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Interval Valued Q-fuzzy Quasi-ideals in a Semigroups

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Abstract. We initiate the study of interval-valued Q-fuzzy quasi-ideal of a semigroup. In Section 2, we list some basic definitions in the later sections. In Section 3, we investigate interval-valued Q-fuzzy subsemigroups and in Section 4, we define interval valued Q-fuzzy quasi-ideals and establish some of their basic properties.

Keywords: interval-valued Q-fuzzy set, interval-valued Q- fuzzy left(right) ideal, interval-valued Q-fuzzy bi-ideal, interval-valued Q-fuzzy quasi-ideal.

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1. Introduction

The theory of fuzzy sets proposed by Zadeh [13] in 1965 has achieved a great success in various fields. Since then, Ahsan and Latif [1] investigated fuzzy quasi-ideals in a semigroup. With the research of fuzzy sets, in 1965, Zadeh [14] introduced the notion of interval-valued fuzzy sets as a generalization of fuzzy sets. After then, Biswas [3] applied it to group theory. Rosenfeld [9] defined fuzzy subgroup and gave some of its properties. Rosenfeld's definition of fuzzy group is a turning point for pure Mathematicians. Since then, the study of fuzzy algebraic structures have been carried out in many directions such as semi-group, groups, rings, near-rings, modules, vector spaces, topology and so on.

Recently, Kang and Hur [4] studied interval-valued fuzzy subgroups and investigated some of its properties. Narayanan and Manikandan [8] studied Interval-valued fuzzy ideals generated by an interval-valued fuzzy subset in semi-groups and investigated some of its properties.

Thillaigovindan and Chinnadurai [11] studied on interval-valued fuzzy quasi-ideals of semi-groups and investigated some of its properties. Solairaju and Nagarajan [10] defined a new structure and constructions of Q-fuzzy group. Kim et al. [5] studied interval-valued fuzzy Quasi-ideals in a semigroups and investigated some of its properties. Murugadas et al. [7] studied interval-valued Q-fuzzy ideals generated by an interval-valued Q-fuzzy subset in ordered semi-groups and investigated some of its properties. Venkatesan and Sriram [12] studied multiplicative operations of IFMs of two operators namely $X_1$and X_2

and investigated its algebraic properties.

In this paper, we initiate the study of interval-valued Q-fuzzy quasi-ideal of a semigroup. In Section 2, we list some basic definitions in the later sections. In Section 3, we investigate interval-valued Q-fuzzy subsemigroups and in Section 4, we define interval-valued Q-fuzzy quasi-ideals and establish some of their basic properties.

2. Preliminaries

In this section, we give to some basic definitions of interval-valued fuzzy set that are necessary for this paper.

Definition 2.1. Let $A, B \in D(I)^X$ and let $\{A_\alpha\}_{\alpha\in\Gamma} \subset D(I)^X$. Then

i) $A \subset B$ iff $A^L \leq B^L$and $A^U \leq B^U$.

ii) $A = B$ iff $A \subset B$and $B \subset A$.

iii) $A^L = [1 - A^U, 1 - A^L]$.

iv) $A \cup B = [A^L \vee B^L, A^U \vee B^U]$.

v) $A \cap B = [A^L \wedge B^L, A^U \wedge B^U]$.

vi) $\bigcup_{\alpha\in\Gamma} A_\alpha = [\bigvee_{\alpha\in\Gamma} {A_\alpha}^L, \bigvee_{\alpha\in\Gamma} {A_\alpha}^U]$.

vii) $\bigcap_{\alpha\in\Gamma} A_\alpha = [\bigwedge_{\alpha\in\Gamma} {A_\alpha}^L, \bigwedge_{\alpha\in\Gamma} {A_\alpha}^U]$.

Definition 2.2. Let S be semigroup and let $\tilde{0} \neq A \in D(I)^S$. Then A is called an:

i) Interval-valued Q-fuzzy semigroup of S if $A^L(xy,q) \geq A^L(x,q) \wedge A^L(y,q)$ and $A^U(xy,q) \geq A^U(x,q) \wedge A^U(y,q)$ for any $x, y \in S, q \in Q$.

ii) Interval-valued Q-fuzzy left ideal of S if $A^L(xy,q) \geq A^L(y,q)$ and $A^U(xy,q) \geq A^U(y.q)$for any $x, y \in S, q \in Q$.

iii) Interval-valued Q-fuzzy right ideal of S if $A^L(xy,q) \geq A^L(x,q)$ and $A^U(xy,q) \geq A^U(x,q)$ for any $x, y \in S, q \in Q$.

iv) Interval-valued Q-fuzzy (two sided) ideal of S if it is both an $IVLI$ and $IVRI$ of S.

We will denote the set of all $IVSGs$ of S as $IVSG(S)$.

It is clear that $A \in IVI(S)$ if and only if $A^L(xy,q) \geq A^L(x,q) \wedge A^L(y,q)$ and $A^U(xy,q) \geq A^U(x,q) \wedge A^U(y,q)$ for any $x, y \in S, q \in Q$ and if $A \in IVLI(S)$, then $A \in IVSG(S)$.

3. Interval-valued Q-fuzzy subsemigroups

In this section, we investigate interval-valued Q-fuzzy subsemigroups and its some algebraic properties.

Definition 3.1. A mapping $A = X \times Q \to D(I)$ is called an interval-valued Q-fuzzy set in X, denoted by $A = A^L \to A^U$ if $A^L, A^U \in I^X$ such that $A^L \leq A^U$, i.e., $A^L(x) \leq A^U(x)$ for each $x \in X$ where $A^L(x,q)$ [respectively $A^U(x,q)$] is called the lower [respectively upper] end point of $x\ to\ A$. For any $[a,b] \in D(I)$, the interval-valued fuzzy set $A\ in\ X$ defined by $A(x) = [A^L(x,q), A^U(x,q)] = [a,b]$ for each $x \in X\ and\ q \in Q$ is denoted by $[\widetilde{a,b}]$ and if $a = b$, then the IVS $[\widetilde{a,b}]$ is denoted by simply $\tilde{a}$. In particular, $\tilde{0}$ and $\tilde{1}$ denote the interval-valued Q-fuzzy empty set and the interval-valued Q-fuzzy whole set in X, respectively. We will denote the set of all IVSs in X as $D(I)^X$. It is clear that set $A = [A^L, A^U] \in D(I)^X$ for each $A \in I^X$.

Definition 3.2. Let $(X,\cdot)$ be a groupoid and let $A,B \in D(I)^X$. Then the interval-valued Q-fuzzy product of A and B, denoted by $(A \circ B)$, is an IVS in X defined as follows :
For each $x \in X,\ q \in Q$

$$(A \circ B)(x,q) = \begin{cases} [a,b], & if\ yz = x, q \in Q, \\ [0,0], & otherwise. \end{cases}$$

where $= \vee_{yz=x}(A^L(y,q) \wedge B^L(z,q)),\ b = \vee_{yz=x}(A^U(y,q) \wedge B^U(z,q))$.
It is clear that for any $A,B,C \in D(I)^X$, if $B \subset C$, then $(A \circ B) \subset (A \circ C)$ and $(B \circ A) \subset (C \circ A)$.

Result 3.3. Let $(S,\ .)$ be a groupoid.
i) If " ." is associative, then so is " $\circ$ " in $D(I)^S$.
ii) If " ." has an identity $e \in S$, then $e_1 \in IVF_P\ (X)$ is an identity of $\circ$ in $D(I)^S$.

Proposition 3.4. Let S be a groupoid, let Q be a non empty set and let $A,B,C \in D(I)^S$. Then
i) $A \circ (B \cup C) = (A \circ B) \cup (A \circ C),\ (B \cup C) \circ A = (B \circ A) \cup (C \circ A)$.
ii) $A \circ (B \cap C) \subset (A \circ B) \cap (A \circ C),\ (B \cap C) \circ A \subset (B \circ A) \cap (C \circ A)$.
Proof: (i) Let $x \in S, q \in Q$ Suppose x is not expressible as $x = yz$.
Then clearly $(A \circ (B \cup C))(x,q) = \tilde{0} = ((A \circ B) \cup (A \circ C))(x,q)$.
Suppose x is expressible as $x = yz$. Then

$$\begin{aligned}
\big(A \circ (B \cup C)\big)(x,q) &= \bigvee_{x=yz} (A^L(y,q) \wedge (B \cup C)^L(z,q)) \\
&= \bigvee_{x=yz} (A^L(y,q) \wedge B^L(z,q)) \vee (A^L(y,q) \wedge C^L(z,q)) \\
&= \bigvee_{x=yz} (A^L(y,q) \wedge B^L(z,q)) \vee \bigvee_{x=yz} (A^L(y,q) \wedge C^L(z,q)) \\
&= (A \circ B)^L(x,q) \vee (A \circ C)^L(x,q) \\
&= ((A \circ B) \cup (A \circ C))^L(x,q)
\end{aligned}$$

Thus $A \circ (B \cup C) = (A \circ B) \cup (A \circ C)$. By the similar arguments, we have $(B \cup C) \circ A = (B \circ A) \cup (C \circ A)$.
ii) Let $x \in S, q \in Q$. Suppose x is not expressible as $x = yz$.
Then clearly $(A \circ (B \cup C))(x,q) = \tilde{0} = ((A \circ B) \cap (A \circ C))(x,q)$.
Suppose x is expressible as $x = yz$. Then

$$\begin{aligned}
\big(A \circ (B \cap C)\big)(x,q) &= \bigvee_{x=yz} (A^L(y,q) \wedge (B \cap C)^L(z,q)) \\
&= \bigvee_{x=yz} (A^L(y,q) \wedge B^L(z,q)) \wedge (A^L(y,q) \wedge C^L(z,q)) \\
&= \bigvee_{x=yz} (A^L(y,q) \wedge B^L(z,q)) \wedge \bigvee_{x=yz} (A^L(y,q) \wedge C^L(z,q)) \\
&= (A \circ B)^L(x,q) \wedge (A \circ C)^L(x,q)
\end{aligned}$$

$= ((A \circ B) \cap (A \circ C))^L(x,q).$

Similarly, we have that $\left(A \circ (B \cap C)\right)^U(x,q) \le \left((A \circ B) \cap (A \circ C)\right)^U(x,q).$
Thus $A \circ (B \cap C) \subset (A \circ B) \cap (A \circ C).$
By the similar arguments, we have $(B \cap C) \circ A \subset (B \circ A) \cap (C \circ A).$

Proposition 3.5. Let S be a semigroup and let $\tilde{0} \neq A \in D(I)^S$. Then $A \in IVSG(S)$ if and only if $(A \circ A) \subset A$.

Result 3.6. Let A be a non-empty subset of a semigroup S.
i) A is a subsemigroup of S if and only if $[\chi A, \chi A] \in$IVSG(S).
ii) $A \in LI(S)$ if and only if $[\chi_A, \chi_A] \in IVLI(S)$.

Result 3.7. Let S be a semigroup and let $\tilde{0} \neq A \in D(I)^S$. Then $A \in IVLI(S)$ if and only if $(\tilde{1} \circ A) \subset A$.

Proposition 3.8. Let S be a semigroup and let $A, B, C \in D(I)^S, q \in Q$. If $A \subset B$, then $(A \circ C) \subset (B \circ C)$ and $(C \circ A) \subset (C \circ B)$.
Proof: Let $x \in S$, $q \in Q$. Suppose (x,q) is not expressible as $(x,q) = (yz,q)$. Then clearly $(A \circ C)(x,q) = \tilde{0} = (B \circ C)(x,q)$. Suppose x is not expressible as $x = yz$. Then

$$\begin{aligned}(A \circ C)^L(x,q) &= \bigvee_{x=yz} (A^L(y,q) \wedge C^L(z,q)) \\ &= \bigvee_{x=yz} (B^L(y,q) \wedge C^L(z,q)) \\ &= (B \circ C)^L(x,q).\end{aligned}$$

Similarly, we have that $(A \circ C)^U(x,q) \le (B \circ C)^U(x,q)$.
Hence $(A \circ C) \subset (B \circ C)$. By the similar arguments, we have $(C \circ A) \subset (C \circ B)$.

4. Interval valued Q-fuzzy quasi-ideals

A nonempty subset A of a semigroup S is called a quasi- ideal of S if $AS \cap SA \subset A$.
We will denote the set of all quasi-ideals of S as QI(S).

Definition 4.1. Let S be a semigroup and let $\tilde{0} \neq A \in D(I)^S$. Then A is called an interval-valued fuzzy quasi-ideal (in short, $IVQI$) of S if $(\tilde{1} \circ A) \cap (A \circ \tilde{1}) \subset A$.
We will denote the set of all IVQIs of S as IVQI(S).

Example 4.2. Let $S = \{a, b, c\}$ be any semigroup with the following multiplication table:
We define a mapping $A : S \to D(I)$ as follows:
$A(a) = [0.1, 0.8]$, $A(b) = [0.1, 0.8]$, $A(c) = [0.3, 0.6]$.
Then we can see that $A \in IVQI(S)$.

Theorem 4.3. Let A be a nonempty subset of a semigroup S. Then $A \in QI(S)$ if and only if $[\chi_A, \chi_A] \in IVQI(S)$.

.	a	b	c
a	a	a	a
b	a	b	b
c	a	a	b

Proof: Suppose $A \in QI(S)$ and let $x \in S$. Suppose $x \in A, q \in Q$. Then clearly $\chi_A(x,q) = 1 \geq \left(\left(\tilde{1} \circ [\chi_A, \chi_A]\right) \cap \left([\chi_A, \chi_A] \circ \tilde{1}\right)\right)^L (x,q)$.
Thus $\left(\tilde{1} \circ [\chi_A, \chi_A]\right) \cap \left([\chi_A, \chi_A] \circ \tilde{1}\right) \subset [\chi_A, \chi_A]$. Suppose $x \notin A$. Then either x is expressible as $x = yz$ or not.
Case (i): Suppose x is not expressible as $x = yz$. Then

$$((\tilde{1} \circ [\chi_A, \chi_A]) \cap ([\chi_A, \chi_A] \circ \tilde{1}))(x,q) = \tilde{1} = [\chi_A, \chi_A](x,q).$$

Case (ii): Suppose x is expressible as $x = yz$.
Since $x \notin A$, either $y \in A$ or $z \notin A$. If $y \in A$ and $z \notin A$, then there cannot be another expression of the form $x = ab$, where $a \notin A$ and $b \in A$ (Assume that there exist $a \notin A$ and $b \in A$ such that $x = ab$. Then $x \in SA \cap AS \subset A$. Thus $x \in A$. This contradicts the fact that $x \notin A$). Thus either $(\tilde{1} \circ [\chi_A, \chi_A])(x,q) = \tilde{0}$ or $([\chi_A, \chi_A] \circ \tilde{1})(x,q) = \tilde{0}$. So $((\tilde{1} \circ [\chi_A, \chi_A]) \cap ([\chi_A, \chi_A] \circ \tilde{1}))(x) = \tilde{0}$.
Then $\left(\left(\tilde{1} \circ [\chi_A, \chi_A]\right) \cap \left([\chi_A, \chi_A] \circ \tilde{1}\right)\right) \subset [\chi_A, \chi_A]$. Hence, in all, $[\chi_A, \chi_A] \in IVQI(S)$.
Conversely, suppose the necessary condition holds.
Let $x \in SA \cap AS$. Then $x \in SA$ and $x \in AS$. Thus there exist $a, a' \in A$ and $s, s' \in S$ such that $x = sa$ and $x = a's'$. So

$$\left(\left(\tilde{1} \circ [\chi_A, \chi_A]\right) \cap \left([\chi_A, \chi_A] \circ \tilde{1}\right)\right)^L (x,q)$$
$$= \left(\tilde{1} \circ [\chi_A, \chi_A]\right)^L (x,q) \wedge \left([\chi_A, \chi_A] \circ \tilde{1}\right)^L (x,q)$$
$$= \bigvee_{x=yz} (\tilde{1}^L(y,q) \wedge \chi_A(z,q)) \wedge \bigvee_{x=yz} ({\chi_A}^L(y,q) \wedge {\chi_S}^L(z,q))$$
$$\geq (\chi_S(s) \wedge \chi_A(a)) \wedge (\chi_A(a') \wedge \tilde{1}^L(s'))$$
$$= 1.$$

Similarly, we have that $((\tilde{1} \circ [\chi_A, \chi_A]) \cap ([\chi_A, \chi_A] \circ \tilde{1}))^U(x) \geq 1$. Then, by the hypothesis, $\chi_A(x) \geq 1$. Thus $x \in A$. So $SA \cap AS \subset A$. Hence $A \in QI(S)$.

Definition 4.4. A nonempty fuzzy set A of a semigroup S is called a Q-fuzzy quasi-ideal of S if $(\chi_S \circ A) \wedge (A \circ \chi_S) \leq A$, where χ_S is the whole fuzzy set defined by $\chi_S(x,q) = 1$ for each $x \in S, q \in Q$.

Remark 4.5. Let S be a semigroup.
i) If A is a Q-fuzzy quasi-ideal of S, then $[A, A] \in IVQI(S)$.
ii) If $A \in IVQI(S)$, then AL and AU are Q-fuzzy quasi-ideals of S.

Proposition 4.6. Let S be a semigroup. Then $IVQI(S) \subset IVSG(S)$.
Proof: Let $A \in IVQI(S)$. Since $A \subset \tilde{1}$, by Proposition 3.8, $A \circ A \subset \tilde{1} \circ A$ and $A \circ A \subset A \circ \tilde{1}$. Then $A \circ A \subset [\tilde{1} \circ A] \cap [A \circ \tilde{1}]$. Since $A \in IVQI(S), (\tilde{1} \circ A) \cap (A \circ \tilde{1}) \subset A$. Thus $A \circ A \subset A$.
Hence, by Proposition 3.5, $A \in IVSG(S)$.

Definition 4.7. Let S be a semigroup and let $\dot{\tilde{0}} \neq A \in D(I)^S$, $q \in Q$.
Then A is called an interval-valued Q-fuzzy bi-ideal (in short, *IVBI*) of S if it satisfies the following conditions: for any $x, y, z \in S, q \in Q$.
i) $A^L(xy, q) \geq A^L(x, q) \wedge A^L(y, q)$ and $A^U(xy, q) \geq A^U(x, q) \wedge A^U(y, q)$.
ii) $A^L(xyz, q) \geq A^L(x, q) \wedge A^L(z, q)$ and $A^U(xyz, q) \geq A^U(x, q) \wedge A^U(z, q)$.
We will denote the set of all IVBIs of S as IVBI(S).

Result 4.7. Let A be a nonempty subset of a semigroup. Then $A \in BI(S)$ if and only if $[\chi_A, \chi_A] \in IVBI(S)$.

Theorem 4.8. Let S be a semigroup and let $\dot{\tilde{0}} \neq A \in D(I)^S$, $q \in Q$.
Then $A \in IVBI(S)$ if and only if $A \circ A \subset A$ and $A \circ \tilde{1} \circ A \subset A$.
Proof: Suppose $A \in IVBI(S)$. From Proposition 3.5, $A \circ A \subset A$. Let $x \in S, q \in Q$
Suppose x is not expressible as $x = yz$. Then clearly $(A \circ \tilde{1} \circ A)(x, q) = \tilde{0}$.
Thus $A \circ \tilde{1} \circ A \subset A$.
Suppose x is expressible as $x = yz$. Then $(A \circ \tilde{1} \circ A)(x, q) \neq \tilde{0}$. Thus
$(A \circ \tilde{1} \circ A)^L(x, q)$

$$= \bigvee_{x = yz} (A^L(y, q) \wedge (\tilde{1} \circ A)^L(z, q)) > 0$$

and

$$= \bigvee_{x = yz} (A^U(y, q) \wedge (\tilde{1} \circ A)^U(z, q)) > 0$$

So $(\tilde{1} \circ A)^L(z, q) > 0$ and $(\tilde{1} \circ A)^U(z, q) > 0$. Then there exist $u, v \in S$ with $z = uv$ such that

$$(\tilde{1} \circ A)^L(z, q) = \vee_{z = uv}(\tilde{1}^L(u, q) \wedge A^L(q, q)) = \vee_{z = uv} A^L(v, q))$$

and

$$= \bigvee_{z = uv} (\tilde{1}^U(u, q) \wedge A^U(v, q)) = \bigvee_{z = uv} A^U(v, q))$$

Since $A \in IVBI(S)$, $A^L(x, q) = A^L(yuv, q) \geq A^L(y, q) \wedge A^L(v, q)$ and $A^U(x, q) = A^U(yuv, q) \geq A^U(y, q) \wedge A^U(v, q)$. Then
$A^L(x, q)$

$$\geq \bigvee_{x = yz} (A^L(y, q) \wedge (\bigvee_{z = uv} A^L(q, q))) = (A \circ \tilde{1} \circ A)^L(x)$$

and

$$\geq \bigvee_{x=yz} (A^U(y,q) \wedge (\bigvee_{z=uv} A^U(q,q))) = (A \circ \tilde{1} \circ A)^U(x)$$

Hence, in all, $A \circ \tilde{1} \circ A \subset A$.
Conversely, suppose the necessary condition holds.
Since $A \circ A \subset A$, it is clear that the following hold:
$A^L(xy,q) \geq A^L(x,q) \wedge A^L(y,q)$ and $A^U(xy,q) \geq A^U(x,q) \wedge A^U(y,q)$.
For any $x, y \in S, q \in Q$. Let $x, y, z \in S$ and let $u = xyz$. Then

$$\begin{aligned} A^L(xyz) = A^L(p) & \\ & \geq (A \circ \tilde{1} \circ A)^L(u) \\ & = \bigvee_{u=st} (A^L(s,q) \wedge (\tilde{1} \circ A)^L(t,q)) \\ & \geq A^L(x,q) \wedge (\tilde{1} \circ A)^L(yz,q) \\ & = A^L(x,q) \wedge \vee_{yz=ab}(\tilde{1}^L(a,q) \wedge A^L(b,q)) \\ & \geq A^L(x,q) \wedge A^L(y,q) \wedge A^L(z,q) \\ & = A^L(x,q) \wedge A^L(z,q). \end{aligned}$$

Similarly, we have that $A^U(xyz,q) \geq A^U(x,q) \wedge A^U(z,q)$.
Hence, $A \in IVBI(S)$.

Proposition 4.9. Let S be a semigroup. Then $IVQI(S) \subset IVBI(S), q \in Q$.
Proof: Let $A \in IVQI(S)$. Then, by Proposition 4.6, $A \in IVSG(S)$.
Thus $A^L(xy,q) \geq A^L(x,q) \wedge A^L(y,q)$ *and* $A^L(xy,q) \geq A^L(x,q) \wedge A^L(y,q)$ for any $x, y \in S, q \in Q$.
So, by Proposition 3.5, $A \circ A \subset A$. It is clear that $A \circ \tilde{1} \subset \tilde{1}$ and $\tilde{1} \circ A \subset \tilde{1}$.
Then, by Proposition 3.8, $A \circ \tilde{1} \circ A \subset \tilde{1} \circ A$ and $A \circ \tilde{1} \circ A \subset A \circ \tilde{1}$.
Thus $A \circ \tilde{1} \circ A \subset [\tilde{1} \circ A] \cap [A \circ \tilde{1}] \subset A$.
Hence, by Theorem 4.8, $A \in IVBI(S)$.

5. Conclusions

In this paper, we initiate the study of interval-valued fuzzy quasi-ideal of a semigroup and investigate interval-valued Q-fuzzy subsemigroups and define interval-valued Q-fuzzy quasi-ideals and establish some of their basic properties.

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