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On Normal Sub-Intuitionistic Fuzzy Multigroups

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Abstract. This paper developed the concept of normal sub-intuitionistic fuzzy Multigroups. It then established that intersection, union, of any two normal sub-intuitionistic fuzzy multigroups is also normal. The inverse of any normal sub-intuitionistic fuzzy multigroup is normal and for any normal sub-intuitionistic fuzzy multigroup, the root (support) set of normal sub-intuitionistic fuzzy multigroup is a normal sub-intuitionistic fuzzy group. It also showed that under the isomorphism function between any two groups, the image of a normal sub-intuitionistic fuzzy multigroup under the isomorphism is a normal sub-intuitionistic fuzzy multigroup and the inverse image of a normal sub-intuitionistic fuzzy multigroup under the isomorphism is a normal sub-intuitionistic fuzzy multigroup.

Keywords: Multiset, fuzzy multiset, intuitionistic fuzzy multiset, multi groups, fuzzy multigroups, intuitionistic fuzzy multi group, isomorphism function, normal sub-intuitionistic fuzzy multigroup

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1. Introduction

Classical set theory introduced by George Cantor has proved itself to be one of the most powerful tools of modern Mathematics. In this set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object are allowed in a set, then the mathematical structure is called as multiset. Thus, a multiset differs from a set in the sense that each element has a multiplicity. An account of the development of multiset theory can be seen in [1, 2, 3, 4]. Most of the real life situations are complex and for modeling them we need a simplification of the complex system. The simplification must be in such a way that the information lost should be minimum. One way to do this is to allow some degree of uncertainty into it. To handle situations like this, many tools were suggested. They include fuzzy sets, rough sets, soft sets etc. Considering the uncertainty factor, Zadeh [5] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. Fuzzy sets were introduced with a view to reconcile mathematical modeling and human knowledge in the engineering sciences. Since then, a considerable body of literature has blossomed around the concept of fuzzy sets in an incredibly wide range of areas, from mathematics and logics to traditional and advanced engineering methodologies. In 1983, Atanassov [6, 7] introduced the concept of Intuitionistic Fuzzy sets. The same time a theory called 'Intuitionistic Fuzzy set theory' was independently introduced by Takeuti and Titani [8] as a theory developed in (a kind of) Intuitionistic logic. An Intuitionistic Fuzzy set is characterized by two functions expressing the degree of membership and the degree of nonmembership of elements of the universe to the Intuitionistic Fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainty and vagueness. It is well-known that in the beginning of the last century L.Brouwer introduced the concept of Intuitionism. The name Intuitionistic Fuzzy set is due to George Gargove, with themotivation that their fuzzification denies the law of excluded middle-one of the main ideas of Intuitionism. As a generalization of multiset, Yager [9] introduced fuzzy multisets and suggested possible applications to relational databases. An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values. The concept of Intuitionistic Fuzzy Multiset is introduced in [10] which have applications in medical diagnosis and robotics. In mathematics, abstract algebra is the study of algebraic structures and more specifically the term algebraic structure generally refers to a set (called carrier set or underlying set) with one or more finitely operations defined on it. Examples of algebraic structures include groups, rings, fields, and lattices. The algebraic structures of Fuzzy multisets are introduced in [11]. Adamu et al. [12] developed the concepts of normal and soft normal groups under multisets and soft multisets context. They defined the concepts of normal submultigroups and soft normal Multigroups and proved some of their related algebraic structures. In this paper we are extending these algebraic structures on intuitionistic fuzzy multisets and intuitionistic fuzzy multigroups by introducing a new concept named normal sub intuitionistic fuzzy multigroups.

2. Preliminaries

Definition 2.1. [13] Let X be a set. A multiset (mset) M drawn from X is represented by a function count M or C_M defined as $C_M : X \longrightarrow \{0,1,2,3,...\}$. For each$x \in X$, $C_M(x)$ is the characteristic value of x in M. Here $C_M(x)$ denotes the number of occurrences of x in M.

Definition 2.2. [14] Let X be a group. A multi set G over X is a multi-group over X if the count of Gsatisfies the following two conditions.

i. $C_G(xy) \geq C_G(x) \wedge C_G(y) \forall x,y \in X$;
ii. $C_G(x^{-1}) \geq C_G(x) \forall x \in X$.

Definition 2.3. [15] If X is a collection of objects, then a fuzzy set A in X is a set of ordered pairs: $A = \{(x, \mu_A(x)) : x \in X, \mu_A : X \longrightarrow [0,1]\}$ where μ_A is called the membership function of A, and is defined from X into $[0,1]$.

Definition 2.4. [16] Let G be a group and $\mu \in FP(G)$ (fuzzy power set of G), then μ is called fuzzy subgroup of G if

i. $\mu(xy) \geq \mu(x) \wedge \mu(y) \forall x, y \in X$ and
ii. $\mu(x^{-1}) \geq \mu(x) \forall x \in X$.

Definition 2.5. [10] Let X be a nonempty set. An Intuitionistic Fuzzy Multiset A denoted by IFMS drawn from X is characterized by two functions : 'count membership' of $A(CM_A)$ and 'count non membership' of $A(CN_A)$ given respectively by $CM_A : X \longrightarrow Q$ and $CN_A : X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$ such that for each$x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $A(CM_A)$ which is denoted by $(\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x))$ where $(\mu_A^1(x) \geq \mu_A^2(x) \geq, \dots, \mu_A^p(x))$ and the corresponding non membership sequence will be denoted by
$(\ \upsilon_A^1(x), \upsilon_A^2(x), \dots, \upsilon_A^p(x))$ such that $0 < \mu_A^i(x) + \upsilon_A^i(x) < 1$ for every $x \in X$ and $i = 1,2, \dots, p$.

An IFMS A is denoted by

$$A = \left\{< x : (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x))\ , (\ \upsilon_A^1(x), \upsilon_A^2(x), \dots, \upsilon_A^p(x)) >: x \in X\right\}$$

Definition 2.6. [10] Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 < \mu_A^i(x) + \upsilon_A^i(x) < 1$ and it is denoted by $L(x : A)$. That is

$$L(x : A) = |\ CM_A(x)\ | = |\ CN_A(x)\ |$$
$$L(x :\ A)\ =\ |CMA(x)|\ =\ |CNA(x)|$$

Definition 2.7. [10] If A and B are IFMSs drawn from X then

$L(x : A, B) = Max\{L(x : A), L(x : B)\}$.
Alternatively we use $L(x)$ for $L(x : A, B)$.

Definition 2.8. [11] For any two IFMSs A and B drawn from a set X, the following operations and relations will hold. Let

$$A = \left\{< x : (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x))\ , (\ \upsilon_A^1(x), \upsilon_A^2(x), \dots, \upsilon_A^p(x)) >: x \in X\right\}$$

And

$$B = \left\{< x : (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x))\ , (\ \upsilon_A^1(x), \upsilon_A^2(x), \dots, \upsilon_A^p(x)) >: x \in X\right\}$$

Then

i. Inclusion

   $A \subset B \Leftrightarrow \mu_A^j(x) \leq \mu_A^j(x)$ and $\upsilon_A^j(x) + \upsilon_A^j(x); j = 1,2, \dots, L(x), x \in X$
   $A = B \Leftrightarrow A \subset B$and $B \subset A$

ii. Complement

$$\neg A = \left\{< x : (\ \upsilon_A^1(x), \upsilon_A^2(x), \dots, \upsilon_A^p(x))\ , (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x)) >: x \in X\right\}$$

iii. Union $(A \cup B)$

   In $A \cup B$the membership and non-membership values are obtained as follows.

$$\mu_{A \cup B}^j(x) = \mu_A^j(x) \vee \mu_B^j(x)$$

$$\upsilon^{j}_{A\cup B}(x) = \upsilon^{j}_{A}(x) \wedge \upsilon^{j}_{A}(x), j = 1,2, \dots, L(x), x \in X.$$

iv. Intersection $(A \cap B)$

In $A \cap B$ the membership and non-membership values are obtained as follows.

$$\mu^{j}_{A\cap B}(x) = \mu^{j}_{A}(x) \wedge \mu^{j}_{B}(x)$$

$$\upsilon^{j}_{A\cap B}(x) = \upsilon^{j}_{A}(x) \vee \upsilon^{j}_{A}(x), j = 1,2, \dots, L(x), x \in X.$$

Definition 2.9. [11] If X and Y are two nonempty sets and $f : X \to Y$ be a mapping. Then

i. The image of the FMS $A \in FM(X)$ under the mapping f is denoted by $f(A)$, or

$$CM_{f[A]}(y) = \begin{cases} \vee_{f(x)=y} CM_A(x) & if\ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

ii. The inverse image of the FMS $B \in FM(Y)$ under the mapping f is denoted by $f^{-1}(B)$ or$f^{-1}[B]$, where $CM_{f^{-1}[B]}(x) = CM_B f[x]$.

Definition 2.10. [11] Let X be a group. A fuzzy multiset G over X is a fuzzy multi group (FMG) over X if the count (count membership) of G satisfies the following two conditions.

i. $CM_G(xy) \geq CM_G(x) \wedge CM_G(y)\ \forall x, y \in X.$

ii. $CM_G(x^{-1}) = CM_G(x)\ \forall x \in X$

Definition 2.11. [12] A submgroup H of a mgroup $M \in MG(X)$ is said to be a normal submgroup iff for any $h \in H^*$,

$C_H(x^{-1}hx) \geq C_H(h), \forall\ x \in M^*.$

Theorem 2.12. [12] Let $M_1$and M_2 be submgroups of a mgroup $M \in MG(X)$such that $M_1, M_2 \in \varDelta\wp(M)$, then

i. $M_1 \cap M_2 \in \varDelta\wp(M)$;

ii. $M_1 \cup M_2 \in \varDelta\wp(M)$.

Theorem 2.13 [12] If $H \in \varDelta\wp(M)$ then H^* is a normal subgroup of M^*

Theorem 2.14. [12] Let X be a group and $M \in MG(X)$. If $M \in \varDelta\wp(M)$, then $M^{-1} \in \varDelta\wp(M)$.

Theorem 2.15. [12] Let X, Ybe two groups and $f : X \longrightarrow Y$ be an isomorphism of groups. Suppose $M_1 \in MG(X)$, $M_2 \in MG(Y)$ and $f(M_1) \subseteq M_2$

i. if $H \in \varDelta\wp(M_1)$, then $f(H) \in \varDelta\wp(M_2)$;

ii. if $H \in \varDelta\wp(M_2)$ then $f^{-1}(H) \in \varDelta\wp(M_1)$.

Definition 2.16. [17] Let X be a group. An intuitionistic fuzzy multiset G over X is an intuitionistic fuzzy multi group (IFMG) over X if the counts (count membership and non-membership) of G satisfies the following two conditions.

i. $CM_G(xy) \geq CM_G(x) \wedge CM_G(y)\ \forall x, y \in X.$

ii. $CM_G(x^{-1}) \geq CM_G(x)\ \forall x \in X$.
iii. $CN_G(xy) \leq CM_G(x) \wedge CM_G(y)\ \forall x, y \in X$.
iv. $CN_G(x^{-1}) \leq CM_G(x)\ \forall x \in X$.

Definition 2.17. [17] Let $G \in FMS(X)$. Then define
$$G^* = \{x \in X : CM_G(x) = CM_G(e) \text{ and } CN_G(x) = CN_G(e)\}.$$

Proposition 2.18. [17] Let $G \in FMG(X)$. Then G^* is a subgroup of X.

Definition 2.19. [17] Let $G \in FMS(X)$. Let $j \in \mathbb{N}$. Then define
$$G^j = \left\{x \in X : \mu_G^j(x) \geq 0, \mu_G^{j+1}(x) = 0 \text{ and } \upsilon_G^j(x) = 0\right\}.$$

Theorem 2.20. [17] Let $G \in FMG(X)$. Then G^j is a subgroup of X iff
$$\mu_G^{j+1}(xy^{-1}) = 0 \text{ and } \upsilon_G^{j+1}(xy^{-1}) = 0\ \forall x, y \in G^j$$

3. Normal sub-intuitionistic fuzzy multigroups

Throughout this section, let X be a group with binary operation and the identity element is e. Also we assume that the intuitionistic fuzzy multisets and intuitionistic fuzzy Multigroups are taken from $IFMS(X)$ and $IFMG(X)$respectively.

Definition 3.1. A sub-intuitionistic fuzzy multigroup H of an intuitionistic fuzzy multigroup $G \in IFMG(X)$is said to be a normal sub-intuitionistic fuzzy Multigroups iff for any $h \in H^*$,

i. $CM_H(x^{-1}hx) \geq CM_H(h), \forall\ x \in G^*$.
ii. $CN_H(x^{-1}hx) \leq\ CN_H(h) \forall\ x \in G^*$.

We denote the set of all normal sub-intuitionistic fuzzy Multigroups of an intuitionistic fuzzymgroup $G \in IFMG(X)$ by $\Delta\wp SIFMG(G)$ and the normality of H over $G \in IFMG(X)$ by$H \trianglelefteq G$

Example 3.2. $(\mathrm{Z}_4, +_4)$ is a group. Then
$$G = \begin{Bmatrix} <2:(0.6,0.4,0.3,0.1),(0.4,0.6,0.7,0.9)>, \\ <1:(0.8,0.7,0.7,0.5,0.1,0.1),(0.2,0.3,0.3,0.5,0.9,0.9)>, \\ <3:(0.8,0.7,0.7,0.5,0.1,0.1),(0.2,0.3,0.3,0.5,0.9,0.9)>, \\ <0:(0.9,0.8,0.7,0.5,0.1,0.1),(0.1,0.8,0.3,0.5,0.9,0.9)>, \end{Bmatrix}$$
is an intuitionistic fuzzy multi group. And
$$H = \begin{Bmatrix} <2:(0.6,0.4,0.3,0.1),(0.4,0.6,0.7,0.9)>, \\ <1:(0.7,0.6,0.5,0.5,0.1,0.1),(0.3,0.4,0.5,0.5,0.9,0.9)>, \\ <3:(0.7,0.6,0.5,0.5,0.1,0.1),(0.3,0.4,0.5,0.5,0.9,0.9)>, \\ <0:(0.9,0.8,0.7,0.5,0.1,0.1),(0.1,0.2,0.3,0.5,0.9,0.9)> \end{Bmatrix}$$

is a normal sub-intuitionistic fuzzy multigroup of the intuitionistic fuzzy multigroup G.

Proposition 3.3. Let $G_1$and G_2 be sub-intuitionistic fuzzymgroup of an intuitionistic fuzzy mgroup $G \in IFMG(X)$ such that $G_1, G_2 \in \Delta\wp SIFMG(G)$, then

i. $G_1 \cap G_2 \in \varDelta\wp SIFMG(G)$;
ii. $G_1 \cup G_2 \in \varDelta\wp SIFMG(G)$.

Proof:

i. $G_1$and G_2 are sub-intuitionistic fuzzy mgroups, we have;

$CM_{G_1}(xy) \geq CM_{G_1}(x) \wedge CM_{G_1}(y), CM_{G_1}(x^{-1}) = CM_{G_1}(x) \;\; \forall\, x, y \in X$,

$CM_{G_2}(xy) \geq CM_{G_2}(x) \wedge CM_{G_2}(y), CM_{G_2}(x^{-1}) = CM_{G_2}(x) \;\; \forall\, x, y \in X$.

$$\therefore CM_{G_1 \cap G_2}(xy) = \bigwedge\{CM_{G_1}(xy), CM_{G_2}(xy)\}$$
$$\geq \bigwedge\{[C_{G_1}(x) \wedge C_{G_1}(y)], [C_{G_2}(x) \wedge C_{G_2}(y)]\}$$
$$= CM_{G_1}(x) \wedge CM_{G_1}(y) \wedge CM_{G_2}(x) \wedge CM_{G_2}(y)$$
$$= [CM_{G_1}(x) \wedge CM_{G_2}(x)] \wedge [CM_{G_1}(y) \wedge CM_{G_2}(y)]$$
$$= CM_{G_1 \cap G_2}(x) \wedge CM_{G_1 \cap G_2}(y) \qquad (1)$$
$$CM_{G_1 \cap G_2}(x^{-1}) = CM_{G_1}(x^{-1}) \wedge CM_{G_2}(x^{-1})$$
$$= CM_{G_1}(x) \wedge CM_{G_2}(x)$$
$$= CM_{G_1 \cap G_2}(x) \qquad (2)$$

Next we also have;

$CN_{G_1}(xy) \leq CN_{G_1}(x) \wedge CN_{G_1}(y), CN_{G_1}(x^{-1}) = CN_{G_1}(x) \;\; \forall\, x, y \in X$,

$CN_{G_2}(xy) \leq CN_{G_2}(x) \wedge CN_{G_2}(y), CN_{G_2}(x^{-1}) = CN_{G_2}(x) \;\; \forall\, x, y \in X$.

$$\therefore CN_{G_1 \cap G_2}(xy) = \bigwedge\{CN_{G_1}(xy), CN_{G_2}(xy)\}$$
$$\leq \bigvee\{[CN_{G_1}(x) \wedge CN_{G_1}(y)], [CN_{G_2}(x) \wedge CN_{G_2}(y)]\}$$
$$= CN_{G_1}(x) \wedge CN_{G_1}(y) \wedge CN_{G_2}(x) \wedge CN_{G_2}(y)$$
$$= [CN_{G_1}(x) \wedge CN_{G_2}(x)] \wedge [CN_{G_1}(y) \wedge CN_{G_2}(y)]$$
$$= CN_{G_1 \cap G_2}(x) \wedge CN_{G_1 \cap G_2}(y) \qquad (3)$$
$$CN_{G_1 \cap G_2}(x^{-1}) = CN_{G_1}(x^{-1}) \wedge CN_{G_2}(x^{-1})$$
$$= CN_{G_1}(x) \wedge CN_{G_2}(x)$$
$$= CN_{G_1 \cap G_2}(x) \qquad (4)$$

Thus, from (1), (2), (3) and (4), we have

$$G_1 \cap G_2 \in IFMG(X) \qquad (5)$$

Next

Since $G_1 \subseteq G$ and $G_2 \subseteq G$, then $G_1 \cap G_2 \subseteq G$

Since $G_1, G_2 \in \varDelta\wp SIFMG(M)$, we have

$CM_{G_1}(x^{-1}y_1x) \geq CM_{G_1}(y_1), \forall\, x \in G^*, y_1 \in {G_1}^*$;

$CM_{G_2}(x^{-1}y_2x) \geq CM_{G_2}(y_2), \forall\, x \in G^*, y_2 \in {G_2}^*$;

Now

$$CM_{G_1 \cap G_2}(x^{-1}yx) = \bigwedge\{CM_{G_1}(x^{-1}yx), CM_{G_2}(x^{-1}yx)\}$$
$$\geq \bigwedge\{CM_{G_1}(y), CM_{G_2}(y)\} \forall\, x \in G^*, y \in (G_1 \cap G_2)^* = G_1^* \cap G_2^*$$

Since $CM_{G_1}(x^{-1}y_1x) \geq CM_{G_1}(y_1)$ and $CM_{G_2}(x^{-1}y_2x) \geq CM_{G_2}(y_2)$

$$\therefore CM_{G_1 \cap G_2}(x^{-1}yx) \geq CM_{G_1}(y) \wedge CM_{G_2}(y) = CM_{G_1 \cap G_2}(y)$$

Thus $CM_{G_1 \cap G_2}(x^{-1}yx) \geq CM_{G_1 \cap G_2}(y), \forall\, x \in G^*, y \in G_1 \cap G_2$ (6)

Similarly,

$CN_{G_1}(x^{-1}y_1x) \leq CN_{G_1}(y_1), \forall\, x \in G^*, y_1 \in {G_1}^*$;

$CN_{G_2}(x^{-1}y_2x) \leq CN_{G_2}(y_2), \forall\, x \in G^*, y_2 \in {G_2}^*$;

Therefore,

$$CN_{G_1\cap G_2}(x^{-1}yx) = \vee\{CN_{G_1}(x^{-1}yx), CN_{G_2}(x^{-1}yx)\}$$
$$\leq \vee\{CN_{G_1}(y), CN_{G_2}(y)\} \forall\, x \in G^*, y \in (G_1 \cap G_2)^* = G_1^* \cap G_2^*$$

Since $CN_{G_1}(x^{-1}y_1x) \leq CN_{G_1}(y_1)$ and $CN_{G_2}(x^{-1}y_2x) \leq CN_{G_2}(y_2)$

$$\therefore CN_{G_1\cap G_2}(x^{-1}yx) \leq CN_{G_1}(y) \wedge CN_{G_2}(y) = N(y)$$

Thus $CN_{G_1\cap G_2}(x^{-1}yx) \leq CN_{G_1\cap G_2}(y), \forall\, x \in G^*, y \in G_1 \cap G_2$ (7)

From equations (5), (6) and (7) we have, $G_1 \cap G_2 \in \Delta\wp SIFMG(M)$.

ii. Since G_1 and G_2 are sub-intuitionistic fuzzy mgroups, we have;

$CM_{G_1}(xy) \geq CM_{G_1}(x) \wedge CM_{G_1}(y), CM_{G_1}(x^{-1}) = CM_{G_1}(x) \;\; \forall\, x, y \in X$,

$CM_{G_2}(xy) \geq CM_{G_2}(x) \wedge CM_{G_2}(y), CM_{G_2}(x^{-1}) = CM_{G_2}(x) \;\; \forall\, x, y \in X$.

$$\therefore CM_{G_1\cup G_2}(xy) = \vee\{CM_{G_1}(xy), CM_{G_2}(xy)\}$$
$$\geq \vee\{[CM_{G_1}(x) \wedge CM_{G_1}(y)], [CM_{G_2}(x) \wedge CM_{G_2}(y)]\}$$
$$= [CM_{G_1}(x) \wedge CM_{G_2}(x)] \vee [CM_{G_1}(y) \wedge CM_{G_2}(y)]$$
$$= [CM_{G_1}(x) \vee CM_{G_2}(x)] \wedge [CM_{G_1}(y) \vee CM_{G_2}(y)]$$

$= CM_{G_1\cup G_2}(x) \wedge CM_{G_1\cup G_2}(y)$ (1)

$$CM_{G_1\cup G_2}(x^{-1}) = CM_{G_1}(x^{-1}) \vee CM_{G_2}(x^{-1})$$

$$= C_{M_1}(x) \vee C_{M_2}(x) = C_{M_1\cup M_2}(x)$$

$\therefore CM_{G_1\cup G_2}(x^{-1}) = CM_{G_1\cup G_2}(x)$ (2)

We also have;

$CN_{G_1}(xy) \geq CN_{G_1}(x) \wedge CN_{G_1}(y), CN_{G_1}(x^{-1}) = CN_{G_1}(x) \;\; \forall\, x, y \in X$,

$CN_{G_2}(xy) \geq CN_{G_2}(x) \wedge CN_{G_2}(y), CN_{G_2}(x^{-1}) = CN_{G_2}(x) \;\; \forall\, x, y \in X$.

$$\therefore CN_{G_1\cup G_2}(xy) = \vee\{CN_{G_1}(xy), CN_{G_2}(xy)\}$$
$$\leq \vee\{[CN_{G_1}(x) \wedge CN_{G_1}(y)], [CN_{G_2}(x) \wedge CN_{G_2}(y)]\}$$
$$= [CN_{G_1}(x) \wedge CN_{G_2}(x)] \vee [CN_{G_1}(y) \wedge CN_{G_2}(y)]$$
$$= [CN_{G_1}(x) \vee CN_{G_2}(x)] \wedge [CN_{G_1}(y) \vee CN_{G_2}(y)]$$

$= CN_{G_1\cup G_2}(x) \wedge CN_{G_1\cup G_2}(y)$ (3)

$$CN_{G_1\cup G_2}(x^{-1}) = CN_{G_1}(x^{-1}) \vee CN_{G_2}(x^{-1})$$
$$= CN_{G_1}(x) \vee CN_{G_2}(x)$$
$$= CN_{G_1\cup G_2}(x)$$

$\therefore CN_{G_1\cup G_2}(x^{-1}) = CN_{G_1\cup G_2}(x)$ (4)

Thus, from (1), (2), (3) and (4), we have

$G_1 \cup G_2 \in IFMGMG(X)$ (5)

Next

Since $G_1 \subseteq G$ and $G_2 \subseteq G$, then $G_1 \cup G_2 \subseteq G$

Since $G_1, G_2 \in \Delta\wp SIFMG(M)$, we have

$CM_{G_1}(x^{-1}y_1x) \geq CM_{G_1}(y_1), \forall\, x \in G^*, y_1 \in {G_1}^*$;

$CM_{G_2}(x^{-1}y_2x) \geq CM_{G_2}(y_2), \forall\, x \in G^*, y_2 \in {G_2}^*$;

$$\therefore C_{M_1\cup M_2}(x^{-1}yx) = \vee\{C_{M_1}(x^{-1}yx), C_{M_2}(x^{-1}yx)\}$$
$$\geq \vee\{CM_{G_1}(y), CM_{G_2}(y)\} \forall\, x \in G^*, y \in (G_1 \cup G_2)^* = G_1^* \cup G_2^*$$

Since $CM_{G_1}(x^{-1}y_1x) \geq CM_{G_1}(y_1)$ and $CM_{G_2}(x^{-1}y_2x) \geq CM_{G_2}(y_2)$

$$\therefore CM_{G_1 \cup G_2}(x^{-1}yx) \geq CM_{G_1}(y) \vee CM_{G_2}(y) = CM_{G_1 \cup G_2}(y)$$

Thus $CM_{G_1 \cup G_2}(x^{-1}yx) \geq CM_{G_1 \cup G_2}(y), \forall\, x \in G^*, y \in G_1 \cup G_2$ (6)

Similarly,

$CN_{G_1}(x^{-1}y_1x) \leq CN_{G_1}(y_1), \forall\, x \in G^*, y_1 \in {G_1}^*$;

$CN_{G_2}(x^{-1}y_2x) \leq CN_{G_2}(y_2), \forall\, x \in G^*, y_2 \in {G_2}^*$;

$$\therefore CN_{G_1 \cup G_2}(x^{-1}yx) = \vee\{CN_{G_1}(x^{-1}yx), CN_{G_2}(x^{-1}yx)\}$$

$$\leq \vee\{CN_{G_1}(y), CN_{G_2}(y)\} \forall\, x \in G^*, y \in (G_1 \cup G_2)^* = G_1^* \cup G_2^*$$

Since $CN_{G_1}(x^{-1}y_1x) \leq CN_{G_1}(y_1)$ and $CN_{G_2}(x^{-1}y_2x) \leq CN_{G_2}(y_2)$

$$\therefore CN_{G_1 \cup G_2}(x^{-1}yx) \leq CN_{G_1}(y) \wedge CN_{G_2}(y) = CN_{G_1 \cup G_2}(y)$$

Thus $CN_{G_1 \cup G_2}(x^{-1}yx) \leq CN_{G_1 \cup G_2}(y), \forall\, x \in G^*, \in G_1 \cup G_2$ (7)

From equations (5), (6) and (7) we have, $G_1 \cup G_2 \in \Delta\wp SIFMG(M)$.

Remark 3.4. Let $\{G_i : G_i \in \Delta\wp SIFMG(M), i \in \Delta\}$ then $\bigcap_{i \in \Delta} G_i \in \Delta\wp SIFMG(M)$ and $\bigcup_{i \in \Delta} M_i \in \Delta\wp(M)$.

Proposition 3.5. If $H \in \Delta\wp SIFMG(G)$ then H^* is a normal sub-intuitionistic group of G^*.

Proof:

Since $H \subseteq G \Longrightarrow H^* \subseteq G^*$

Thus, H^* is a sub-intuitionistic group of G^*

Since $H \in \Delta\wp SIFMG(G)$, then for any $x \in G^*$and $h \in H^*$

$CM_H(x^{-1}hx) \geq CM_H(h) > 0$

$\therefore\ CM_H(x^{-1}hx) > 0$

Similarly,

$CN_H(x^{-1}hx) \leq CN_H(h) = 0$

$\therefore\ CM_H(x^{-1}hx) = 0$

$\Longrightarrow (x^{-1}hx) \in H^*$

$\therefore H^*$is a normal sub-intuitionistic group of G^*.

Proposition 3.6. Let $G \in FMG(X)$and $H \in \Delta\wp SIFMG(G)$. Then H^{j*} is a normal subgroup of G^* iff

$$\mu_H^{j+1}(x^{-1}yx) = 0 \ and\ \upsilon_H^{j+1}(x^{-1}yx) = 0\ \forall x \in G^*\ and\ y \in H^{j*}.$$

Proof: Let $x \in G^*\ and\ y \in H^{j*}$. It implies that

$\mu_H^{j}(x^{-1}yx) \geq 0$ And $\mu_H^{j+1}(x^{-1}yx) = 0$ (by definition 2.19)

$\upsilon_H^{j}(x^{-1}yx) = 0$ and $\upsilon_H^{j+1}(x^{-1}yx) = 0$

Assume $\mu_H^{j+1}(x^{-1}yx) = 0\ and\ \upsilon_H^{j+1}(x^{-1}yx) = 0\ \forall x \in G^*\ and\ y \in H^{j*}$

Then by the above proposition,

$\mu_H^{j}(x^{-1}yx) > 0$ and $\upsilon_H^{j}(x^{-1}yx) = 0$

$\Longrightarrow x^{-1}yx \in H^{j*}$. Then H^{j*} is a normal subgroup of G^*. Hence the proof

Conversely,

$H^{j*}$is a normal subgroup ofG^*. Then$x \in G^*, y \in H^{j*} \Longrightarrow x^{-1}yx \in H^{j*}$

$\Longrightarrow \mu_H^{j+1}(x^{-1}yx) = 0\ and\ \upsilon_H^{j+1}(x^{-1}yx) = 0$. Hence the proof.

Proposition 3.7. Let X be a group and $G \in IFMG(X)$. If $G \in \varDelta\wp SIFMG(G)$, then $G^{-1} \in \varDelta\wp SIFMG(G)$.

Proof: Since $G \in \varDelta\wp SIFMG(G)$, then we have

$CM_M(x^{-1}yx) \geq CM_M(y), \forall\, x \in X$ and $y \in G^*$

Also

$$CM_{G^{-1}}(xy) = CM_G[(xy)^{-1}] = CM_G(xy) \geq CM_G(x) \wedge CM_G(y)$$

$$= CM_G(x^{-1}) \wedge CM_G(y^{-1})$$
$$= CM_{G^{-1}}(x) \wedge CM_{G^{-1}}(y)$$
$$\therefore CM_{G^{-1}}(xy) \geq CM_G(x^{-1}) \wedge CM_G(y^{-1})$$
$$CM_{G^{-1}}(x^{-1}) = CM_G[(x^{-1})^{-1}] = CM_G(x)$$
$$= CM_G(x^{-1}) = CM_{G^{-1}}(x)$$
$$\therefore CM_{G^{-1}}(x^{-1}) = CM_{G^{-1}}(x)$$

Thus, $G^{-1} \in IFMG(X)$

$$CM_{G^{-1}}(x^{-1}yx) = CM_G[(x^{-1}yx)^{-1}] = CM_G(x^{-1}yx) \geq CM_G(y), \forall\, x \in X$$

And $y \in G^*$

$$\therefore CM_{G^{-1}}(x^{-1}yx) = CM_G[(x^{-1}yx)^{-1}]$$
$$= CM_G(x^{-1}yx) \geq CM_G(y) = CM_{G^{-1}}(y)$$

$\Longrightarrow CM_{G^{-1}}(x^{-1}yx) \geq CM_{G^{-1}}(y)(1)$

We also have;

$CN_G(x^{-1}yx) \geq CN_G(y), \forall\, x \in X$and $y \in G^*$

And

$CN_{G^{-1}}(xy) = CN_G[(xy)^{-1}] = CN_G(xy) \geq CN_G(x) \wedge CN_G(y)$

$= CN_G(x^{-1}) \wedge CN_G(y^{-1})$

$= CN_{G^{-1}}(x) \wedge CN_{G^{-1}}(y)$

$\therefore CN_{G^{-1}}(xy) \geq CN_G(x^{-1}) \wedge CN_G(y^{-1})$

$CN_{G^{-1}}(x^{-1}) = CN_G[(x^{-1})^{-1}] = CN_G(x)$

$= CN_G(x^{-1}) = CN_{G^{-1}}(x)$

$\therefore CN_{G^{-1}}(x^{-1}) = CN_{G^{-1}}(x)$

Thus, $G^{-1} \in IFMG(X)$

$$CN_{G^{-1}}(x^{-1}yx) = CN_G[(x^{-1}yx)^{-1}] = CN_G(x^{-1}yx) \geq CN_G(y), \forall\, x \in X$$

and $y \in G^*$

$$\therefore CN_{G^{-1}}(x^{-1}yx) = N[(x^{-1}yx)^{-1}]$$
$$= CN_G(x^{-1}yx) \geq CN_G(y) = CN_{G^{-1}}(y)$$

$\Longrightarrow CN_{G^{-1}}(x^{-1}yx) \geq CN_{G^{-1}}(y)(2)$

Hence from (1) and (2) we have $G^{-1} \in \varDelta\wp SIFMG(G)$

Proposition 3.8. Let X, Y be two groups and $f : X \longrightarrow Y$ be an isomorphism of groups. Suppose $G_1 \in IFMG(X)$, $G_2 \in IFMG(Y)$ and $f(G_1) \subseteq G_2$

i. if $H \in \varDelta\wp SIFMG(G_1)$, then $f(H) \in \varDelta\wp SIFMG(G_2)$.

ii. if$H \in \varDelta\wp SIFMG(G_2)$ then $f^{-1}(H) \in \varDelta\wp SIFMG(G_1)$.

Proof: i. Let $H \in \varDelta\wp SIFMG(G_1)$. Clearly H is a sub-intuitionistic fuzzy mgroup of G_1(by definition).

We show that $f(H)$ is a sub-intuitionistic fuzzy of G_2.

$CM_{f(H)}(xy) = \vee_{f(z)=xy} CM_H(z)$ Since $f^{-1}(xy) \neq \emptyset$ (by definition 2.9. (i))
But f is 1-1 and onto (by hypothesis), thus $f^{-1}(xy) = \{z\}$.
In particular,

$$CM_{f(H)}(xy) = CM_H(z) = CM_H\big(f^{-1}(xy)\big) = CM_H(f^{-1}(x)f^{-1}(y)) \quad (1)$$

(f is an isomorphism). But

$$CM_H(f^{-1}(x)f^{-1}(y)) \geq CM_H\big(f^{-1}(x)\big) \wedge CM_H\big(f^{-1}(y)\big) \quad (2)$$

(H is submgroup of M_1). But

$$CM_H\big(f^{-1}(x)\big) \wedge CM_H\big(f^{-1}(y)\big) = CM_{(f^{-1})^{-1}(H)}(x) \wedge CM_{(f^{-1})^{-1}(H)}(y) \quad (3)$$

(by definition 2.9.(ii)) and

$$CM_{(f^{-1})^{-1}(H)}(x) \wedge CM_{(f^{-1})^{-1}(H)}(y) = CM_{f(H)}(x) \wedge CM_{f(H)}(y) \quad (4)$$

Thus using (1), (2), (3) and (4), we have:

$$CM_{f(H)}(xy) \geq CM_{f(H)}(x) \wedge CM_{f(H)}(y) \quad (5)$$

$CM_{f(H)}(x^{-1}) = \vee_{f(y)=x^{-1}} CM_H(y)$ (Since $f^{-1}(x^{-1}) \neq \emptyset$
(by definition 2.9.(i))
But f is 1-1 and onto (by hypothesis), thus $f^{-1}(x^{-1}) = \{y\}$.
In particular,

$$CM_{f(H)}(x^{-1}) = CM_H(y) = CM_H\big(f^{-1}(x^{-1})\big) = CM_H\left(\big(f^{-1}(x)\big)^{-1}\right) \quad (6)$$

But

$$CM_H\left(\big(f^{-1}(x)\big)^{-1}\right) \geq CM_H\big(f^{-1}(x)\big) = CM_{(f^{-1})^{-1}(H)}(x) = CM_{f(H)}(x) \quad (7)$$

Thus from (6) and (7) we have

$$CM_{f(H)}(x^{-1}) \geq CM_{f(H)}(x) \quad (8)$$

Also

$$CN_{f(H)}(xy) = CN_H(z) = CN_H\big(f^{-1}(xy)\big) = CN_H(f^{-1}(x)f^{-1}(y)) \quad (9)$$

(f is an isomorphism). But

$$CN_H(f^{-1}(x)f^{-1}(y)) \leq CN_H\big(f^{-1}(x)\big) \wedge CN_H\big(f^{-1}(y)\big) \quad (10)$$

(H is submgroup of M_1). But

$$CN_H\big(f^{-1}(x)\big) \wedge CN_H\big(f^{-1}(y)\big) = CN_{(f^{-1})^{-1}(H)}(x) \wedge CN_{(f^{-1})^{-1}(H)}(y) \quad (11)$$

(by definition 2.9.(ii)) and

$$CN_{(f^{-1})^{-1}(H)}(x) \wedge CN_{(f^{-1})^{-1}(H)}(y) = CN_{f(H)}(x) \wedge CN_{f(H)}(y) \quad (12)$$

Thus using (9), (10), (11) and (12), we have:

$$CN_{f(H)}(xy) \leq CN_{f(H)}(x) \wedge CN_{f(H)}(y) \quad (13)$$

$$CN_{f(H)}(x^{-1}) = CN_H(y) = CN_H\big(f^{-1}(x^{-1})\big) = CN_H\left(\big(f^{-1}(x)\big)^{-1}\right) \quad (14)$$

But $CN_H\left(\big(f^{-1}(x)\big)^{-1}\right) \leq CM_H\big(f^{-1}(x)\big) = CN_{(f^{-1})^{-1}(H)}(x) = CN_{f(H)}(x)$ (15)

Thus from (14) and (15) we have

$$CN_{f(H)}(x^{-1}) \leq CN_{f(H)}(x) \quad (16)$$

From (5), (8),(13) and (16) we have $f(H) \in IFMG(Y)$ (17)
But $f(G_1) \subseteq G_2$, since $H \subseteq G_1$, then $f(H) \subseteq G_2$
Hence $f(H)$ is a sub-intuitionistic mgroup of G_2 (18)
Now we show that $f(H) \trianglelefteq G_2$.

i.e. $CM_{f(H)}(x^{-1}yx) \geq CM_{f(H)}(y)$ for all $x \in G_2^*$ and $y \in \left(f(H)\right)^*$
and $CN_{f(H)}(x^{-1}yx) \leq CM_{f(H)}(y)$ for all $x \in G_2^*$ and $y \in \left(f(H)\right)^*$
Now

$$CM_{f(H)}(x^{-1}yx) = \vee_{f(z)=x^{-1}yx} CM_H(z) = CM_H\left(f^{-1}(x^{-1}yx)\right) \quad (19)$$

(Since f is 1-1 and onto by definition)
But $CM_H\left(f^{-1}(x^{-1}yx)\right) = CM_H\left(f^{-1}(x^{-1})f^{-1}(y)f^{-1}(x)\right)$
and

$$CM_H\left(f^{-1}(x^{-1})f^{-1}(y)f^{-1}(x)\right) = CM_H\left(\left(f^{-1}(x)\right)^{-1} f^{-1}(y)f^{-1}(x)\right) \quad (20)$$

(since f is an isomorphism by hypothesis)
But

$$CM_H\left(\left(f^{-1}(x)\right)^{-1} f^{-1}(y)f^{-1}(x)\right) \geq CM_H\left(f^{-1}(y)\right) = CM_{f(H)}(y) \quad (21)$$

From (19), (20) and (21), we have

$$CM_{f(H)}(x^{-1}yx) \geq CM_{f(H)}(y) \quad (22)$$

Similarly,

$$CN_{f(H)}(x^{-1}yx) = \vee_{f(z)=x^{-1}yx} CN_H(z) = CN_H\left(f^{-1}(x^{-1}yx)\right) \quad (23)$$

(Since f is 1-1 and onto by definition)
But $CN_H\left(f^{-1}(x^{-1}yx)\right) = CN_H\left(f^{-1}(x^{-1})f^{-1}(y)f^{-1}(x)\right)$
and

$$CN_H\left(f^{-1}(x^{-1})f^{-1}(y)f^{-1}(x)\right) = CN_H\left(\left(f^{-1}(x)\right)^{-1} f^{-1}(y)f^{-1}(x)\right) \quad (24)$$

(since f is an isomorphism by hypothesis)
But

$$CN_H\left(\left(f^{-1}(x)\right)^{-1} f^{-1}(y)f^{-1}(x)\right) \leq CN_H\left(f^{-1}(y)\right) = CN_{f(H)}(y) \quad (25)$$

From (23), (24) and (25), we have

$$CN_{f(H)}(x^{-1}yx) \leq CN_{f(H)}(y) \quad (26)$$

Hence, using (18), (22) and (26) we deduce that $f(H) \in \Delta\wp SIFMG(G_2)$

ii. Let H be a sub-intuitionistic fuzzy mgroup of G_2.
We show that $f^{-1}(H)$ is a sub-intuitionisticfuzzy mgroup of G_1.
But $CM_{f^{-1}(H)}(xy) = CM_H[f(xy)]$ (by definition)
$= CM_H[f(x)f(y)]$ ($f$is an isomorphism)
$\geq CM_H[f(x)] \wedge CM_H[f(y)]$ (by definition)
$= CM_{f^{-1}(H)}(x) \wedge CM_{f^{-1}(H)}(y)$

$$\therefore CM_{f^{-1}(H)}(xy) \geq CM_{f^{-1}(H)}(x) \wedge CM_{f^{-1}(H)}(y) \quad (27)$$

$CM_{f^{-1}(H)}(x^{-1}) = CM_H[f(x^{-1})]$ (by definition)
$= CM_H\left[\left(f(x)\right)^{-1}\right]$ ($f$is an isomorphism)
$= CM_H[f(x)]$ ($H$is a submgroup)
$= CM_{f^{-1}(H)}(x)$ (by definition)

$$\text{Thus } CM_{f^{-1}(H)}(x^{-1}) = CM_{f^{-1}(H)}(x) \quad (28)$$

$CN_{f^{-1}(H)}(xy) = CN_H[f(xy)] = CN_H[f(x)f(y)]$ ($f$is an isomorphism)

$\leq CN_H[f(x)] \wedge CN_H[f(y)]$ (by definition)

$= CN_{f^{-1}(H)}(x) \wedge CN_{f^{-1}(H)}(y)$

$\therefore CN_{f^{-1}(H)}(xy) \quad \leq CN_{f^{-1}(H)}(x) \wedge CN_{f^{-1}(H)}(y)$ (29)

$CN_{f^{-1}(H)}(x^{-1}) = CN_H[f(x^{-1})]$ (by definition)

$= CN_H\left[\left(f(x)\right)^{-1}\right]$ ($f$is an isomorphism)

$= CN_H[f(x)]$ ($H$is a submgroup)

$= CN_{f^{-1}(H)}(x)$ (by definition)

Thus $CN_{f^{-1}(H)}(x^{-1}) = CN_{f^{-1}(H)}(x)$ (30)

Now for all $x \in M_1^*$ and $y \in [f^{-1}(H)]^*$ we have

$CM_{f^{-1}(H)}(x^{-1}yx) = CM_H[f(x^{-1}yx)]$ (by definition)

$= CM_H[(f(x))^{-1}f(y)f(x)]$ (f is an isomorphism)

$\geq CM_H[f(y)]$ ($H \in \varDelta\wp SIFMG(G_2)$by hypothesis)

$= CM_{f^{-1}(H)}(y)$ (by definition)

$\therefore CM_{f^{-1}(H)}(x^{-1}yx) \geq CM_{f^{-1}(H)}(y)$ (31)

$CM_{f^{-1}(H)}(x^{-1}yx) = CM_H[f(x^{-1}yx)]$ (by definition)

$= CN_H[(f(x))^{-1}f(y)f(x)]$ (f is an isomorphism)

$\leq CM_H[f(y)]$ ($H \in \varDelta\wp SIFMG(G_2)$by hypothesis)

$= CN_{f^{-1}(H)}(y)$ (by definition)

$\therefore CN_{f^{-1}(H)}(x^{-1}yx) \leq CN_{f^{-1}(H)}(y)$(6)

Hence from (27) to (31) we have $f^{-1}(H)$ is a normal subintuitionistic fuzzy mgroup of G_1.

4. Conclusion

The paper developed the concept of normal sub-intuitionistic fuzzy Multigroup with some of its related algebraic structures such as intersection, union, of any two normal sub-intuitionistic fuzzy multigroups are also normal. It also showed that the inverse of any normal sub-intuitionistic fuzzy multigroup is normal and for any normal sub-intuitionistic multigroup, the root (support) set of normal sub-intuitionistic fuzzy multigroup is a normal sub-intuitionistic fuzzy group. Finally, it showed that under the isomorphism function between any two groups, the image of a normal sub-intuitionistic fuzzy multigroup under the isomorphism is a normal sub-intuitionistic fuzzy multigroup and the inverse image of a normal sub-intuitionistic fuzzy multigroup under the isomorphism is a normal sub-intuitionistic fuzzy multigroup.

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