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Seventh and Eighth Order Triangular Sum Labeling of Graphs

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Abstract. A (p, q) graph G is said to admit a seventh (or eighth) order triangular sum labeling if its vertices can be labeled by non negative integers such that induced edge labels obtained by the sum of the labels of end vertices are the seventh (or eighth) order triangular numbers. A graph G which admits a seventh (or eighth) order triangular sum labeling is called a seventh (or eighth) order triangular sum graph. In this paper we prove that double star $K_{1,n,n}$, Tg_n and Coconut tree admit seventh and eighth order triangular sum labelings.

Keywords: Star graph, coconut tree, seventh and eighth order triangular sum graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The graphs considered here are finite, connected, undirected and simple. The vertex set and the edge set of graph are denoted by V(G) and E(G) respectively. For various graph theoretic notations and terminology we follow Harary [3] and for number theory we follow Burton [1]. We will give the brief summary of definitions which are useful for the present investigations. In [8] they have proved star and bistar related graphs are divisor cordial graphs. Some authors in [10] have discussed Star in Coloring of some star familes. For a dynamic survey of various graph labeling problem along with an extensive bibliography we refer to Gallioan [4].

Definition 1.1. [2,7] A triangular number is a number obtained by adding all positive integers less than or equal to a given positive integer n. If the n^{th} triangular number is denoted by A_n, then $A_n = 1 + 2 + \ldots + n = \frac{1}{2} n (n+1)$.

The triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, . . .

Definition 1.2. [6, 9] A triangular sum labeling of a graph G is a one to one function f: V(G) $\rightarrow$W (where W is the set of all non-negative integers) that induces a bijection f^+ :

E(G) $\rightarrow$ {A_1, A_2, . . ., A_q} of the edges of G defined by $f^+(uv) = f(u) + f(v)$ $\forall e = uv \in E(G)$. The graph which admits such labeling is called a triangular sum graph.

Definition 1.3. [5] A second order triangular number is a number obtained by adding all the squares of positive integer less than or equal to a given positive integer n. If the n^{th} second order triangular number is denoted by B_n , then

$$B_n = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

$$= \frac{1}{6} n(n+1)(2n+1)$$

The second order triangular numbers are 1, 5,14, 30,55,. …

Definition 1.4. A second order triangular sum labeling of a graph G is a one to one function f: V(G) $\rightarrow$ W that induces a bijection f^+: E(G) $\rightarrow$ {B_1, B_2, . . ., B_n} of the edges of G defined by $f^+(uv)$ = f(u) + f(v), $\forall$ e = uv$\in$ E(G). The graph which admits such labeling is called a second order triangular sum graph.

Definition 1.5. A seventh order triangular number is a number obtained by adding all the seventh powers of positive integers less than or equal to a given positive integer n. If the n^{th} seventh order triangular number is denoted by G_n, then

$$G_n = 1^7 + 2^7 + 3^7 + \ldots + n^7$$

$$= \frac{1}{24} n^2 (n+1)^2 \left[3n^4 + 6n^3 - n^2 - 4n + 2\right]$$

The seventh order triangular numbers are 1, 129, 2316, 18700, 96825, 376761, 1200304, 3297456, 8080425, 18080425, 37567596, . . .

Definition 1.6. A seventh order triangular sum labeling of a graph G is a one to one function f: V(G) $\rightarrow$ W that induces a bijection f^+: E(G) $\rightarrow$ {G_1, G_2, . . ., G_q} of the edges of G defined by $f^+(uv)$ = f(u) + f(v), $\forall$ e = uv$\in$ E(G). The graph which admits such labeling is called a seventh order triangular sum graph.

Definition 1.7. An Eighth order triangular number is a number obtained by adding all the eighth powers of positive integers less than or equal to a given positive integer n. If the n^{th} eighth order triangular number is denoted by H_n, then

$$H_n = 1^8 + 2^8 + \ldots + n^8$$

$$= \frac{n(n+1)(2n+1)}{90}[5n^6 + 15n^5 + 5n^4 - 15n^3 - n^2 + 9n - 3]$$

The eighth order triangular numbers are 1, 257, 6818, 72354, 462979, 2142595, ….

Definition 1.6. An Eighth order triangular sum labeling of a graph G is a one-to-one function f: V(G) $\rightarrow$ W that induces a bijection f^+: E(G) $\rightarrow$ {H_1, H_2, …, H_q} of the edges of G defined by $f^+(uv)$ = f(u) + f(v), $\forall$e = uv$\in$ E(G). The graph which admits such labeling is called a eighth order triangular sum graph.

2. Main result

Theorem 2.1. The star graph $K_{1,n}$ admits seventh order triangular sum labeling.

Proof: Let u be the central vertex and let $u_1, u_2, \ldots, u_n$ be the pendant vertices of the star $K_{1,n}$. Then the vertex set $V(K_{1,n}) = \{u, u_i / 1 \le i \le n\}$ and the edge set $E(K_{1,n}) = \{uu_i / 1 \le i \le n\}$. Clearly $K_{1,n}$ has n + 1 vertices and n edges. Define $f : V(K_{1,n}) \to W$ by

$f(u) = 0$

$$f(u_i) = G_i = \frac{i^2 (i+1)^2}{24}[3i^4 + 6i^3 - i^2 - 4i + 2],\ 1 \le i \le n.$$

Then f induces a bijection $f^+ : E(G) \to \{G_1, G_2, \ldots, G_n\}$ given by

$$f^+(uu_i) = f(u) + f(u_i) = 0 + G_i = G_i = \frac{i^2 (i+1)^2}{24}[3i^4 + 6i^3 - i^2 - 4i + 2],\ 1 \le i \le n.$$

Clearly, the induced edge labels are the first n seventh order triangular numbers. Hence $K_{1,n}$ admits a seventh order triangular sum labeling.

Example 2.2. A seventh order triangular sum labeling of $K_{1,5}$ is shown in figure 2.1.

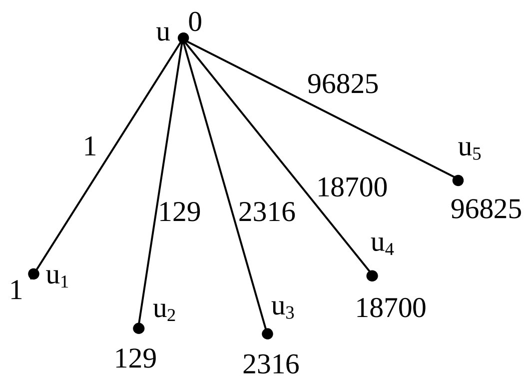

Figure 2.1: $K_{1,5}$ with a seventh order triangular sum labeling

Theorem 2.3. The double star $K_{1,n,n}$ admits seventh order triangular sum labeling.

Proof: Let G be the double star $K_{1,n,n}$. Let $V(G) = \{u, u_i, v_i / 1 \le i \le n\}$ be the vertex set and $E(G) = \{uu_i, u_iv_i / 1 \le i \le n\}$ be the edge set of $K_{1,n,n}$. Then G has 2n + 1 vertices and 2n edges. Define $f : V(G) \to W$ by

$f(u) = 0$

$f(u_i) = G_i,\ 1 \le i \le n$

$f(v_i) = G_{n+i} - f(u_i),\ 1 \le i \le n$

Then f induces a bijection $f^+ : E(G) \to \{G_1, G_2, \ldots, G_{2n}\}$ given by

$f^+(uu_i) = f(u) + f(u_i) = 0 + G_i = G_i,\ 1 \le i \le n$

$f^+(u_iv_i) = f(u_i) + f(v_i) = f(u_i) + G_{n+I} - f(u_i) = G_{n+i},\ 1 \le i \le n.$

Thus, the induced edge labels are the first 2n seventh order triangular numbers. Hence G admits a seventh order triangular sum labeling.

Example 2.4. A seventh order triangular sum labeling of $K_{1,4,4}$ is shown in figure 2.2.

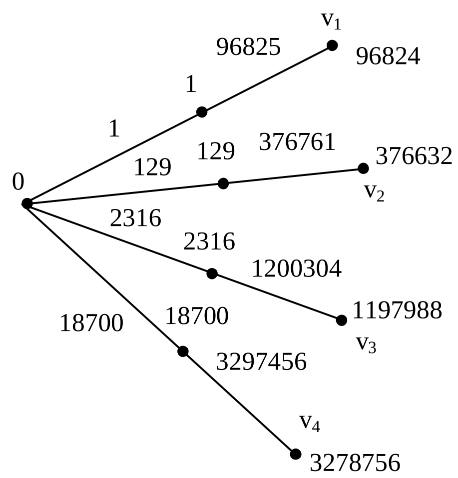

Figure 2.2: K $_{1, 4, 4}$ with a seventh order triangular sum labeling

Theorem 2.5. The bistar$B_{m,n}$ admits seventh order triangular sum labeling.

Proof: Let u and v be the vertices of K_2. Join m pendent vertices $u_1, u_2, \ldots, u_m$ at u and join n pendent vertices $v_1, v_2, \ldots, v_n$ at v. The resultant graph is the bistar G = $B_{m,n}$ with vertex set V(G) = $\{u, v, u_i, v_j \ / \ 1 \leq i \leq m, 1 \leq j \leq n\}$ and edge set E(G) $\{uu_i, vv_j, uv \ / \ 1 \leq i \leq m, 1 \leq j \leq n\}$. Clearly G has m + n + 2 vertices and m + n + 1 edges. Define f: V(G) $\rightarrow$ W by

$f(u) = 0$

$f(v) = 1$

$f(u_i) = G_{i+1}, \ 1 \leq i \leq m$

$f(v_j) = G_{m+1+j} - f(v), 1 \leq j \leq n$

Then f induces a bijection $f^+ : E(G) \rightarrow \{G_1, G_2, \ldots, G_{m+n+1}\}$ given by

$f^+(uu_i) = f(u) + f(u_i) = 0 + G_{i+1} = G_{i+1}, \quad 1 \leq i \leq m$

$f^+(uv) = f(u) + f(v) = 1 = G_1$

$f^+(vv_j) = f(v) + f(v_j) = f(v) + G_{m+1+j} - f(v) = G_{m+1+j}, \ 1 \leq j \leq n.$

Thus, the induced edge labels are the first m + n + 1 seventh order triangular numbers. Hence $B_{m, n}$ admits a seventh order triangular sum labeling.

Example 2.6. A seventh order triangular sum labeling of $B_{4,3}$ is shown in figure 2.3.

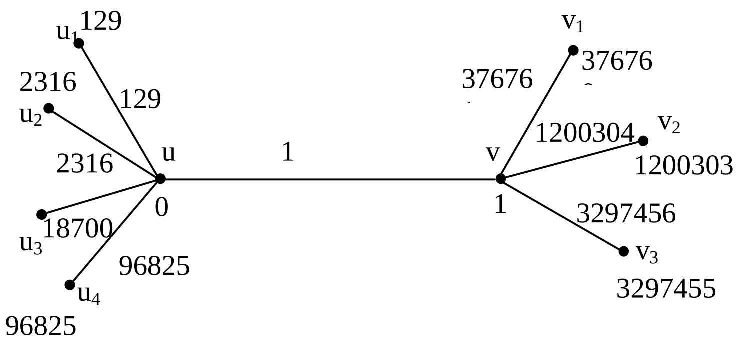

Figure 2.3: $B_{4,3}$ with a seventh order triangular sum labeling

Theorem 2.7. Y_{n+1} admits a seventh order triangular sum labeling if $n \geq 3$.

Proof: Let P_n be the path $u_1u_2 \ldots u_n$. Let v be the vertex adjacent to u_2. The resultant graph G is Y_{n+1} with vertex set $V(G) = \{u_i, v / 1 \leq i \leq n\}$ and the edge set $E(G) = \{u_iu_{i+1}, vu_2 / 1 \leq i \leq n-1\}$. Clearly the graph G has n + 1 vertices and n edges. Define $f : V(G) \rightarrow W$ by

$f(u_1) = 1 = G_1$

$f(v) = 129 = G_2$

$f(u_2) = 0$

$f(u_i) = G_{i+1} - f(u_{i-1}),\ 3 \leq i \leq n$

Then f induces a bijection $f^+: E(G) \rightarrow \{G_1, G_2, \ldots, G_n\}$ given by

$f^+(u_1u_2) = f(u_1) + f(u_2) = 1 = G_1$

$f^+(u_2v) = f(u_2) + f(v) = 0 + 129 = G_2$

$f^+(u_2u_3) = f(u_2) + f(u_3) = 0 + 23161 = G_3$

$f^+(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) = f(u_i) + G_{i+1} - f(u_i) = G_{i+1}, 3 \leq i \leq n$

Thus, the induced edge labels are the first n seventh order triangular numbers. Hence G admits seventh order triangular sum labeling.

Example 2.8. A seventh order triangular sum labeling of Y_{5+1} is shown in figure 2.4.

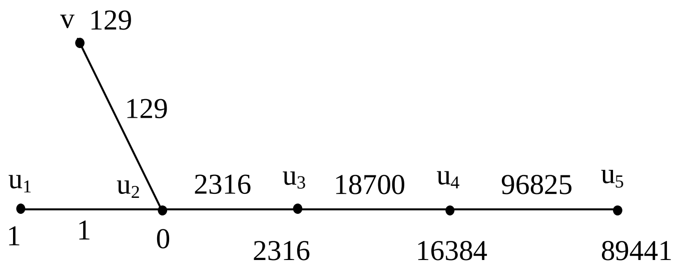

Figure 2.4: Y_{5+1} with a seventh order triangular sum labeling

Theorem 2.9. Tg_n admits seventh order triangular sum labeling.

Proof: Let $u_1u_2 \ldots u_n$ be the path of length n. For $1 \leq j \leq n-2$, let v_j and w_j be the vertices adjacent to u_{j+1}. The resultant graph is called a Twig graph Tg_n, with vertex set $V(Tg_n) = \{u_i, v_j, w_j / 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and the edge set $E(Tg_n) = \{u_iu_{i+1}, v_ju_{j+1}, w_ju_{j+1} / 1 \leq i \leq n-1, 1 \leq j \leq n-2\}$. Clearly the graph Tg_n has 3n - 4 vertices and 3n - 5 edges. Define $f: V(G) \rightarrow W$ by

$f(u_2) = 0$

$f(u_3) = G_1$

$f(u_i) = G_{i-2} - f(u_{i-1}),\ 4 \leq i \leq n$

$f(u_1) = G_{3n-5}$

$f(v_i) = G_{n-2+i} - f(u_{i+1}), 1 \leq i \leq n-2$

$f(w_i) = G_{3n-5-i} - f(u_{i+1}), 1 \leq i \leq n-2.$

Then f induces a bijection $f^+ : E(G) \rightarrow \{G_1, G_2, \ldots, G_{3n-5}\}$ given by

$f^+(u_1u_2) = f(u_1) + f(u_2) = G_{3n-5}$

$f^+(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) = G_{i-1}, 2 \leq i \leq n-1$

$f^+(v_iu_{i+1}) = f(v_i) + f(u_{i+1}) = G_{n-2+i} - f(u_{i+1}) + f(u_{i+1}) = G_{n-2+i}, 1 \leq i \leq n-2.$

$f^+(w_iu_{i+1}) = f(w_i) + f(u_{i+1}) = G_{3n-5-i} + f(w_i) - f(w_i) = G_{3n-5-i}$, $1 \leq i \leq n - 2$.
Thus, the induced edge labels are the first 3n–5 seventh order triangular numbers. Hence Tg_n admits a seventh order triangular sum labeling.

Example 2.10. A seventh order triangular sum labeling of Tg_5 is shown in figure 2.5

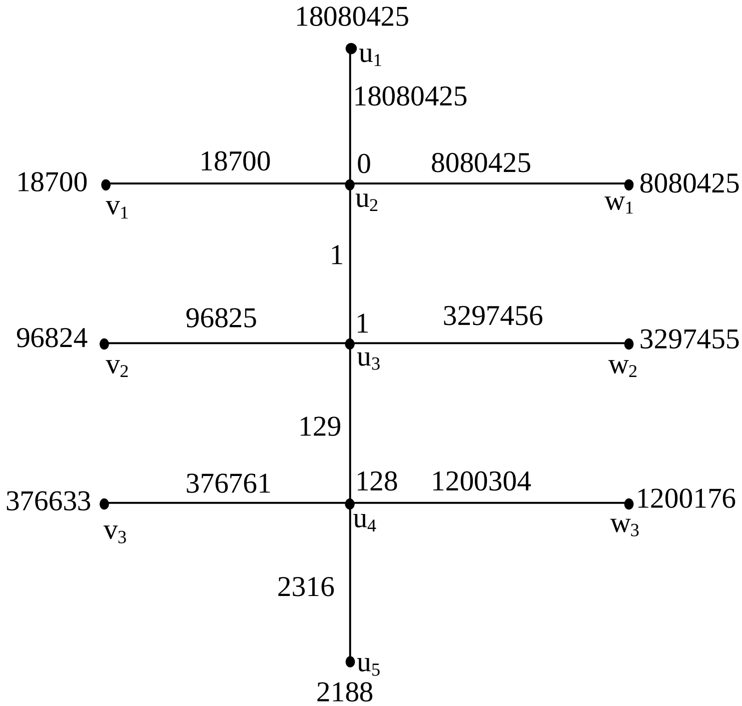

Figure 2.5: Tg_5 with a seventh order triangular sum labeling

Theorem 2.11. The Coconut tree T(m, n) admits a seventh order triangular sum labeling.
Proof: Let $u_1, u_2, \ldots, u_n$ be the end vertices of star $K_{1, n}$ with central vertex u. Let $v_1, v_2, \ldots, v_m$ be the path P_m. Identifying v_1 with u. The resultant graph G is a coconut tree T(m,n) with vertex set $V(G) = \{v_i, u_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and edge set $E(G) = \{v_iv_{i+1}, v_1u_j, /1 \leq j \leq n, 1 \leq i \leq m - 1\}$. Clearly G has n + m vertices and n + m – 1 edges. Define f: V(G) → W by

$f(v_1) = 0$
$f(v_i) = G_{i-1} - f(v_{i-1})$, $2 \leq i \leq m$
$f(u_j) = G_{m+j-1}$, $1 \leq j \leq n$.

Then f induces a bijection $f^+ : E(G) \rightarrow \{G_1, G_2, \ldots, G_{m+n-1}\}$ given by
$f^+(v_iv_{i+1}) = f(v_i) + f(v_{i+1}) = f(v_i) + G_i - f(v_i) = G_i$, $1 \leq i \leq m - 1$
$f^*(v_1u_j) = f(v_1) + f(u_j) = 0 + G_{m+j-1} = G_{m+j-1}$, $1 \leq j \leq n$.
Thus, the induced edge labels are the first n + m – 1 seventh order triangular numbers.
Hence G admits a seventh order triangular sum labeling.

Example 2.12. A seventh order triangular sum labeling of T (4, 5) is shown in figure 2.6.

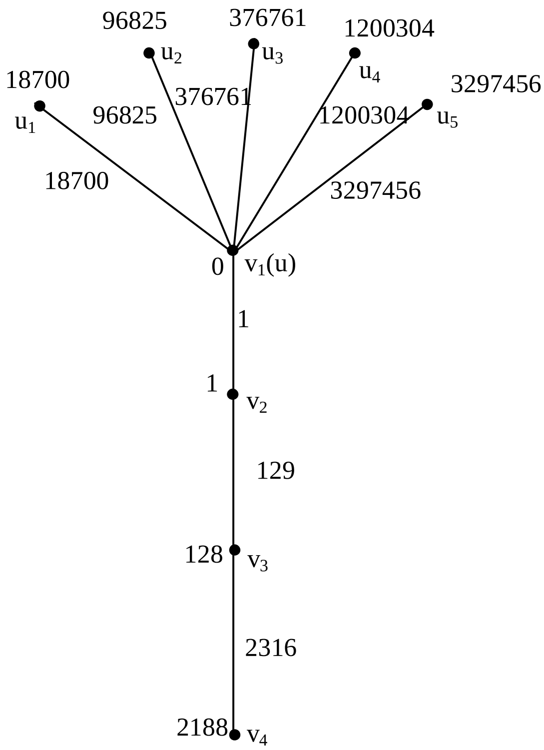

Figure 2.6: T (4, 5) with a seventh order triangular sum labeling

Theorem 2.13. The star graph $K_{1,n}$ admits an eighth order triangular sum labeling.
Proof: Let u be the central vertex and let $u_1, u_2, \ldots, u_n$ be the pendant vertices of the star $K_{1,n}$. Then the vertex set $V(K_{1,n}) = \{u, u_i \ / \ 1 \le i \le n\}$ and the edge set $E(K_{1,n}) = \{uu_i \ / \ 1 \le i \le n\}$. Clearly $K_{1,n}$ has n + 1 vertices and n edges. Define $f : V(K_{1,n}) \to W$ by

$f(u) = 0$
$f(u_i) = H_i, \ 1 \le i \le n$.

Then f induces a bijection $f^+ : E(G) \to \{H_1, H_2, \ldots, H_n\}$ given by
$f^+(uu_i) = f(u) + f(u_i) = 0 + H_i = H_i, \ 1 \le i \le n$.

Cleary, the induced edge labels are the first n eighth order triangular numbers. Hence $K_{1,n}$ admits an eighth order triangular sum labeling.

Example 2.14. An eighth order triangular sum labeling of $K_{1,5}$ is shown in Figure 2.7.

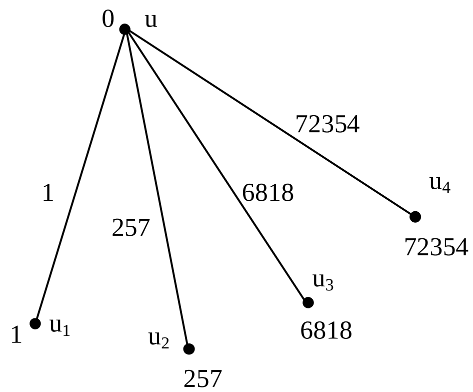

Figure 2.7: $K_{1,4}$ with an eighth order triangular sum labeling

Theorem 2.15. The double star $K_{1,n,n}$ admits an eighth order triangular sum labeling.
Proof: Let G be the double star $K_{1,n,n}$. Let $V(G) = \{u, u_i, v_i / 1 \leq i \leq n\}$ be the vertex set and $E(G) = \{uu_i, u_iv_i / 1 \leq i \leq n\}$ be the edge set of $K_{1,n,n}$. Then G has 2n + 1 vertices and 2n edges. Define $f : V(G) \rightarrow W$ by

$f(u) = 0$

$f(u_i) = H_i, \ 1 \leq i \leq n$

$f(v_i) = H_{n+i} - f(u_i), 1 \leq i \leq n$

Then f induces a bijection $f^+ : E(G) \rightarrow \{H_1, H_2, . . ., H_{2n}\}$ given by

$f^+(uu_i) = f(u) + f(u_i) = 0 + H_i = H_i, 1 \leq i \leq n$

$f^+(u_iv_i) = f(u_i) + f(v_i) = f(u_i) + H_{n+I} - f(u_i) = H_{n+i}, \ 1 \leq i \leq n.$

Thus, the induced edge labels are the first 2n eighth order triangular numbers. Hence G admits an eighth order triangular sum labeling.

Example 2.16. An eighth order triangular sum labeling of double star $K_{1,3,3}$ is shown in figure 2.8.

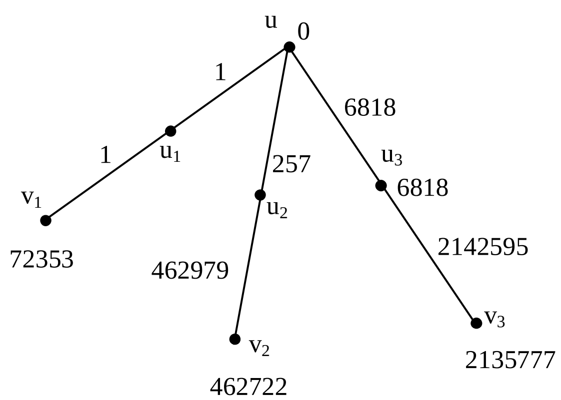

Figure 2.8: Double star $K_{1,3,3}$ with an eighth order triangular sum labeling

Theorem 2.17. Y_{n+1} admits an eighth order triangular sum labeling.

Proof: Let P_n be the path $u_1u_2 \ldots u_n$. Let v be the vertex adjacent to u_2. The resultant graph G is Y_{n+1} with vertex set $V(G) = \{u_i, v / 1 \leq i \leq n\}$ and the edge set $E(G) = \{u_iu_{i+1}, vu_2 /1 \leq i \leq n - 1\}$. Clearly the graph G has n + 1 vertices and n edges. Define f: $V(G) \rightarrow W$ by

$f(u_1) = 1$

$f(u_2) = 0$

$f(v) = 257$

$f(u_i) = H_i - f(u_{i-1}),\ 3 \leq i \leq n.$

Then f induces a bijection $f^+ : E(G) \rightarrow \{H_1, H_2, \ldots, H_n\}$ given by

$f^+(u_1u_2) = f(u_1) + f(u_2) = 1 = H_1$

$f^+(u_2v) = f(u_2) + f(v) = 257 = H_2$

$f^+(u_2u_3) = f(u_2) + f(u_3) = 0 + 6818 = H_3$

$f^+(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) = f(u_i) + H_{i+1} - f(u_i) = H_{i+1}, 3 \leq i \leq n.$

Cleary, the induced edge labels are the first n eighth order triangular numbers. Hence G admits an eighth order triangular sum labeling.

Example 2.18. An eighth order triangular sum labeling of $\mathbf{Y_{5+1}}$is shown in figure 2.9.

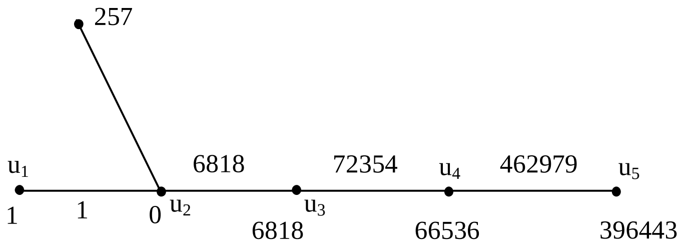

Figure 2.9: Y_{5+1} with an eighth order triangular sum labeling

Theorem 2.19. Tg_n admits an eighth order triangular sum labeling.

Proof: Let $u_1u_2 \ldots u_n$ be the path of length n. For $1 \leq j \leq n - 2$, let v_j and w_j be the vertices adjacent to u_{j+1}. The resultant graph is called a Twig graph Tg_n, with vertex set $V(Tg_n) = \{u_i, v_j, w_j / 1 \leq i \leq n, 1 \leq j \leq n - 2\}$ and the edge set $E(Tg_n) = \{u_iu_{i+1}, v_ju_{j+1}, w_ju_{j+1} / 1 \leq i \leq n - 1, 1 \leq j \leq n - 2\}$. Clearly the graph Tg_n has 3n – 4 vertices and 3n – 5 edges. Define f: $V(G) \rightarrow W$ by

$f(u_2) = 0$

$f(u_3) = H_1$

$f(u_i) = H_{i-2} - f(u_{i-1}),\ 4 \leq i \leq n$

$f(u_1) = H_{3n-5}$

$f(v_i) = H_{n-2+i} - f(u_{i+1}), 1 \leq i \leq n - 2$

$f(w_i) = H_{3n-5-i} - f(u_{i+1}), 1 \leq i \leq n - 2.$

Then f induces a bijection $f^+ : E(G) \rightarrow \{H_1, H_2, \ldots, H_{3n-5}\}$ given by

$f^+(u_1u_2) = f(u_1) + f(u_2) = H_{3n-5}$

$f^+(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) = H_{i-1},\ 2 \leq i \leq n - 1$

$f^+(v_iu_{i+1}) = f(v_i) + f(u_{i+1}) = H_{n-2+i},\ 1 \leq i \leq n - 2.$

$f^+(w_iu_{i+1}) = f(w_i) + f(u_{i+1}) = H_{3n-5-i},\ 1 \leq i \leq n - 2.$

Cleary, the induced edge labels are the first 3n–5 eighth order triangular numbers. Hence Tg_n admits an eighth order triangular sum labeling.

Example 2.20. An eighth order triangular sum labeling of Tg_4 is shown in figure 2.10.

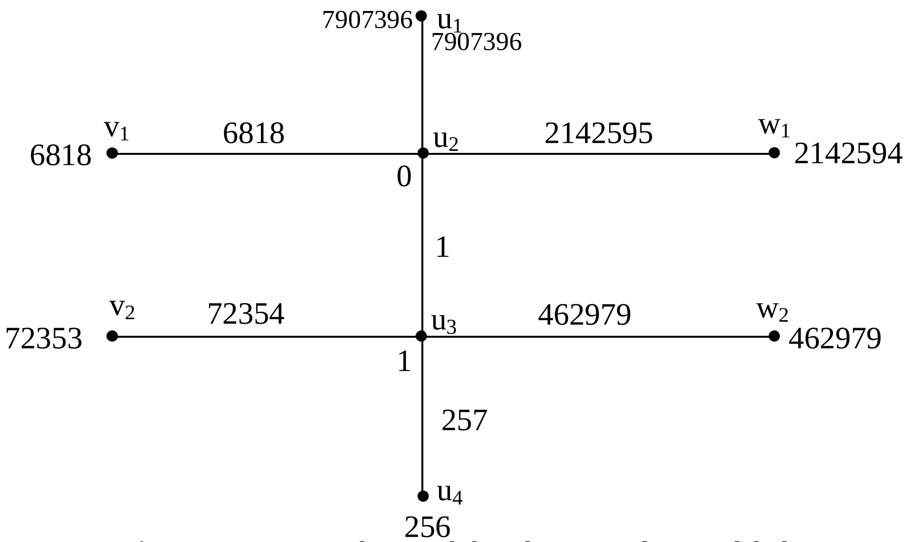

Figure 2.10: Tg_4 with an eighth order triangular sum labeling

Theorem 2.21. The coconut tree T(m, n) is an eighth order triangular sum labeling.

Proof: Let $u_1, u_2, \ldots, u_n$ be the end vertices of star $K_{1,\,n}$ with central vertex u. Let $v_1, v_2, \ldots, v_m$ be the path P_m. Identifying v_1 with u. The resultant graph G is a coconut tree T(m,n) with vertex set $V(G) = \{v_i, u_j / 1 \le i \le m, 1 \le j \le n\}$ and edge set $E(G) = \{ v_iv_{i+1}, v_1u_j, / 1 \le j \le n, 1 \le i \le m - 1\}$. Clearly G has n + m vertices and n + m - 1 edges. Define f: $V(G) \to W$ by

$f(v_1) = 0$

$f(v_i) = H_{i-1} - f(v_{i-1}), \quad 2 \le i \le m$

$f(u_j) = H_{n+j-1}, \quad 1 \le i \le n.$

Then f induces a bijection $f^+ : E(G) \to \{H_1, H_2, \ldots, H_{m+n-1}\}$ given by

$f^+(v_iv_{i+1}) = f(v_i) + f(v_{i+1}) = H_i, \ 1 \le i \le m - 1$

$f^*(v_1u_j) = f(v_1) + f(u_j) = H_{m+j-1}, 1 \le j \le n.$

Cleary, the induced edge labels are the first n + m – 1 eighth order triangular numbers. Hence G admits an eighth order triangular sum labeling.

Example 2.22. An eighth order triangular sum labeling of T(4, 4) is shown in Figure 2.11.

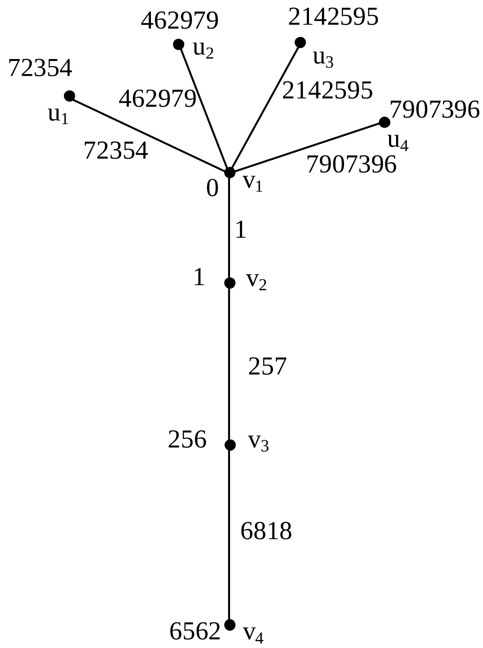

Figure 2.11: Coconut tree T(4, 4) with an eighth order triangular sum labeling

3. Conclusion

In this paper, we prove the seventh and eighth order triangular sum labeling of star graph $K_{1,n}$, double star $K_{1,n,n}$, bistar $B_{m,n}$, Y_{n+1}, Tg_n and Coconut tree T(m, n).

REFERENCES

1. D.M. Burton, Elementary number theory, Brown Publishers, Second edition (1990).
2. J. Devaraj, S.P.Reshma and C.Sunitha, On triangular sum graphs of some graphs, *International Journal of Advanced and Innovative Research*, 6 (2017) 8.
3. F.Harrary, Graph theory, Narosa publishing House (2001).
4. J.A.Gallioan, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 17 (2010) # DS6.
5. S.Murugesan, D.Jayaraman and J.Shiama, Second and third order triangular sum labeling of graphs, *Intern. Journal of Mathematical Archive*, 4(2) (2013) 55-62.
6. M.A.Seoud and M.A Salim, Further results on triangular sum graphs, *International Mathematical Form*, 7(48) (2012) 2393-2405.
7. P.Shankaran, On triangular sum labelings of graphs, *Global Research and Development Journal for Engineering*, 3 (2018) 29-33.
8. S.K.Vaidya and N.H.Shah, Some star and bistar related divisor cordial graphs, *Annals of Pure and Applied Mathematics*, 3(1) (2013) 67-77.
9. S.K.Vaidya, U.M.Prajapatti and P.L.Vihol, Some important results on triangular sum graphs, *Applied Mathematical Sciences*, 3(36) (2009) 1763-1772.
10. D.Vijayalakshmi and S.Priyanka, Star-in-coloring of some star graph families, *Annals of Pure and Applied Mathematics*, 14(3) (2017) 461-464.