

$$2(10K + 7)^{2m+1} = 5^A(5^{B-A} - 1). \tag{6}$$

It follows from (6) that $A > 0$ is impossible. Thus $A = 0$. When $A = 0$, then $B = x$, and from (6) we obtain

$$2(10K + 7)^{2m+1} = 5^x - 1. \tag{7}$$

Since $5^x - 1 = 5^x - 1^x$, we can write (7) as

$$2(10K + 7)^{2m+1} = 5^x - 1^x = (5 - 1)(5^{x-1} + 5^{x-2} + \cdots + 5^1 + 1).$$

The product $2(10K + 7)^{2m+1}$ is a multiple of 2 only, whereas $5^x - 1^x$ contains the factor 4. This is a contradiction implying that our assumption is false.

When z^2 has a last digit which equals 4, then $5^x + (10K + 7)^y = z^2$ has no solutions.

This concludes the proof of Theorem 4.1. □

Theorem 4.2. Suppose that $K \geq 0$ is an integer, and x, y, z are positive integers. If z^2 ends in the digit 6, then no value K satisfies $5^x + (10K + 7)^y = z^2$.

Proof: We shall assume that for some value K, there exist positive integers x, y, z such that z^2 ends in the digit 6 and reach a contradiction.

The square z^2 has a last digit equal to 6 only when $(10K + 7)^y$ has a last digit equal to 1. Thus $y = 4n$ where $n \geq 1$ is an integer. Then, by our assumption $5^x + (10K + 7)^y = 5^x + (10K + 7)^{4n} = z^2$ yields

$$5^x = z^2 - (10K + 7)^{4n} = z^2 - (10K + 7)^{2(2n)} = \left(z - (10K + 7)^{2n}\right)\left(z + (10K + 7)^{2n}\right).$$

Denote

$$z - (10K + 7)^{2n} = 5^C, \qquad z + (10K + 7)^{2n} = 5^D, \qquad C < D, \qquad C + D = x,$$

where C, D are non-negative integers. Then $5^D - 5^C$ yields

$$2(10K + 7)^{2n} = 5^C(5^{D-C} - 1). \tag{8}$$

It follows from (8) that $C > 0$ is impossible. Hence $C = 0$. When $C = 0$, then $D = x$, and we have from (8)

$$2(10K + 7)^{2n} = 5^x - 1. \tag{9}$$

In (9), the difference $5^x - 1$ can be written as $5^x - 1^x$, and

$$2(10K + 7)^{2n} = 5^x - 1^x = (5 - 1)(5^{x-1} + 5^{x-2} + \cdots + 5^1 + 1).$$

The product $2(10K + 7)^{2n}$ is a multiple of 2 only, whereas $5^x - 1^x$ is a multiple of 4. This contradiction implies that our assumption is false.

When z^2 has a last digit which is equal to 6, then $5^x + (10K + 7)^y = z^2$ has no solutions.

The proof of Theorem 4.2 is complete. □

Remark 4.1. In accordance with the preface of this section, and as a direct consequence of both Theorems 4.1 and 4.2, it now follows for all integers $K \geq 0$, that the equation $5^x + (10K + 7)^y = z^2$ has no solutions.

Concluding remark for $M = 3,7$. The author and many others have considered the equation $p^x + q^y = z^2$ in which p and q are primes. In this article $p = 5$, where q is defined as $q = 10K + 3$ or $q = 10K + 7$ for all integers $K \geq 0$. By using all values $K \geq 0$, the results established therefore include all primes q whose last digit is equal to 3 and equal to 7. Moreover, the results are valid for all composites whose last digits are 3 and 7.

5. On $5^x + (10K + 1)^y = z^2$

In this section we consider $5^x + (10K + 1)^y = z^2$ when $K \geq 1$ is an integer. We will show that if $y = 1$ and K assumes odd values, then for each value $x \geq 1$ the equation has infinitely many solutions.

Theorem 5.1. If $5^x + (10K + 1)^y = z^2$ has a solution with positive integers x, y, z, then K and y are odd.

Proof: The prime 5 is of the form $5 = 4N + 1$ $(N = 1)$. Then, for all values $x \geq 1$, $5^x = (4N + 1)^x$ is of the form $4A + 1$. Moreover, for all values $x \geq 1$ the power 5^x has a last digit equal to 5. For all values $y \geq 1$, the power $(10K + 1)^y$ has a last digit equal to 1. In any solution of $5^x + (10K + 1)^y = z^2$, the last digit of z^2 is therefore equal to 6. Since z^2 is even, then $z = 2T$ and $z^2 = 4T^2$ where T is an integer.

For all values $K \geq 1$, and for all even values y, the power $(10K + 1)^y$ is of the form $4B + 1$. Then

$$5^x + (10K + 1)^y = (4A + 1) + (4B + 1) = 4(A + B) + 2 \neq 4T^2 = z^2,$$

and $5^x + (10K + 1)^y = z^2$ has no solutions.
Hence, if $5^x + (10K + 1)^y = z^2$ has a solution, then $y = 2n + 1$ where $n \geq 0$ is an integer.

Suppose that K is even. For all even values $K \geq 2$, the value $10K + 1$ is of the form $4U + 1$. For all odd values $y = 2n + 1$, then the power $(10K + 1)^y = (4U + 1)^{2n+1}$ is of the form $4V + 1$. We have

$$5^x + (10K + 1)^y = (4A + 1) + (4V + 1) = 4(A + V) + 2 \neq 4T^2 = z^2,$$

and $5^x + (10K + 1)^y = z^2$ has no solutions.
Thus, if $5^x + (10K + 1)^y = z^2$ has a solution, then $K \geq 1$ is odd.

The proof of Theorem 5.1 is complete. □

Theorem 5.2. If $K \geq 1$ is odd and $y = 1$, then for each and every integer $x \geq 1$, $5^x + (10K + 1)^y = z^2$ has infinitely many solutions.

Proof: By Theorem 5.1, it follows that in any solution of the equation, the last digit of z^2 is equal to 6. A square $(10C + R)^2$ has a last digit equal to 6 when $R = 4$ or when $R = 6$. For our purpose, it will clearly suffice to consider one of the two possibilities say $R = 6$.

Let $x \geq 1$ be a fixed value. Let C denote the smallest possible value for which $(10C + 6)^2 > 5^x$. Then there exists an odd value K satisfying the equation

$$5^x + (10K + 1)^1 = (10C + 6)^2 = z^2. \qquad (10)$$

Evidently, for any fixed value x with each value $C + 1, C + 2, \ldots, C + Q, \ldots$, there exists a respective odd value K such that (10) holds. The infinitude of solutions is established.

For each and every value $x \geq 1$, $5^x + (10K + 1)^1 = z^2$ has infinitely many solutions.

This concludes the proof of Theorem 5.2. □

We now exhibit some solutions for odd and even values x which illustrate Theorem 5.2. When K is odd, $y = 1$, $x = 1, 3$ and $z = 10C + 6$ we have

Solution 1.	$5^1 + 31^1 = 6^2$	$x = 1$,	$K = 3$,	$C = 0$.
Solution 2.	$5^1 + 251^1 = 16^2$	$x = 1$,	$K = 25$,	$C = 1$.
Solution 3.	$5^1 + 671^1 = 26^2$	$x = 1$,	$K = 67$,	$C = 2$.
Solution 4.	$5^3 + 131^1 = 16^2$	$x = 3$,	$K = 13$,	$C = 1$.
Solution 5.	$5^3 + 551^1 = 26^2$	$x = 3$,	$K = 55$,	$C = 2$.
Solution 6.	$5^3 + 1171^1 = 36^2$	$x = 3$,	$K = 117$,	$C = 3$.

When K is odd, $y = 1$, $x = 2, 4, 6$ and $z = 10C + 6$ we have

$$5^2 + 11^1 = 6^2, \qquad 5^4 + 51^1 = 26^2, \qquad 5^6 + 251^1 = 126^2,$$
$$5^2 + 231^1 = 16^2, \qquad 5^4 + 671^1 = 36^2, \qquad 5^6 + 2871^1 = 136^2.$$

In the above solutions, the values $10K + 1$ consist of primes and composites.

The following question may now be raised.

Question 1. Does $5^x + (10K + 1)^y = z^2$ have solutions for odd values K and odd values $y > 1$?

6. On $5^x + (10K + 9)^y = z^2$

First, we prove that $5^x + (10K + 9)^y = z^2$ has solutions only if K and y are both odd. Secondly, we show that when $y = 1$, then for each and every value $x \ge 1$ the equation has infinitely many solutions. We remark that these results resemble the results obtained in Section 5.

Theorem 6.1. If $5^x + (10K + 9)^y = z^2$ has a solution with positive integers x, y, z, then K and y are odd.

Proof: For all values $x \ge 1$, the power 5^x has a last digit equal to 5, whereas for all values $y \ge 1$ the power $(10K + 9)^y$ has a last digit which is respectively equal to 9 or to 1. Therefore, the sum $5^x + (10K + 9)^y$ has a last digit which is respectively equal to 4 or to 6. Hence, in any solution of $5^x + (10K + 9)^y = z^2$, the power z^2 is even. Thus, $z = 2T$, $z^2 = 4T^2$ where T is an integer.

Let y be even. For all values $x \ge 1$ and $y = 2n$ where $n \ge 1$ is an integer, it is easily seen that the sum $5^x + (10K + 9)^{2n}$ is a multiple of 2 only. Since z^2 is a multiple of 4, therefore when $y = 2n$, it follows that $5^x + (10K + 9)^y = z^2$ has no solutions. Hence, if $5^x + (10K + 9)^y = z^2$ has a solution, then $y = 2n + 1$ where $n \ge 0$ is an integer.

Let $y = 2n + 1$. For all values $x \ge 1$, the power 5^x is of the form $4H + 1$. When K is even, then $10K + 9$ is of the form $4G + 1$, and $(10K + 9)^{2n+1} = (4G + 1)^{2n+1}$ has the form $4Q + 1$. Then

$$5^x + (10K + 9)^y = 5^x + (10K + 9)^{2n+1} = (4H + 1) + (4Q + 1) = 4(H + Q) + 2 \ne 4T^2 = z^2,$$

and the equation has no solutions. Hence K is not even, and K is odd as asserted.

This concludes the proof of Theorem 6.1. □

Theorem 6.2. If K is odd and $y = 1$, then for each and every value $x \ge 1$, $5^x + (10K + 9)^y = z^2$ has infinitely many solutions.

Proof: Since $y = 1$, it follows that the last digit of z^2 is equal to 4. A square $(10W + R)^2$ has a last digit equal to 4 only when $R = 2$ or when $R = 8$. For our purpose, it suffices to consider one of the two cases, say $R = 2$.

Let $x \ge 1$ be a fixed value. Let W denote the smallest possible value for which $(10W + 2)^2 > 5^x$. Then there exists an odd value K which satisfies the equation

$$5^x + (10K + 9)^1 = (10W + 2)^2 = z^2. \quad (11)$$

Certainly, for any fixed value x with each value $W + 1, W + 2, \ldots, W + J, \ldots$, there exists a respective odd value K such that (11) is satisfied. An infinitude of solutions then exists.

For each and every value $x \ge 1$, $5^x + (10K + 9)^1 = z^2$ has infinitely many solutions.

The proof of Theorem 6.2 is complete. □

We now demonstrate some solutions of $5^x + (10K + 9)^1 = z^2$ when $x = 2, 4$, K is odd, $y = 1$ and $z = 10W + 2$.

Solution 7.	$5^2 + 119^1 = 12^2$	$x = 2$,	$K = 11$,	$W = 1$.
Solution 8.	$5^2 + 459^1 = 22^2$	$x = 2$,	$K = 45$,	$W = 2$.
Solution 9.	$5^2 + 999^1 = 32^2$	$x = 2$,	$K = 99$,	$W = 3$.
Solution 10.	$5^4 + 399^1 = 32^2$	$x = 4$,	$K = 39$,	$W = 3$.
Solution 11.	$5^4 + 1139^1 = 42^2$	$x = 4$,	$K = 113$,	$W = 4$.
Solution 12.	$5^4 + 2079^1 = 52^2$	$x = 4$,	$K = 207$,	$W = 5$.

In the above six solutions, the values $10K + 9$ are composites.

The following question may now be raised.

Question 2. Does $5^x + (10K + 9)^y = z^2$ have solutions for odd values K and odd values $y > 1$?

Final remark. In this article, we have established for all integers $K \geq 0$ that $5^x + (10K + 3)^y = z^2$, $5^x + (10K + 7)^y = z^2$ and $5^x + 5^y = z^2$ have no solutions in positive integers x, y, z. For $5^x + (10K + 1)^y = z^2$ and $5^x + (10K + 9)^y = z^2$, it has been determined for all values $x \geq 1$ with $y = 1$, that infinitely many solutions exist. The results achieved are primarily and in principle based upon our new method which uses the last digits of the powers in the equations. This elementary tool, together with other simple and known basic facts enabled these results. We presume that more equations may be solved in this manner.

In all the articles of the author which are cited in this paper, this new method has been utilized very recently (2019 – 2020). It is therefore obvious, that no other references exist on this subject.

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