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On ve-degree Indices and their Polynomials of Dominating Oxide Networks

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

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Abstract. Recently, the ve-degree concept is defined in Graph Theory. We introduce the first and second hyper-ve-degree indices of a molecular graph. Considering these hyper-ve-degree indices, we define the first and second hyper-ve-degree polynomials of a graph. We compute the first and second ve-degree Zagreb indices and their polynomials of dominating oxide networks. Also we compute the first and second hyper-ve-degree indices and their polynomials of dominating oxide networks.

Keywords: ve-degree indices, hyper-ve-degree indices, dominating oxide network

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*35

1. Introduction

Chemical Graph Theory is a branch of Graph Theory whose focus of interest is to finding topological indices of chemical graphs, which correlate well with chemical properties of the chemical molecules. A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. Several topological indices have been considered in Theoretical chemistry, especially in QSAR and QSPR research, see [1].

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The set of all vertices which adjacent to v is called open neighborhood of v and denoted by $N(v)$. The closed neighborhood set of v is the set $N[v] = N(v) \cup \{v\}$. Let S_v denote the sum of the degrees of all vertices adjacent to a vertex v. In [2], Chellali et al. defined the ve-degree concept in graph theory as follows:

Definition 1. The ve-degree $d_{ve}(v)$ of a vertex v in a graph G is the number of different edges that incident to any vertex from the closed neighborhood of v.

Recently, Ediz [3] introduced the first ve-degree Zagreb beta index of a graph G and it is defined as

$$Ve_1(G) = S^{\beta}(G) = \sum_{uv \in E(G)} \left(d_{ve}(u) + d_{ve}(v)\right). \tag{1}$$

Also Ediz [3] introduced the second ve-degree Zagreb index of a graph G and it is defined as

$$Ve_2(G) = S^{\mu}(G) = \sum_{uv \in E(G)} d_{ve}(u) d_{ve}(v). \tag{2}$$

Considering the ve-degree indices, we propose the first and second ve-degree polynomials of a graph G as

$$Ve_1(G,x) = \sum_{uv \in E(G)} x^{d_{ve}(u)+d_{ve}(v)}. \tag{3}$$

and $$Ve_2(G,x) = \sum_{uv \in E(G)} x^{d_{ve}(u)d_{ve}(v)}. \tag{4}$$

We introduce the first and second hyper-ve-degree indices of a graph G as

$$HVe_1(G) = \sum_{uv \in E(G)} \left(d_{ve}(u) + d_{ve}(v)\right)^2 \tag{5}$$

and $$HVe_2(G) = \sum_{uv \in E(G)} \left(d_{ve}(u) d_{ve}(v)\right)^2. \tag{6}$$

Considering the hyper-ve-degree indices, we propose the first and second hyper-ve-degree polynomials of a graph G as

$$HVe_1(G,x) = \sum_{uv \in E(G)} x^{\left[d_{ve}(u)+d_{ve}(v)\right]^2}. \tag{7}$$

and $$HVe_2(G,x) = \sum_{uv \in E(G)} x^{\left[d_{ve}(u)d_{ve}(v)\right]^2}. \tag{8}$$

The third ve-degree index of a graph G is defined as

$$Ve_3(G) = \sum_{uv \in E(G)} \left|d_{ve}(u) - d_{ve}(v)\right|. \tag{9}$$

Considering the third ve-degree index, we introduce the third ve-degree polynomial as

$$Ve_3(G,x) = \sum_{uv \in E(G)} x^{\left|d_{ve}(u)-d_{ve}(v)\right|}. \tag{10}$$

Recently, some ve-degree topological indices were studied, for example, in [4,5, 6,7] and also several topological indices were studied, for example in [8,9,10,11,12,13]. Recently, some polynomials were studied, for example, in [14,15,16,17,18,19,20, 21,22,23,24,25].

We consider the family of dominating oxide networks [26,27]. In this paper, we determine the first and second ve-degree indices, the first and second hyper-ve-degree indices, the third ve-degree index for dominating oxide networks.

2. Results for dominating oxide networks

The family of dominating oxide networks is symbolized by $DOX(n)$. The molecular structure of a dominating oxide network is presented in Figure 1.

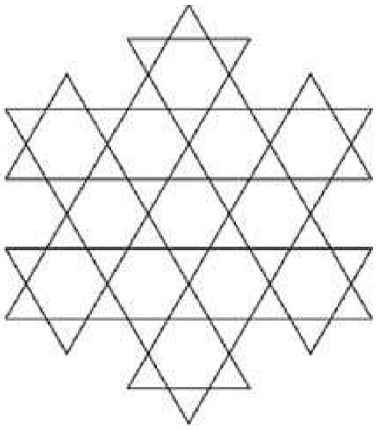

Figure 1: The structure of a dominating oxide network

In [8], Ediz obtained the partition of the edges with respect to their sum degree of end vertices of dominating oxide networks in Table 1.

(S_u, S_v)	(8, 12)	(8, 14)	(12, 12)	(12, 14)	(14, 16)	(16, 16)
Number of edges	$12n$	$12n–12$	6	$12n–12$	$24n–24$	$54n^2– 114n+60$

Table 1:

Also he obtained the ve-degree partition of the end vertices of edges for dominating oxide networks in Table 2.

$(d_{ve}(u), d_{ve}(v))$	(7, 10)	(7, 12)	(10, 10)	(10, 12)	(12, 14)	(14, 14)
Number of edges	$12n$	$12n–12$	6	$12n–12$	$24n–24$	$54n^2– 114n+60$

Table 2: The ve-degree of the end vertices of edges for *DOX* networks

In the following theorem, we compute the values of $Ve_1(DOX(n))$ and $Ve_2(DOX(n))$ for dominating oxide networks.

Theorem 1. The first ve-degree Zagreb beta index and second ve-degree Zagreb index of a dominating oxide network $DOX(n)$ are given by

(i) $Ve_1(DOX(n)) = 1512n^2 – 1872n + 684$

(ii) $Ve_2(DOX(n)) = 10584n^2 – 15024n + 5880$.

Proof: Let G be the graph of a dominating oxide network $DOX(n)$.

(i) Using equation (1) and Table 2, we deduce

$$Ve_1\left(DOX\left(n\right)\right) = \sum_{uv\in E(G)} \left(d_{ve}\left(u\right)+d_{ve}\left(v\right)\right)$$

$= (7+10)12n + (7+12)(12n – 12) + (10+10)6 + (10+12)(12n – 12)$
$+ (12+14)(24n – 24) + (14+14)(54n^2 – 114n + 60)$

$= 1512n^2 – 1872n + 684.$

(ii) Using equation (2) and Table 2, we deduce

$$Ve_2\left(DOX\left(n\right)\right)=\sum_{uv\in E(G)} d_{ve}\left(u\right)d_{ve}\left(v\right)$$

$= (7\times10)12n + (7\times12)(12n – 12) + (10\times10)6+(10\times12)(12n – 12)$
$+ (12\times14)(24n – 24) + (14\times14)(54n^2 – 114n + 60)$
$= 10584n^2 – 15024n + 5880$

In the following theorem, we determine the value of $Ve_1(DOX(n), x)$ and $Ve_2(DOX(n), x)$ for dominating oxide networks.

Theorem 2. The first and second ve-degree polynomials of a dominating oxide network $DOX(n)$ are given by

(i) $Ve_1(DOX(n), x) = 12nx^{17} + (12n – 12)\, x^{19} + 6x^{20} + (12n – 12)\, x^{22} + (24n – 24)x^{26} + (54n^2 – 114n + 60)\, x^{28}$

(ii) $Ve_2(DOX(n, x)) = 12nx^{70} + (12n – 12)\, x^{84} + 6x^{100} + (12n – 12)\, x^{120} + (24n – 24)x^{168} + (54n^2 – 114n + 60)\, x^{196}$

Proof: Let G be the graph of a dominating oxide network $DOX(n)$.

(i) By using equation (3) and Table 2, we derive

$$Ve_1\left(DOX\left(n\right),x\right)=\sum_{uv\in E(G)} x^{d_{ve}(u)+d_{ve}(v)}$$

$= 12nx^{17} + (12n – 12)\, x^{19} + 6x^{20} + (12n – 12)\, x^{22} + (24n – 24)x^{26}$
$+ (54n^2 – 114n + 60)\, x^{28}.$

(ii) By using equation (4) and Table 2, we derive

$$Ve_2\left(DOX\left(n\right),x\right)=\sum_{uv\in E(G)} x^{d_{ve}(u)d_{ve}(v)}$$

$= 12nx^{70} + (12n – 12)\, x^{84} + 6x^{100} + (12n – 12)\, x^{120} + (24n – 24)x^{168}$
$+ (54n^2 – 114n + 60)\, x^{196}.$

In the following theorem, we compute the first and second hyper-ve-degree indices of a dominating oxide network $DOX(n)$.

Theorem 3. The first and second hyper-ve-degree indices of a dominating oxide network $DOX(n)$ are given by

(i) $HVe_1\,(DOX(n)) = 42336\, n^2 – 59544n – 23076$

(ii) $HVe_2\,(DOX(n)) = 2074464\, n^2 – 3470448n – 1430112.$

Proof: Let G be the graph of dominating oxide network $DOX(n)$.

(i) By using equation (5) and Table 2, we deduce

$HVe_1(DOX(n)) = (7+10)^2 12n + (7+12)^2\, (12n – 12) + (10+10)^2\, 6+(10+12)^2(12n–12)$
$+ (12+14)^2\, (24n – 24) + (14+14)^2\, (54n^2 – 114n + 60)$
$= 42336\, n^2 – 59544n – 23076$

(ii) By using equation (6) and Table 2, we derive

$HVe_2(DOX(n)) = (7\times10)^2 12n + (7\times12)^2\, (12n – 12) + (10\times10)^2\, 6+(10\times12)^2(12n–12)$
$+ (12\times14)^2\, (24n – 24) + (14\times14)^2\, (54n^2 – 114n + 60)$
$= 2074464\, n^2 – 3470448n – 1430112.$

In the following theorem, we determine the value of $HVe_1(DOX(n), x)$ and $HVe_2(DOX(n), x)$ for dominating oxide networks.

Theorem 4. The first and second hyper-ve-degree polynomials of a dominating oxide network are given by

(i) $HVe_1(DOX(n), x) = 12nx^{289} + (12n - 12)x^{361} + 6x^{400} + (12n - 12)x^{484} + (24n - 24)x^{676} + (54n^2 - 114n + 60)x^{784}$.

(ii) $HVe_2(DOX(n), x) = 12nx^{70^2} + (12n-12)x^{84^2} + 6x^{100^2} + (12n-12)x^{120^2} + (24n-24)x^{168^2} + (54n^2 - 114n + 60)x^{196^2}$.

Proof: Let G be the graph of a dominating oxide network $DOX(n)$.

(i) By using equation (7) and Table 2, we derive

$$HVe_1(DOX(n), x) = \sum_{uv \in E(G)} x^{[d_{ve}(u)+d_{ve}(v)]^2}$$

$$= 12nx^{(7+10)^2} + (12n-12)x^{(7+12)^2} + 6x^{(10+10)^2} + (12n-12)x^{(10+12)^2} + (24n-24)x^{(12+14)^2} + (54n^2 - 114n + 60)x^{(14+14)^2}$$

$$= 12nx^{289} + (12n - 12)x^{361} + 6x^{400} + (12n - 12)x^{484} + (24n - 24)x^{676} + (54n^2 - 114n + 60)x^{784}.$$

(ii) By using equation (8) and Table 2, we derive

$$HVe_2(DOX(n), x) = \sum_{uv \in E(G)} x^{[d_{ve}(u)d_{ve}(v)]^2}$$

$$= 12nx^{70^2} + (12n-12)x^{84^2} + 6x^{100^2} + (12n-12)x^{120^2} + (24n-24)x^{168^2} + (54n^2 - 114n + 60)x^{196^2}.$$

In the following theorem, we compute the third ve-degree index and its polynomial of dominating oxide network.

Theorem 5. The third ve-degree index and its polynomial of a dominating oxide network are given by

(i) $Ve_3(DOX(n)) = 168n - 132$.

(ii) $Ve_3(DOX(n), x) = (12n - 12)x^5 + 12nx^3 + (36n - 36)x^2 + (54n^2 - 114n + 60)x^0$.

Proof: Let G be the graph of a dominating oxide network $DOX(n)$.

(i) By using equation (9) and Table 2, we deduce

$$Ve_3(DOX(n)) = \sum_{uv \in E(G)} \left| d_{ve}(u) - d_{ve}(v) \right|$$

$$= 12n\times 3 + (12n - 12)5 + 6\times 0 + (12n-12)2 + (24n-24)2 + (54n^2 - 114n + 60)0$$

$$= 168n - 132.$$

(ii) By using equation (10) and Table 2, we derive

$$Ve_3\left(DOX(n),x\right)=\sum_{uv\in E(G)} x^{\left|d_{ve}(u)-d_{ve}(v)\right|}$$

$$= 12x^3 + (12n - 12)x^5 + 6x^0 + (12n - 12)x^2 + (24n - 24)x^2 + (54n^2 - 114n + 60)x^0.$$

$$= (12n-12)x^5+12nx^3 + (36n - 36)x^2 + (54n^2 - 114n + 60)x^0.$$

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